

T³-Teachers Teaching with Technology™

Professional Development Services from Texas Instruments

Exploring Mathematics with TI-Nspire™ Technology

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Beginning with the TI-Nspire™ Math and Science Learning Handheld

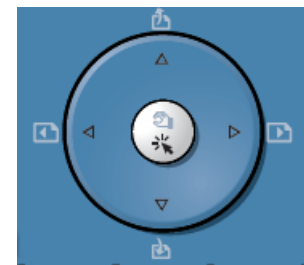
Materials

- TI-Nspire™ Math and Science Learning Handheld

Overview

Explore the functionality of the TI-Nspire™ math and science learning handheld.


Key Name	Function
 Click Key	Selects an object on the screen.  +  grabs an object on the screen.
Nav Pad	Press the arrow keys to move the cursor/pointer.
 Escape Key	Removes menus or dialog boxes from the screen.
 Tab Key	Moves to the next entry field.
 Home Key	Displays the home menu.
 Menu Key	Displays application or context menu.





Nav Pad Cursor Controls

Figures 1 and 2

The Calculator Application

- Turn on the TI-Nspire™ learning handheld. If the screen shown in Figure 3 is not displayed, press  and choose the Calculator application (Figure 3).

Note: To select a menu option, you can highlight the option and press  or . Alternately, you can press the number key for that option.

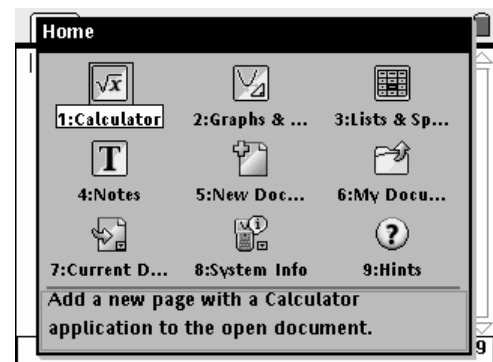


Figure 3

The Graphs & Geometry Application Basic Graphing

- Press $\left[\text{Home} \right]$, then press $\left[2 \right]$ to choose 2:Graphs & Geometry (Figure 11).
 - The Graphs and Geometry selection is now page two of the document.

Note: The graphing window shown is the default window setting with a screen aspect ratio of one.

- The function notation $f1(x)$ is shown in the entry line (Figure 12).

- Type $2x^2-x+1$ and press $\left[\text{enter} \right]$ (Figure 13).

- Rescaling the Axes.
 - Press $\left[\text{esc} \right]$.
 - Place the cursor on one of the axes so that the hand appears and the tic marks flash (Figure 14).

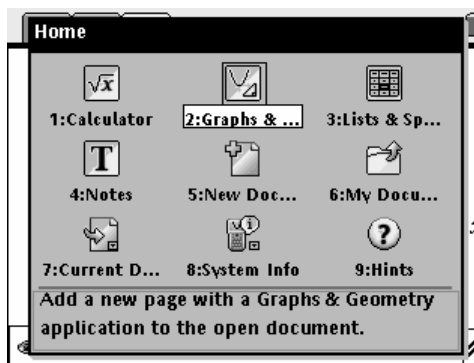


Figure 11

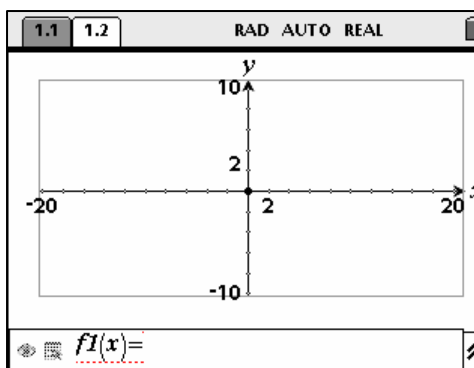


Figure 12

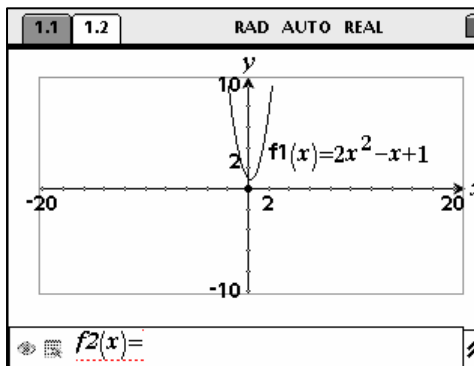


Figure 13

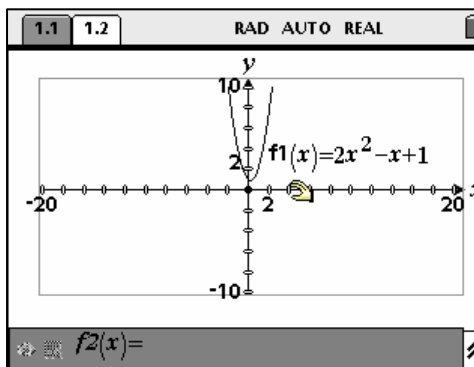


Figure 14

- Press ctrl $\left(\frac{\text{ctrl}}{\text{tick}}$) to grab a tick mark. Use the Nav Pad arrow keys to increase or decrease the maximum or minimum value and the scaling of the axes (Figure 15).
- This method will rescale both axes and will maintain the aspect ratio of the default window.

Alternate Graphing Method

1. Open a new Graphs & Geometry application page.
2. From the graphs screen, press menu .
 - Press 1 for Tools and 5 for 5:Text (Figure 16).
3. Click anywhere on the screen to open a text box.
 - In the text box, type $2x^2-x+1$ and press enter (Figures 17 and 18).

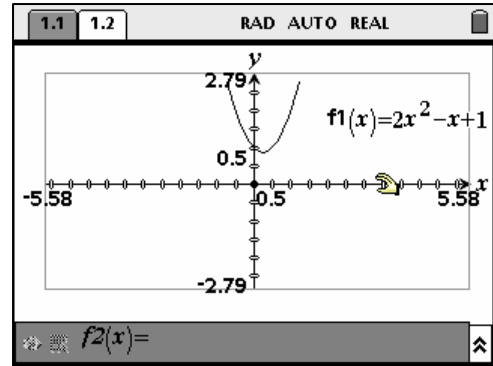


Figure 15

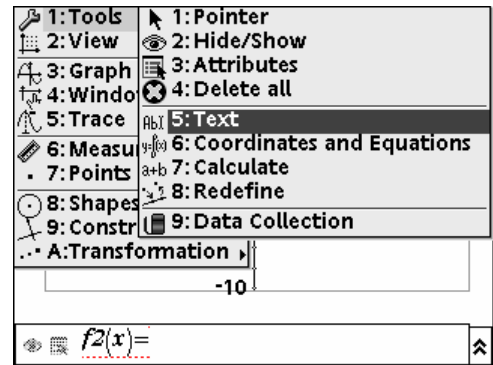


Figure 16

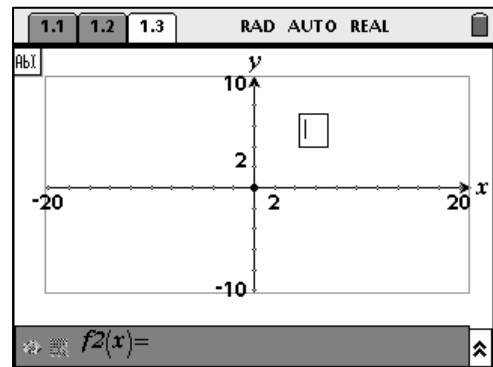


Figure 17

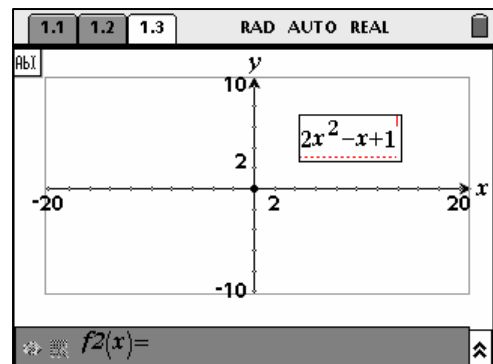


Figure 18

4. Press **(esc)** to exit the text mode.
5. Use the Nav Pad arrow keys to move the cursor over the equation until the hand appears (Figure 19).
 - Press **(ctrl)** **($\frac{\square}{\square}$)** or hold down **($\frac{\square}{\square}$)** until the hand closes.

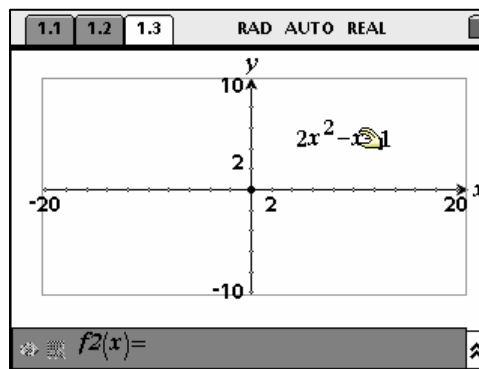


Figure 19

6. Drag the equation to the x-axis.
 - When the graph appears, press **(\approx)** **(enter)** (Figures 20 and 21).

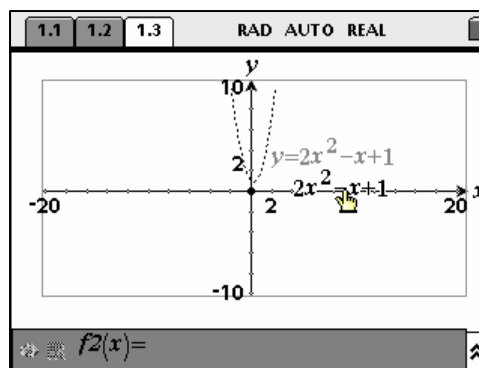


Figure 20

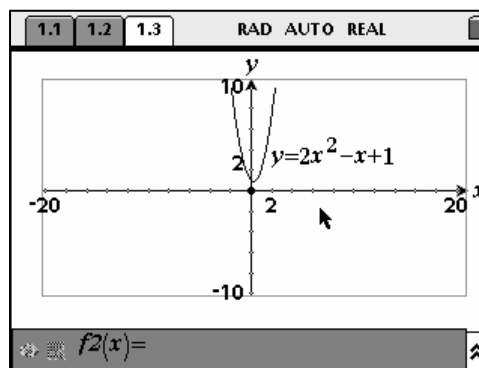


Figure 21

Document Model

While creating your calculations and graphs, you should have noticed the tabs at the top of your screen.

Moving Between Pages

1. Press **(ctrl)** and then **(left arrow)** (on the Nav Pad) to move back one page (Figure 22).

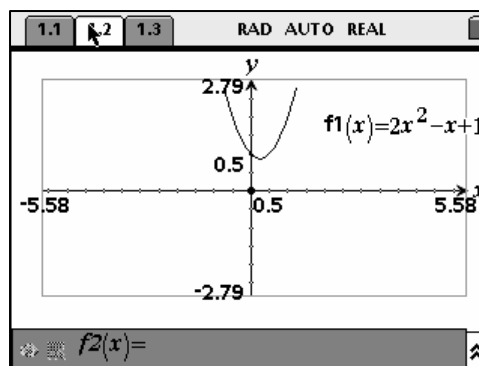


Figure 22

- To view all of the pages of the problem, press **ctrl** and then **▲** (on the Nav Pad). See Figure 23.

Note: This screen allows you to move from one page to another. To open the page that you wish to view, highlight the page by moving left or right with the Nav Pad. When the appropriate page is highlighted, press **enter** **clear**

Saving a Document

- Press **ctrl** **⏠** to choose Tools.
- Press **1** for 1:File and **4** for 4:Save As...(Figure 24).
- When the Save As... dialog box appears, enter Parabola for the file name (Figure 25).
- Press **tab** until OK is highlighted and press **enter** to save the document.
- Check to see if the document was saved.
- Press **⏠** **6** to select 6:My Documents (Figure 26).

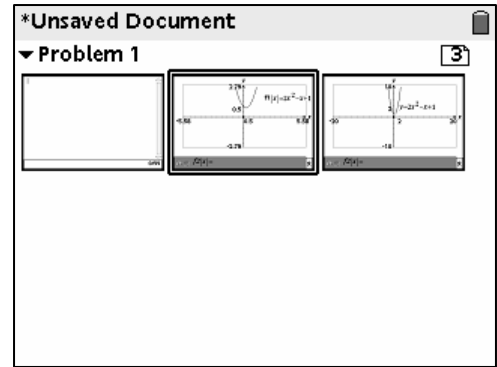


Figure 23

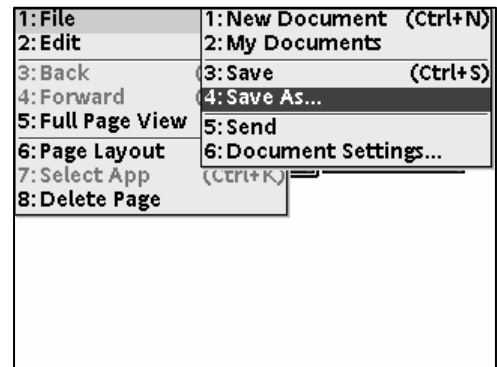


Figure 24

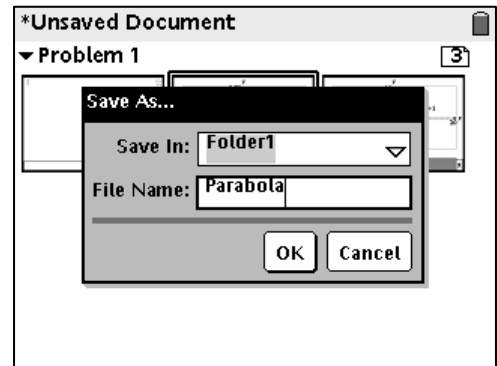



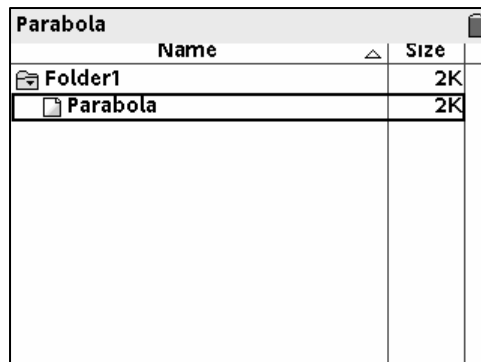
Figure 25



Figure 26

7. The folder will appear showing the document that you have just saved (Figure 27).
8. To reopen the document, highlight the file you wish to open and press .

Note: All work done on the TI-Nspire™ learning handheld is contained within a document. Each document contains one or more problems with a maximum of 30 problems. Each problem contains one or more pages with a maximum of 50 pages. Functions, stored variable, and data are retained throughout a problem



Parabola		
Name	Size	
Folder1	2K	
Parabola	2K	

Figure 27

Basic Graphing and Interpretation Tools for the TI-Nspire™ Math and Science Learning Handheld



Materials

- TI-Nspire™ math and science learning handheld

Overview

Graphing and Analyzing Functions

Open a New Document

1. Press  and  for 5:New Document (Figure 1).

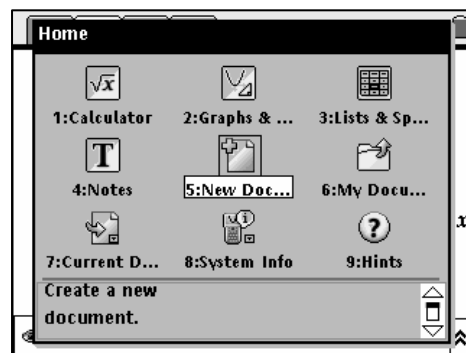



Figure 1

2. Press  for 2:Add Graphs & Geometry (Figure 2).

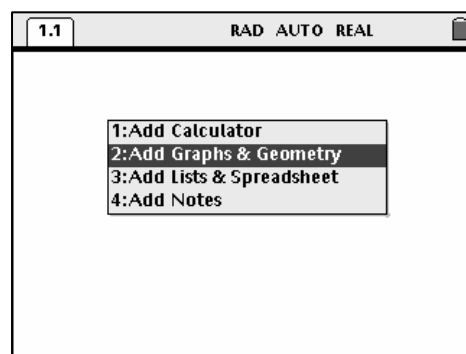




Figure 2

Graphing Functions

1. If the Equation Entry Line window is in the “white” background, then you may simply start typing a function from the keyboard immediately (Figure 3).

Note: A grayed out Equation Entry Line is shown in Figure 4. In this case, either press **tab** or move the pointer to this window, and press  to activate the entry line.

2. Type a function definition into the Equation Entry Line (Figure 5). Once the function definition is typed, press  to graph the function (Figure 6).

- As the graph appears on the screen, a copy of the function definition appears on the screen. The function entry window scrolls to the next open slot in the function definition table, ready for the next function to be entered and graphed (Figure 6).

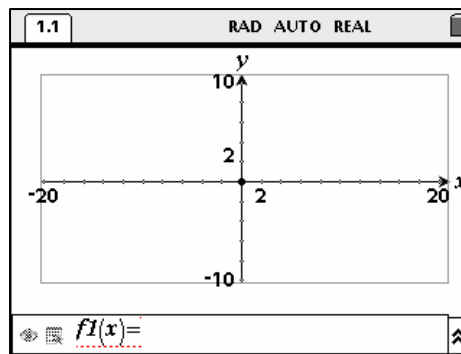


Figure 3

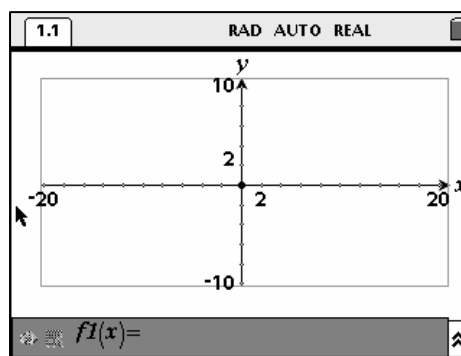


Figure 4

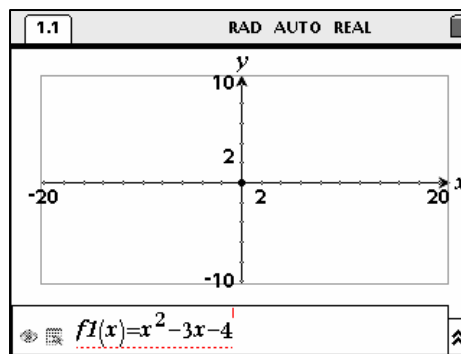


Figure 5

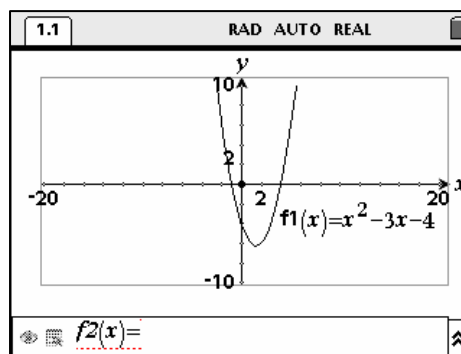


Figure 6

3. After the graph appears, readjust the viewing window.
 - Press **esc** to get the pointer (Figure 7).
 - Place the cursor over one of the four axis values.

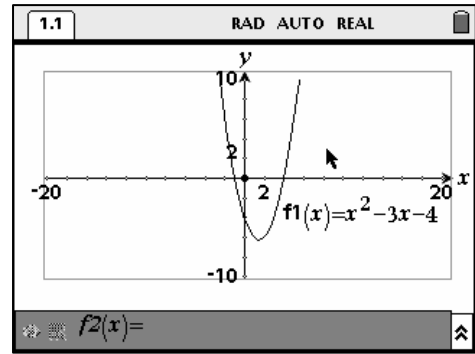


Figure 7

- Press **enter** or **2nd** twice (Figure 8).

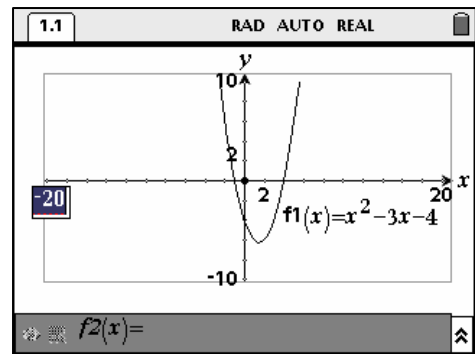


Figure 8

- Enter a new value and press **enter** (Figure 9).

Note: This method of adjusting the viewing window will not maintain the aspect ratio of the default window.

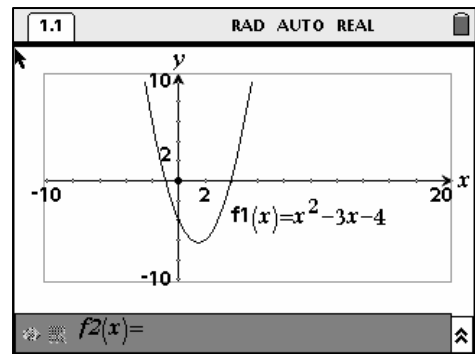


Figure 9

4. The function definition for **f1(x)** appeared on the screen near the graph. This definition can be dragged to any convenient location on the screen, edited directly, or hidden from view.
 - Move the active pointer (or cursor) over the function until the hand appears (Figure 10).
 - Press **ctrl** **2nd** to grab the function.

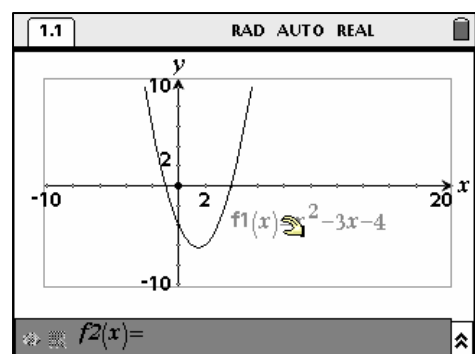


Figure 10

- Drag the function to any location on the screen (Figure 11).




- Press  or  to drop the function.

Note: If the pointer (an arrow) is not visible, press



Changing the Function Definition

There are two ways to change a function definition after it is graphed. The first, and probably the most obvious, is to return to the Equation Entry Line and change the definition.

1. If this entry line is “white,” simply press  to scroll to the previous definition, **f1(x)** in this case (Figure 12). The cursor appears at the right end of the entered function definition.
2. Use the arrow keys to scroll across the definition or  to erase unwanted characters.
3. Type the new values into the definition, and press  to draw the new graph (Figures 13 and 14).

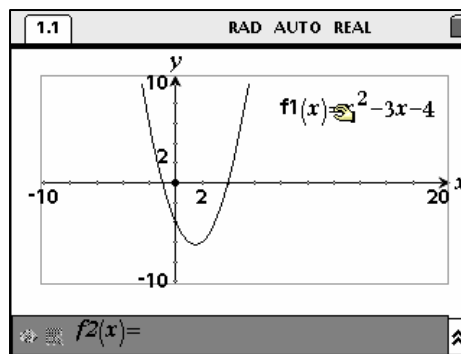


Figure 11

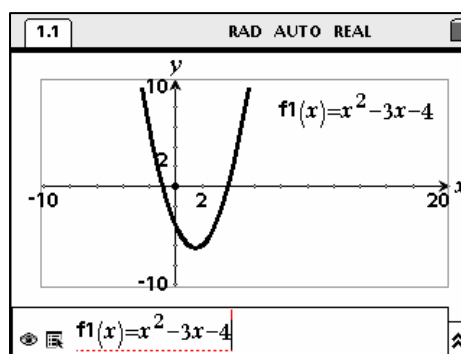


Figure 12

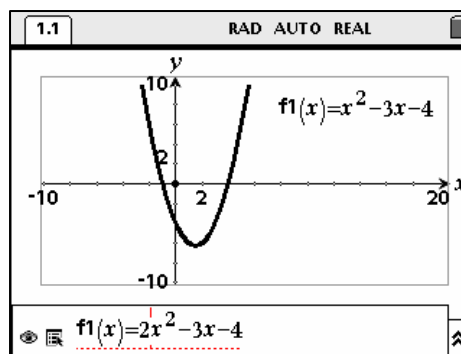


Figure 13

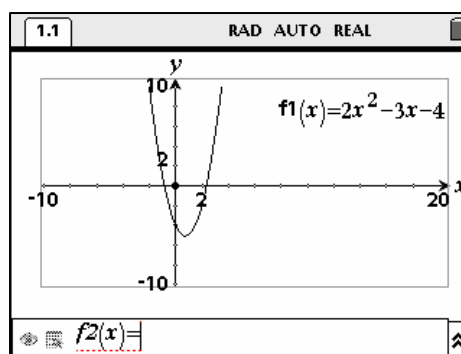


Figure 14

A Different Method to Change the Definition of a Function

1. Press esc to make pointer active (Figure 15).
 - Hover over the function definition until a hand appears.

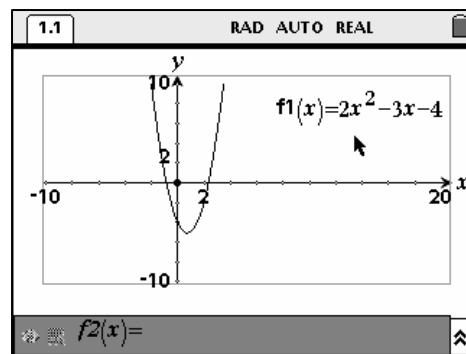


Figure 15

2. Press enter or $\text{2nd} + \text{enter}$ twice to open an edit box around the function definition (Figure 16).
 - The edit cursor is at the right end of this box.

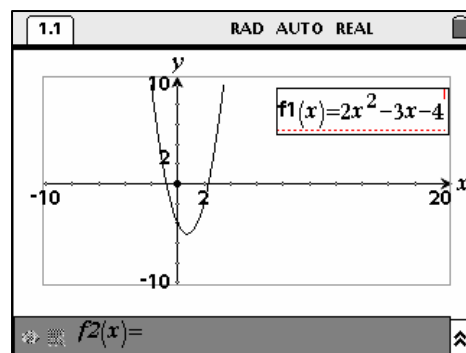


Figure 16

3. Once the edit box is open, use the arrow keys or clear to enter new values and press enter to graph the function (Figures 17 and 18).
 - The edit cursor is at the right end of this box.

Note: This technique will work for any function definition appearing on the screen.

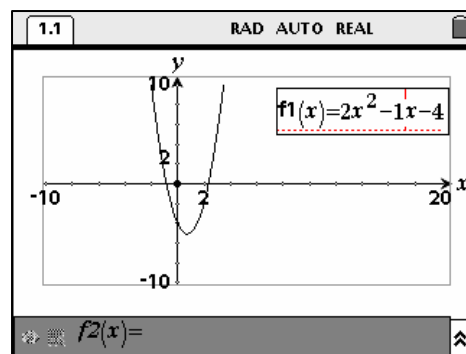


Figure 17

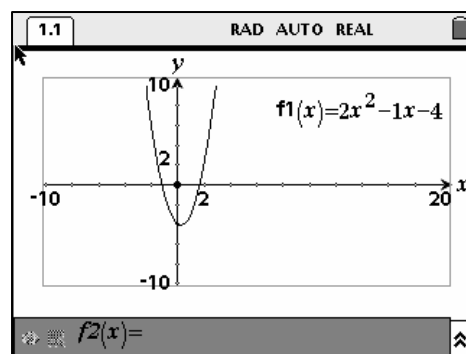





Figure 18

Entering Multiple Function Definitions


Entering additional function definitions can be accomplished in several ways. One way is to simply type a new function in the Equation Entry Line (Figure 19).

1. Type a new function definition into any open “slot” in the Equation Entry Line (Figure 19).

- Press  to graph the new equation (Figure 20).

2. Move your equations so that both can be seen separately (Figure 21).
3. The list of functions can be scrolled with the  and  keys.
4. The order of entry is not important but, after each entry, the list will scroll to the next available “slot” (Figure 21).

A second way to enter another function is to clear the entry line and enter a new function.

1. Use  to clear the entry line (Figure 22).

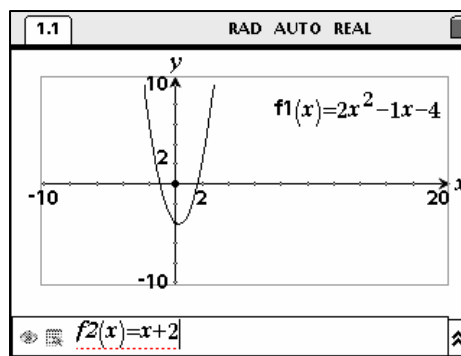


Figure 19

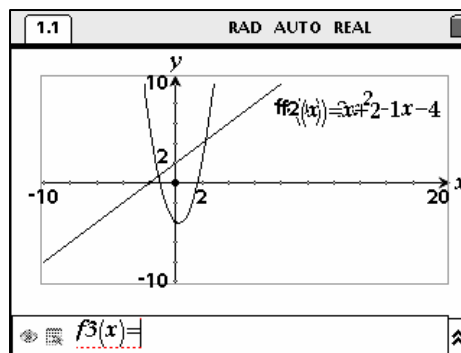


Figure 20

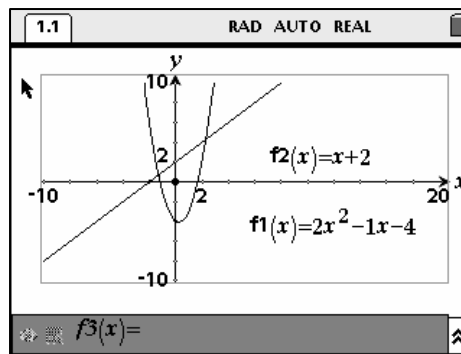


Figure 21

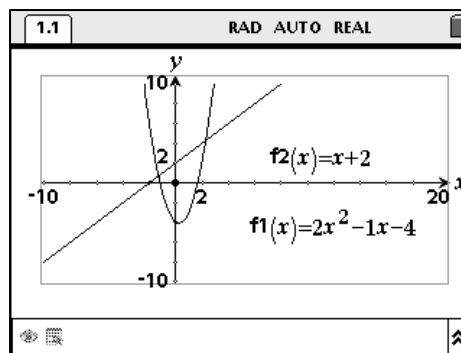





Figure 22

2. Type the function, e.g., $g(x)=4+2x-x^2$, and press  (Figures 23 and 24).

Scaling by Dragging an Axis

Note: As previously indicated, the viewing window may be changed by editing the four axis values.

1. Place the cursor on the x -axis so that the hand appears and the tic marks flash (Figure 25).
2. To “grab” only the x -axis tic marks, for altering only the x -axis scale, press the  key and then the  key.
3. Use the NavPad cursor control’s arrow keys to increase or decrease the maximum or minimum value and the scaling of the chosen axis (Figure 26).
 - Note: This method will not maintain the aspect ratio of the default window.

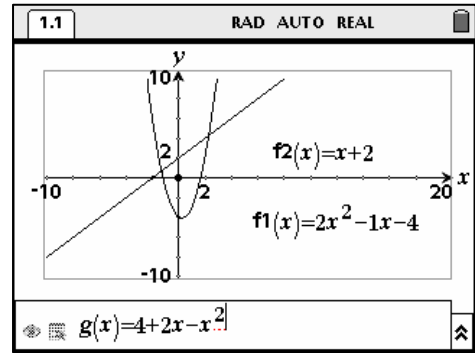


Figure 23

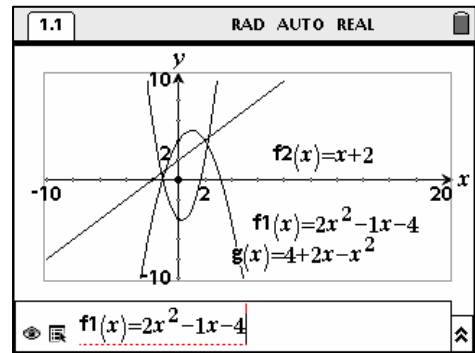


Figure 24

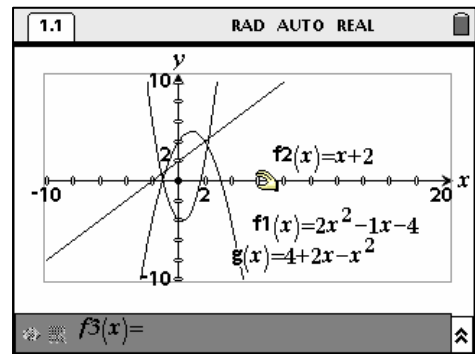


Figure 25

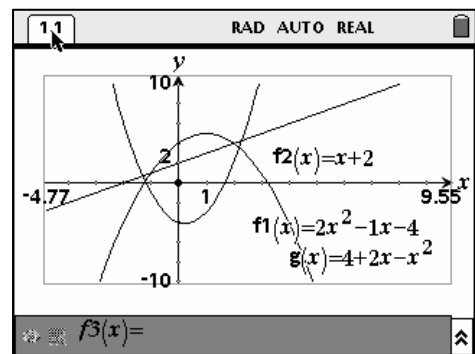




Figure 26

A third way to add more functions and their graphs to the screen is by direct entry on the Graphs & Geometry screen.

1. Press , and  for 2:Graphs & Geometry to add a new page (Figure 27).

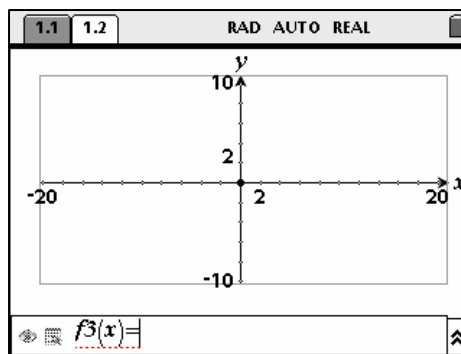








Figure 27

2. Press   and then  for 4:Hide Entry Line (Figure 28).
 - Alternate way to hide the entry line: Press , then  for 2:View, and  for 3:Hide Entry Line.

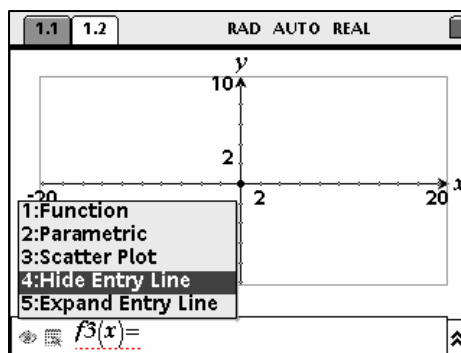




Figure 28

3. Press , then 1 for 1:Tools, and 5 for 5: Text (Figure 29). Move to any location on the screen and press  for a text box.

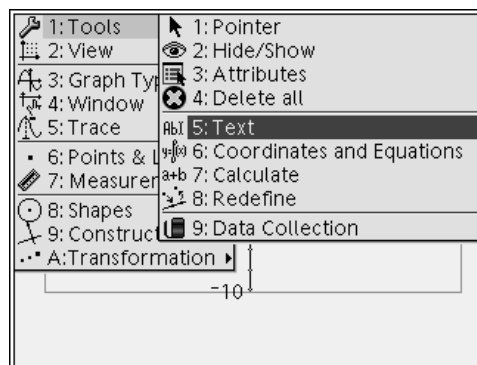



Figure 29

4. Type the definition of any function on the screen such as $x^2 + 2$. Press  (Figure 30).

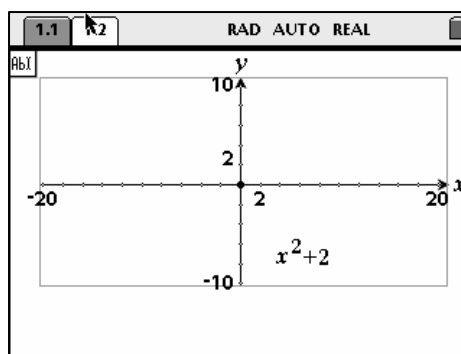






Figure 30

5. With the cursor bar over the definition, press **ctrl**  to grab the definition.
 - Drag the function definition onto the x - or y -axis.
 - As the definition hovers over the axis, a “ghost” of the function’s graph will appear (Figure 31).

6. Press **enter** , and the function graph will appear as a solid line (Figure 32).

Actively Manipulating Functions

First, move and resize the graphing window for this function. To move the graphing window:

1. Place the cursor in the first quadrant. Press **esc**  in case some other tool is active.
2. Press **ctrl**  to “grab” the graph screen (Figure 33).

3. Use the NavPad arrow keys to move the graph on the screen (Figure 34).

4. Resize the graphing window by dragging the endpoints of an axis.

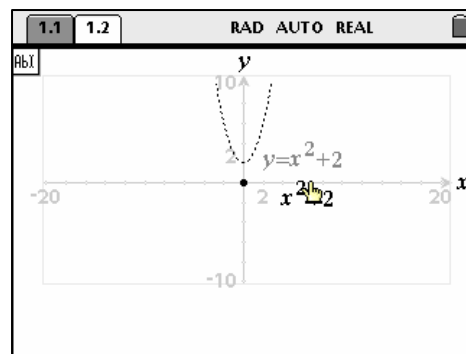


Figure 31

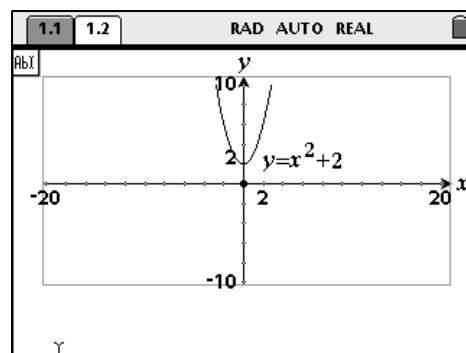


Figure 32

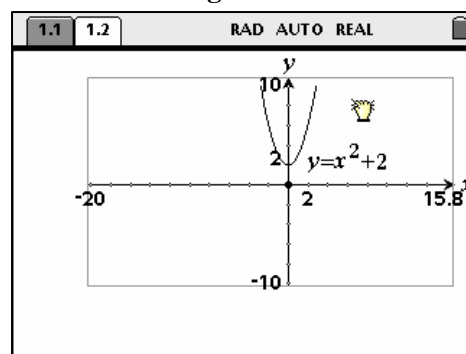


Figure 33

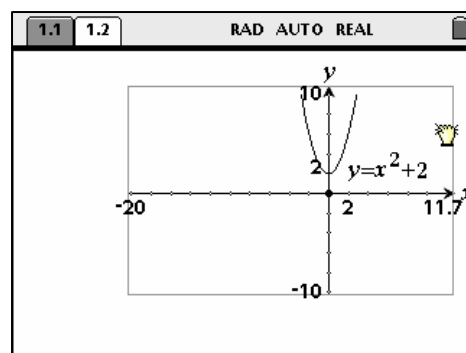


Figure 34

- Place the cursor at the end of one of the axes (Figure 35).

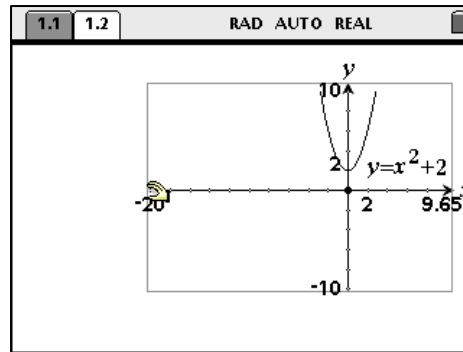


Figure 35

- Press **ctrl** and the **arrow** key to grab the arrow at the end of the axis.
 - Use the NavPad arrow keys to increase or decrease the length of the axis.
 - This will change the maximum or minimum value but not the scaling of the axis (Figure 36).

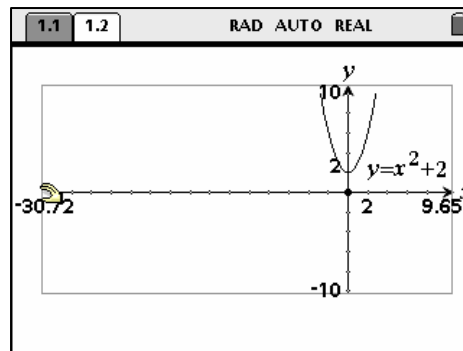


Figure 36

As you move the pointer close to the graph of a function, two different types of cursors appear. The two-directional arrow stretches the function (Figure 37).

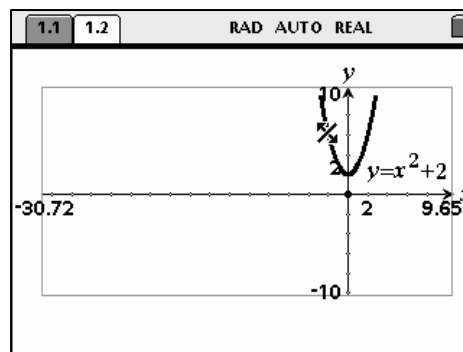


Figure 37

The four-directional arrow translates the function (Figure 38).

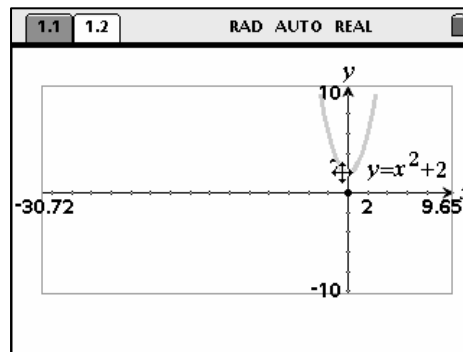


Figure 38

- Place the pointer near the graph until the two-directional arrow appears. Press ctrl $\left(\frac{1}{x}\right)$ and use the NavPad arrow keys to stretch the function.
 - Note that the position of the vertex does not change. The new function definition is displayed as the graph is stretched (Figure 39).
- Place the pointer near the vertex of the graph until the four-directional arrow appears.
 - Press ctrl $\left(\frac{1}{x}\right)$ and use the NavPad arrow keys to translate the function.
 - The new function definition is displayed as the graph is translated (Figure 40).

Tracing on a Function

Press $\left(\frac{1}{x}\right)$ and $\left(\frac{1}{x}\right)$ for 2:Graphs & Geometry to add a new page.

- Enter the exponential function $f_3(x) = 1500(1+.05)^x$ describing, for example, how \$1500 grows at 5% in a system of yearly compounding interest over a number of years x . Press $\left(\frac{1}{x}\right)$ (Figures 41 and 42).

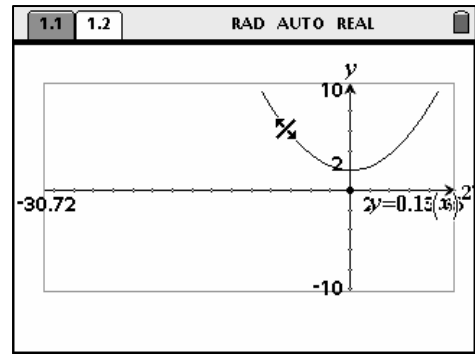


Figure 39

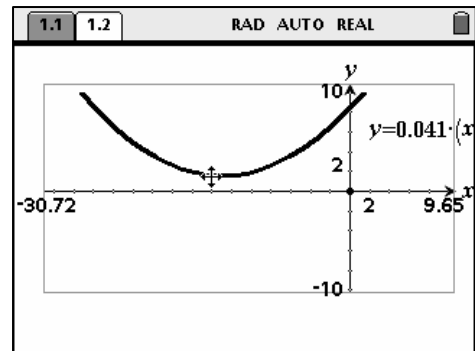


Figure 40

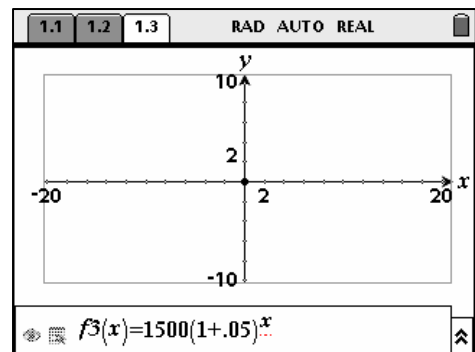


Figure 41

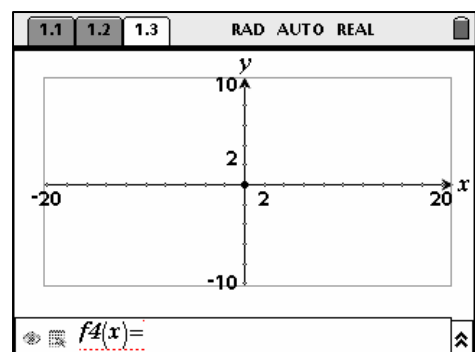


Figure 42

Obviously, the graph is not displayed in this viewing window.

1. Press **menu**, choose **4** for 4:Window, and **1** for 1:Window Settings (Figure 43).

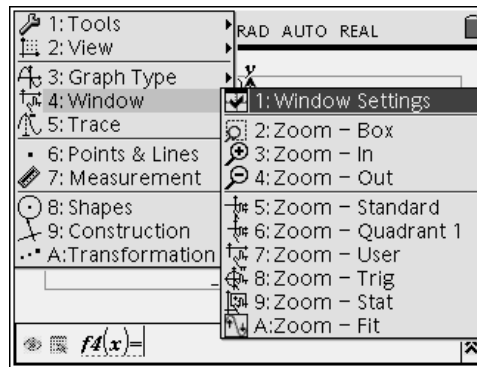


Figure 43

2. Enter the window parameters shown in Figure 44.

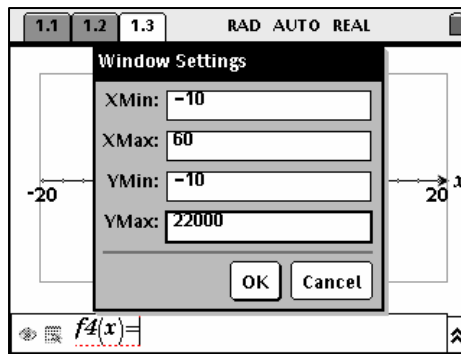


Figure 44

3. Press **tab** to highlight OK, and press **enter** to display a graph that shows growth over the next 60 years (Figure 45).

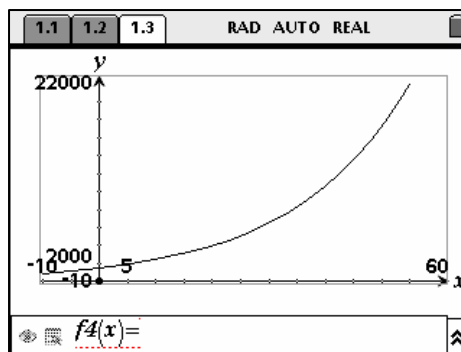


Figure 45

4. Press menu 5 for 5:Trace. Use the left and right NavPad arrows to trace on the function (Figures 46 and 47).

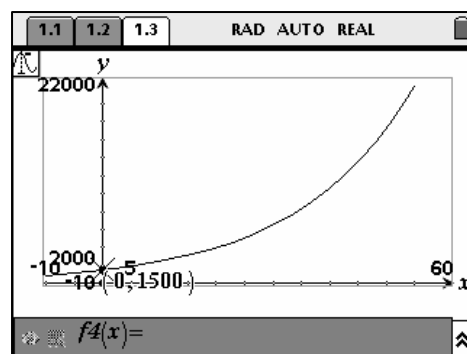


Figure 46

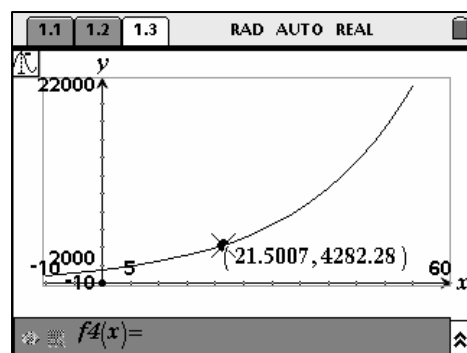


Figure 47

5. To find the function value at a specific value of x , enter the number while in the trace mode and press enter (Figures 48 and 49).

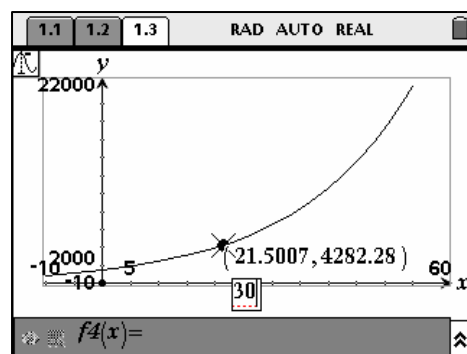


Figure 48

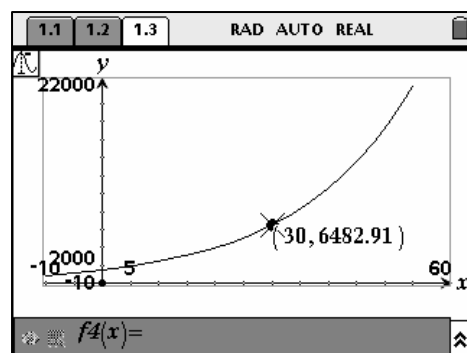







Figure 49

- To mark a point on the graph of the function while in the trace mode, press  at the desired location (Figure 50).

Decimal places may be added or removed from a number on the screen.

- Press  to exit the tracing mode.
- Hover over a number with the pointer until a hand appears and the number begins to blink (Figure 51).

- Press   and choose  for 2:Attributes.
 - The Attributes box appears (Figures 52 and 53).

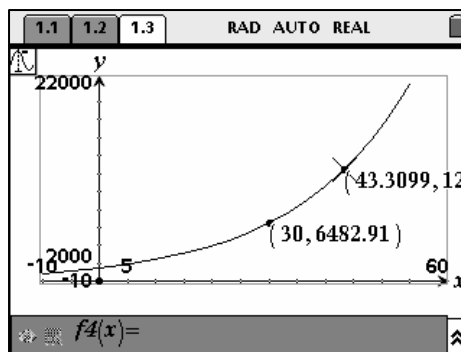


Figure 50

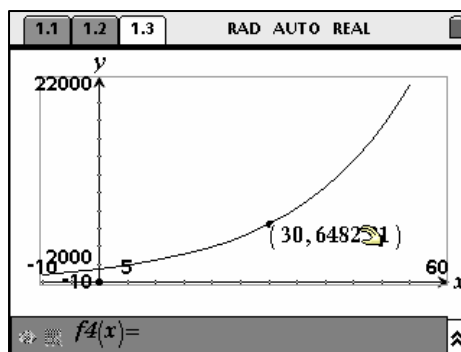


Figure 51

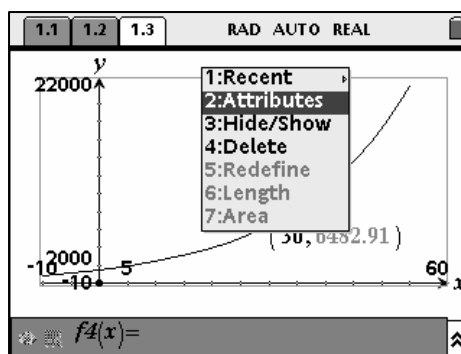


Figure 52

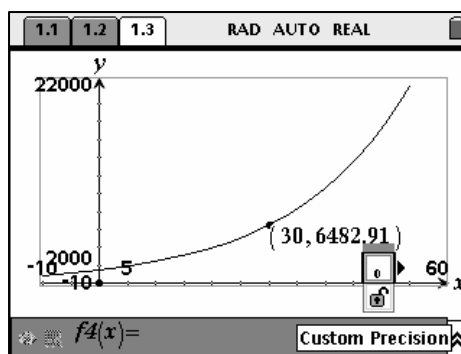






Figure 53

4. Use the NavPad left and right arrow keys to decrease or increase the number of decimal places displayed (Figures 54).
 - Press  to exit the Attributes option.

Note: Sometimes when you attempt to add decimal places to a number, nothing appears to happen. This means that there are no rounded digits hidden.

Change the x-value of the marked trace point on the graph to a different year, in this case the 25th year.

1. Hover the pointer over the x -coordinate of the point, and press  or  twice to open a text box around the number (Figure 55).
2. Enter the new value for x and press  (Figures 56 and 57).
3. After 25 years of yearly compounding at 5% interest, our original investment of \$1500 will grow to be \$5,079.53.

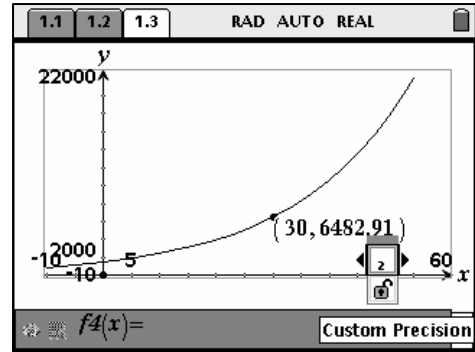


Figure 54

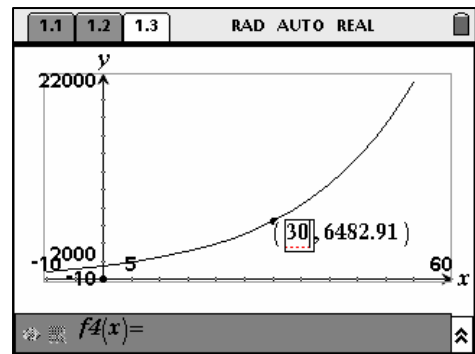


Figure 55

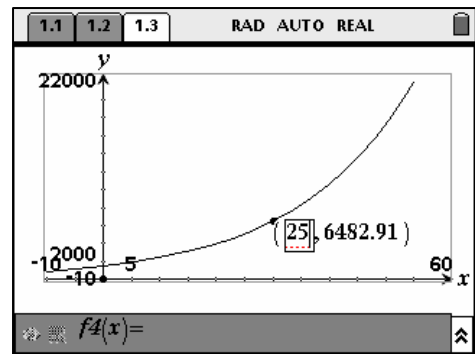


Figure 56

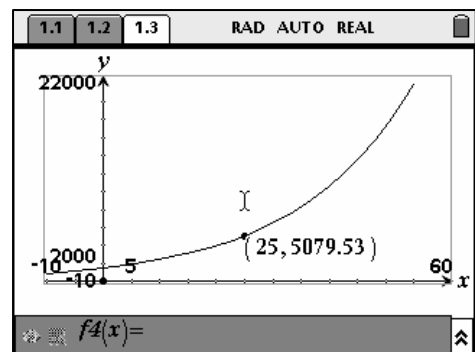









Figure 57

The editing technique used to change the input value of the x -coordinate of the trace point can also be used for the y -coordinate. This has interesting mathematical implications. We can now ask “What value of $f(x)$ will produce a specific value of x ?” This can be a more important question than “What value of x will produce a specific value of $f(x)$?” For example, when will the value of the investment reach \$10,000?

1. Hover the pointer over the y -coordinate of the point and press the  or  key twice to open a text box around the number (Figure 58).
2. Enter the new value for y and press  (Figure 59).
 - Note: This is a discrete problem, so the answer makes mathematical sense but not actual sense in this problem setting.

Save the document.

1. Press   for the tools menu.
2. Press 1 to select 1:File, and 4 for 4:Save As... (Figure 60).
3. Enter the file name basic2 (Figure 61).
4. Press the  key to highlight OK and press .

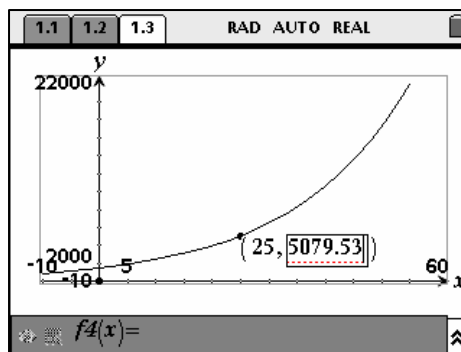


Figure 58

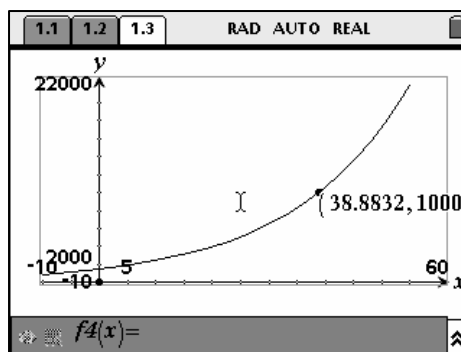


Figure 59

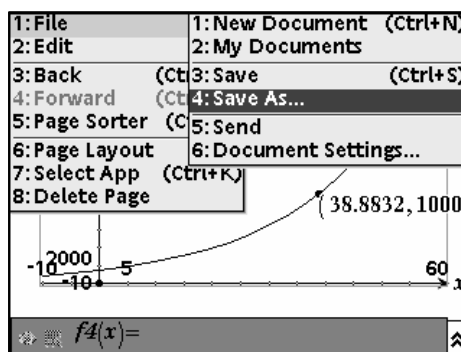


Figure 60

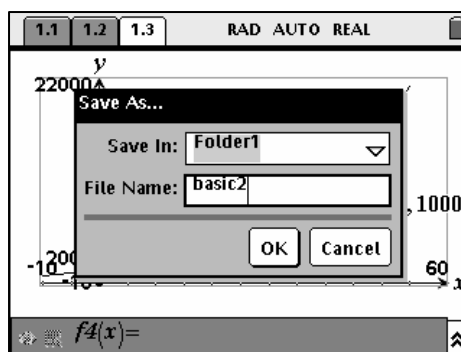







Figure 61

Growth at Various Interest Rates

- Add a new document.
 - Press , and then press  to choose 5:New Document.
 - Press  to choose 2:Graphs & Geometry.
- Compare the growth of our original \$1500 under various interest rates (5%, 6%, and 7%) by graphing all three models on the same axes.
 - Enter and graph $f_1(x) = 1500(1+.05)^x$, $f_2(x) = 1500(1+.06)^x$, and $f_3(x) = 1500(1+.07)^x$.
 - Choose an appropriate window (Figure 62).

- Press  and choose  for 5:Trace (Figure 63).

- Press the right or left NavPad arrow to scroll on the first function (Figure 64).
- Press the up or down NavPad arrow to move from function to function.

- Press the up or down NavPad arrow until the trace points appear on all three functions (Figure 65).
 - Note: Once the three trace points are displayed, pressing the right or left NavPad arrow key moves all three points simultaneously.

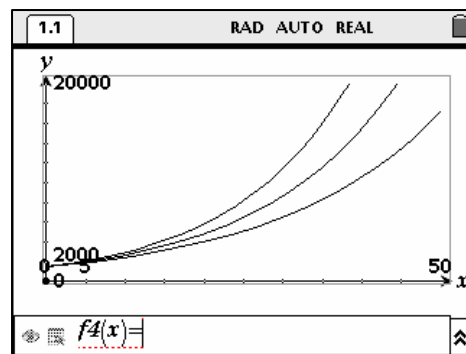


Figure 62

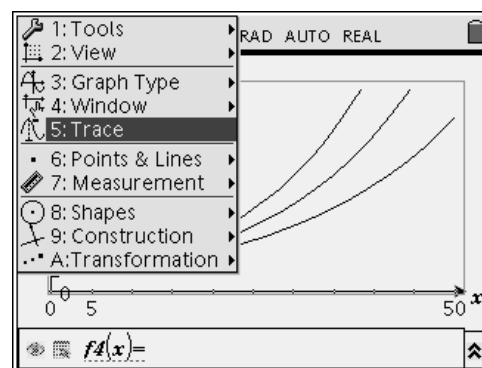


Figure 63

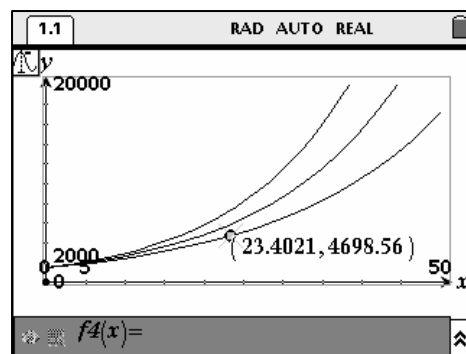


Figure 64

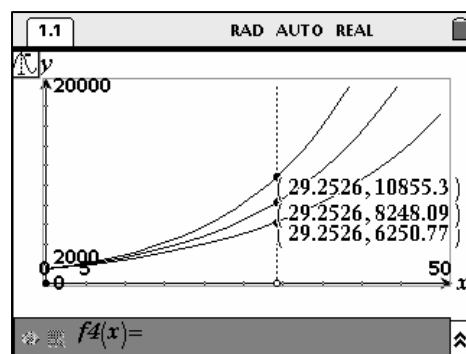


Figure 65

What are the various input values for x that produce a given output value for all of the defined functions, $f_1(x)$, $f_2(x)$, and $f_3(x)$? This amounts to simultaneously solving all the graphed functions. For this problem, how many years will it take to reach a savings amount of \$10,000 at each interest rate?

1. Press esc .
2. Press tab to move to the equation entry line.
3. Enter $f_4(x) = 10000$ and press enter to graph (Figure 66 and 67).

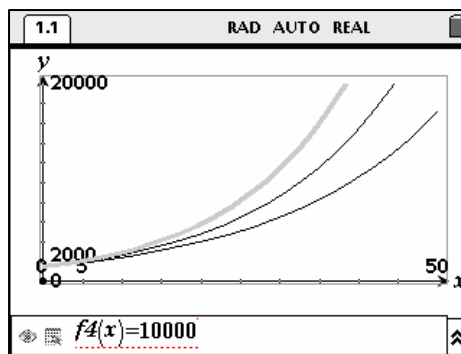


Figure 66

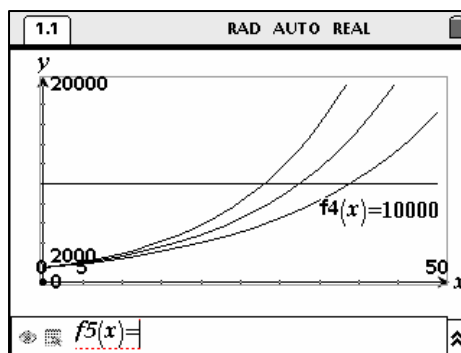


Figure 67

4. Press menu , and choose 6 for 6:Points & Lines, and then 3 for 3:Intersection Point(s). See Figure 68.

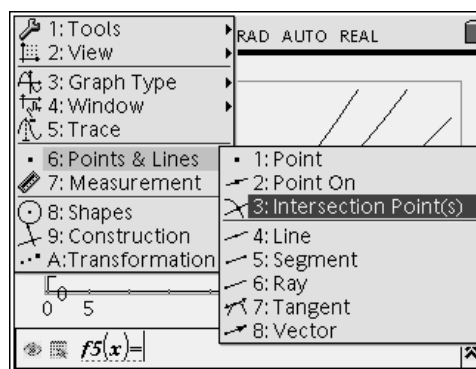


Figure 68

5. Move the cursor to the intersection of horizontal line and one of the functions. Both should appear dashed.
 - Press point or enter to mark the point of intersection (Figure 69).

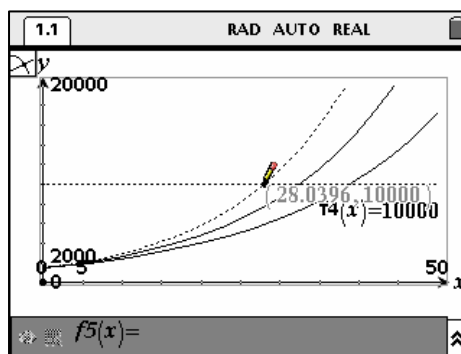



Figure 69

6. Repeat this procedure for each of the other points of intersection (Figures 70).

7. Move the coordinates of the points of intersection to display all three clearly.

- Move the cursor to hover over an ordered pair.
- Press **ctrl** , and drag the ordered pair.
- Repeat for the other two ordered pairs. (Figure 71).

8. Compare the number of years for each interest rate.

Note: Again, this result is mathematical, and not realistic given the discrete nature of yearly compounding interest in this problem setting.

Change $f_4(x)$, and determine how many years it will take to reach a savings amount of \$15,000 at each interest rate (Figures 72 and 73).

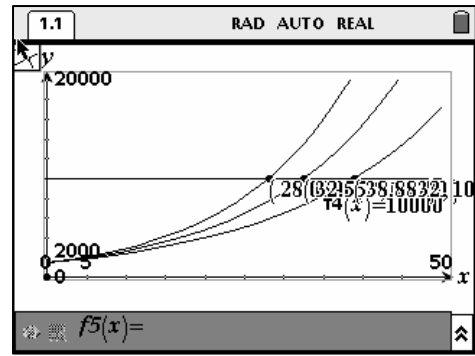


Figure 70

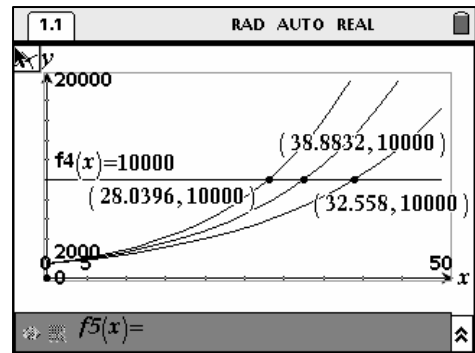


Figure 71

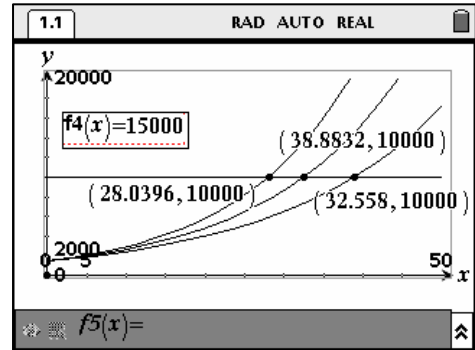


Figure 72

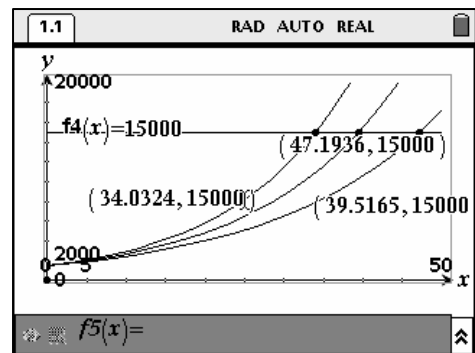


Figure 73

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Basic Geometry

Materials

- TI-Nspire™ Math and Science Learning Handheld



Overview

Our experience has shown that people who have had prior experience with any interactive geometry software (handheld or computer) will learn to use the TI-Nspire™ learning handheld more easily than others. It is important to master some basic geometry skills in order to facilitate learning.


Introduction

The following problems related to triangles will introduce you to the Graphs & Geometry application on the TI-Nspire™ math and science learning handheld.

Opening a New Graphs & Geometry Document

- Press   for Home, 5:New Document (Figure 1).

Note: If a Save option appears, select Yes or No to save the document that was open when the TI-Nspire™ handheld was last used.

- Press  for 2:Add Graphs & Geometry (Figure 2).

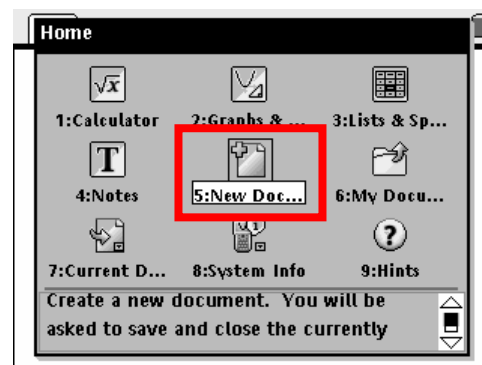


Figure 1

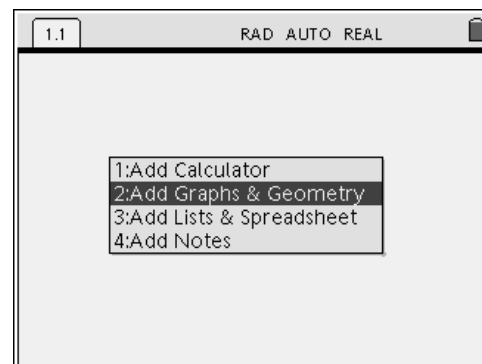


Figure 2

Hiding the Axes and the Entry Line

1. Press **menu** **2** for Menu 2:View.
2. Press **1** or **enter** to choose 1:Hide Axes (Figures 3 and 4).
3. Press **menu** **2** for Menu 2:View.

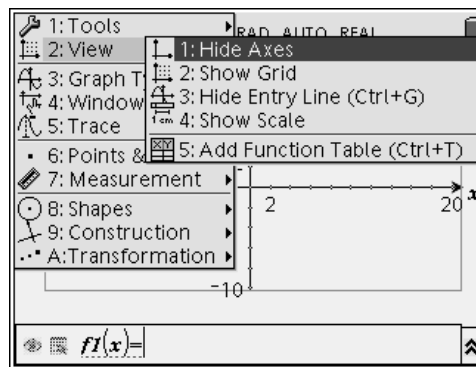


Figure 3

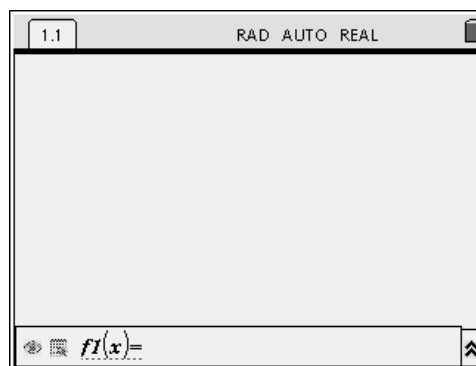


Figure 4

4. Press **3** to choose 3:Hide Entry Line (Figure 5 and 6).

Note: An alternate way to select a menu option is to press **menu** and then use the NavPad cursor controls to highlight the desired option. Then, press the center **enter** key or the **enter** key to select the option.

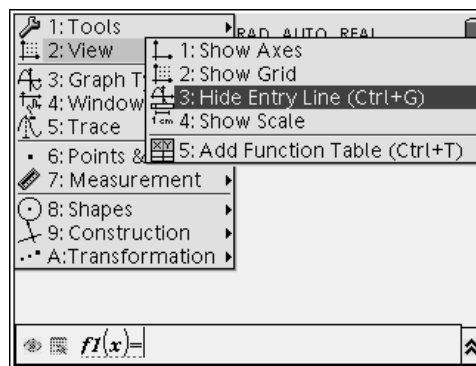


Figure 5

Exploring the Page

1. Use the NavPad cursor controls to explore moving around on the page.

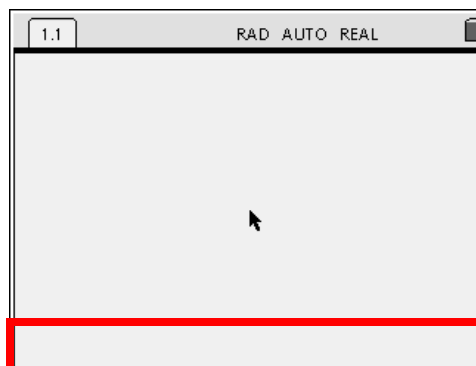


Figure 6

2. Press **(menu)**, and look at the options in the following submenus:
 - Menu 6:Points & Lines
 - Menu 7:Measurement
 - Menu 8:Shapes
 - Menu 9:Constructions
 - Menu A:Transformation
3. After pressing the **▶** key to investigate submenu options in a particular menu selection, press the **(esc)** key to exit that list of menu options.
4. Press **(esc)** a second time to exit the Menu structure entirely.

Interior Angles of a Triangle

Construct a triangle, $\triangle ABC$.

1. Choose the Triangle tool by pressing **(menu)** **8** **2** for Menu 8:Shapes, 2:Triangle (Figure 7).
2. Using the NavPad cursor controls, position the pointer to construct a triangle near the center of the screen.
3. Press the **(point)** key or the **(enter)** key to place the first vertex on the screen.
 - One way to label vertices and points is to type the label immediately after pressing **(point)** or **(enter)** to place the vertex or point on the screen. (Point should be flashing.)
 - For capital letters, press the **(CAPS)** key before pressing the letter key (Figure 8).

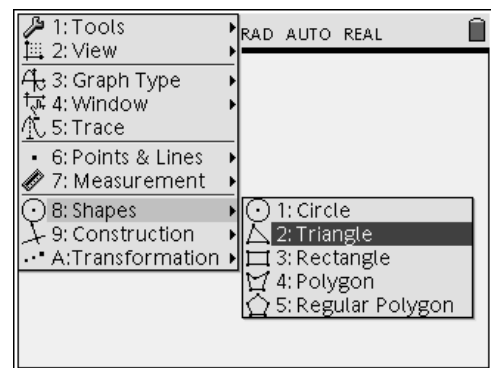


Figure 7

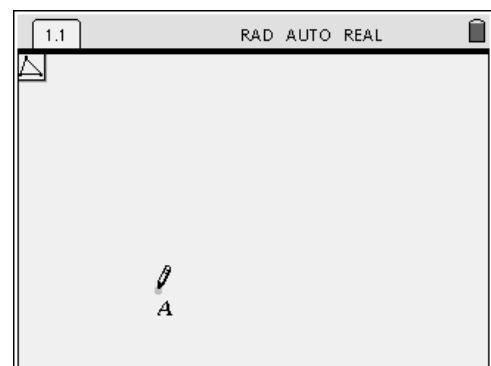











Figure 8

4. Using the NavPad again, move the cursor to a second location for the next vertex.
5. Press  or  to place the second vertex.
6. Type the label immediately.
7. Repeat these steps to create a third non-collinear location for the third vertex, and type its label.
8. Move away from the label, and press the  key to deselect the triangle tool.
 - If you didn't label one or more of the vertices when constructing the triangle (as shown in Figure 9), you can label the vertex using the Text tool.
9. Press    for Menu 1:Tools, 5:Text (Figure 10).
10. Use the NavPad cursor controls to move close to the vertex.
 - The vertex should blink. Be sure that the entire triangle does not blink.
11. Press  or  to open an edit box (Figure 11).
12. Press the appropriate letter key and  to close the edit box.

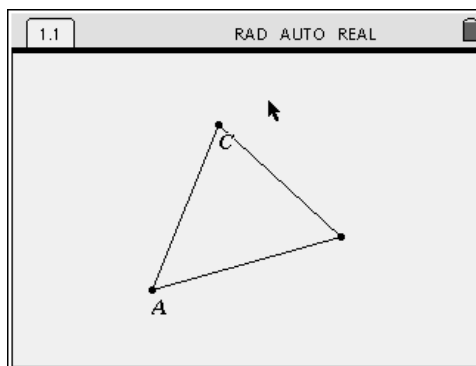


Figure 9

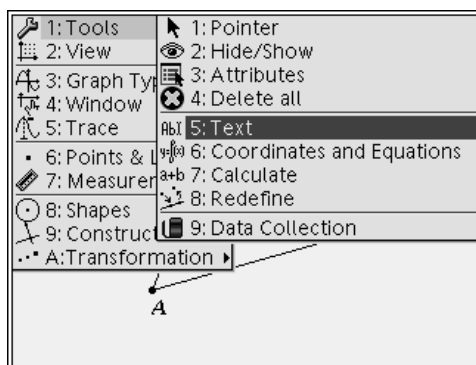


Figure 10

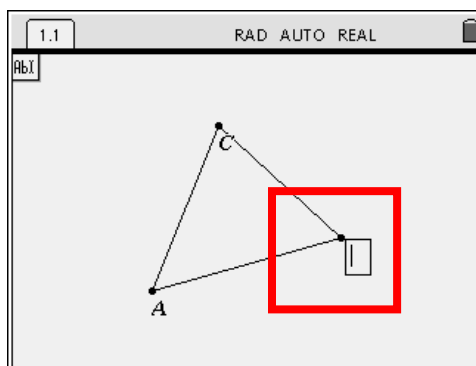



Figure 11


13. Press the **(esc)** key to deselect the Edit tool (Figure 12).


Dragging a Label

Drag a label to another location on the screen.

1. Use the NavPad cursor controls to move close to the object that is to be moved, e.g., label 'C'.
2. When an open hand appears, press the **(ctrl)** key followed by the **()** key to “close” the hand.


Note: If the open hand does not appear when the arrow key is close to the object to be moved, press the **(esc)** key.

Note: When the open hand appears, holding down the click key **()** will also “close” the hand.

3. Drag the label to the desired location, and press **()** (Figure 13).
 - If the label appears shaded after moving away from the label, press the **(esc)** key.

Changing Document Settings

If the Settings information at the top of the page indicates RAD for Radian (see Figure 13 above), change this document setting to Degree.

1. Press **()** **(8)** for Home 8: System Info.
2. Press **(1)** for 1: Document Settings (Figure 14).
3. Press the **(tab)** key to move to Angle.

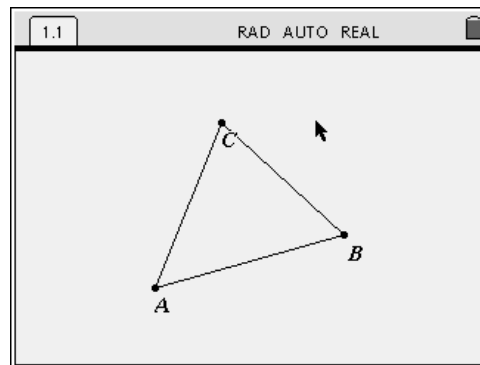


Figure 12

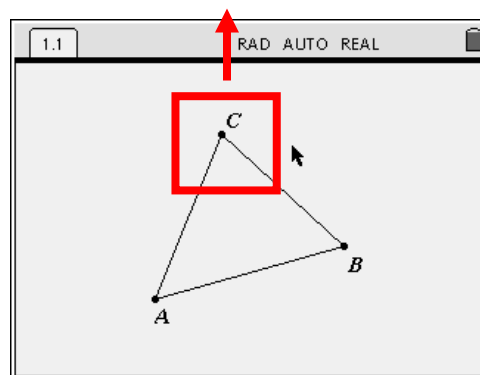


Figure 13

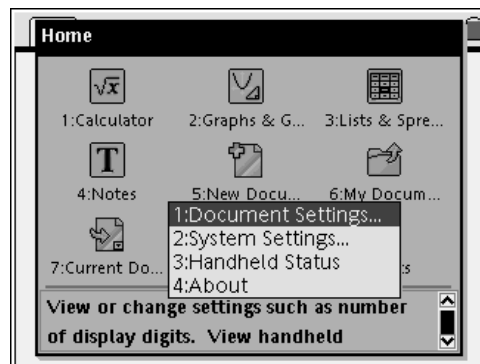











Figure 14

- The box to the right of the word Angle should be highlighted (outlined).
- Press the down on the NavPad cursor control to highlight Degree.
 - Press  (Figure 15).
 - Press  again to exit Document Settings.
 - Alternately, press the  key until the box containing OK is highlighted, and then press  to exit.

Measuring the Three Interior Angles

- Press    for Menu 7:Measurement, 4:Angle (Figure 16).
 - Move the cursor on the screen with the NavPad to select each vertex.
 - The vertices must be selected in either a clockwise or counter-clockwise direction, just as angles are named.
- Note: To select a vertex, use the NavPad cursor controls to move close to a vertex. When the “pencil” shape on the screen changes to a “hand” shape, press  or  to select the vertex (Figure 17).

Note: Regardless of direction, the vertex of the angle to be measured must be selected second. Not only is the angle drawn, but the angle being measured is temporarily marked.

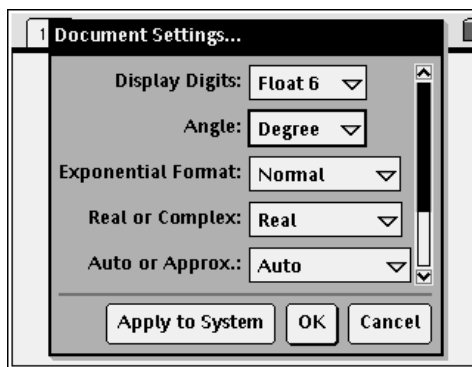


Figure 15

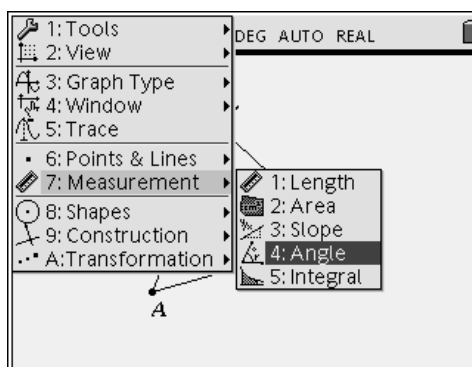


Figure 16

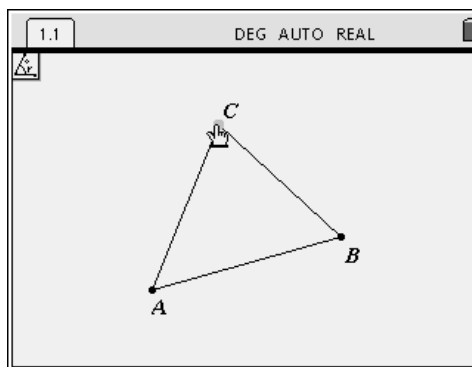


Figure 17

3. After pressing \approx or $\frac{\square}{\square}$ at the third vertex, drag the measured angle value to an appropriate location on the screen, and press \approx (Figure 18).

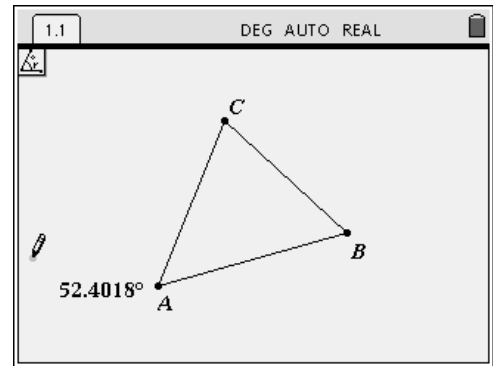


Figure 18

4. Repeat to measure the other two angles (Figure 19).

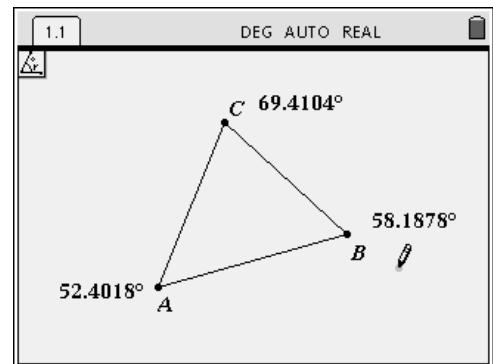


Figure 19

Calculating the Sum of the Three Angles

Note: For a calculation, an expression or statement must first be entered on the page.

1. Press menu $\{1\}$ $\{5\}$ for Menu 1:Tools, 5:Text.
2. Position the pointer in an empty location on the screen.
3. Press \approx or $\frac{\square}{\square}$ to open the edit box (Figure 20).
4. Type the statement $s = a + b + c$ on the screen using the keypad, and press \approx to affix the statement to the screen (Figure 21).

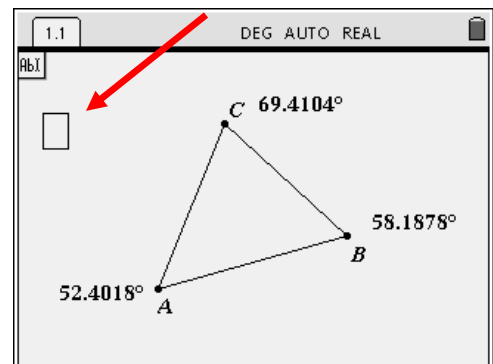


Figure 20

Note: You cannot put an equals sign at the end of this type of statement, but you can start with a word or letter like 's' as long as it is not a reserved word like 'cos.'

5. Press menu $\{1\}$ $\{7\}$ for Menu 1:Tools, 7:Calculate to select the Calculation tool.

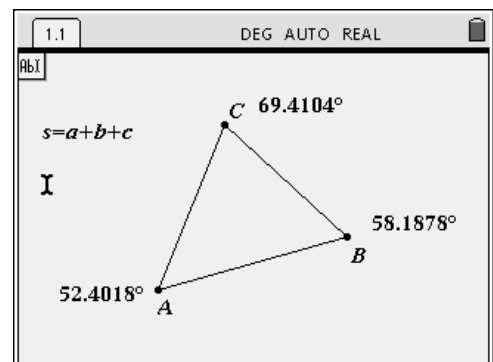





Figure 21

6. Point at the statement $s = a + b + c$ and press  to select this calculation rule (Figure 22).



- As you move away from the calculation rule and near an angle measurement, a box will appear with a message asking for the first variable in the calculation.

- In this example, the message is ‘Variable a?’ (Figure 23).

7. Move near the first measurement.


8. When a “hand” appears, select the angle measurement by pressing  or  (Figure 24).

- When you move away from the first angle measurement, the second variable value is requested.

9. Move to the second angle measurement and, when the hand appears, select it by pressing  or .

10. Repeat this procedure to select the remaining angle measurement.

- A sum will appear on the screen as you move away from the last measurement selected.

11. Press  to affix the sum to a convenient location on the screen (Figure 25).

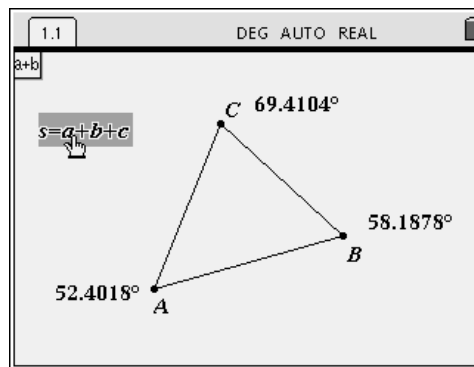


Figure 22

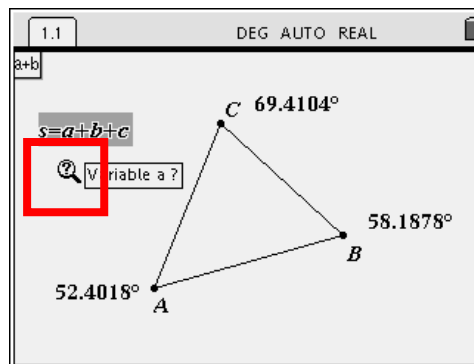


Figure 23

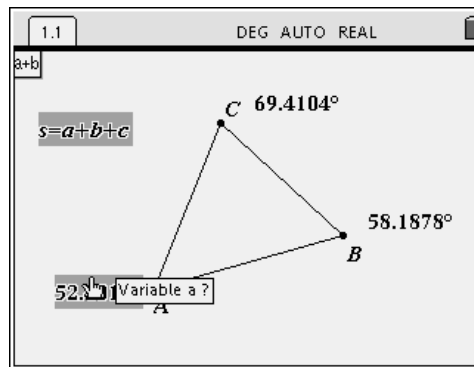


Figure 24

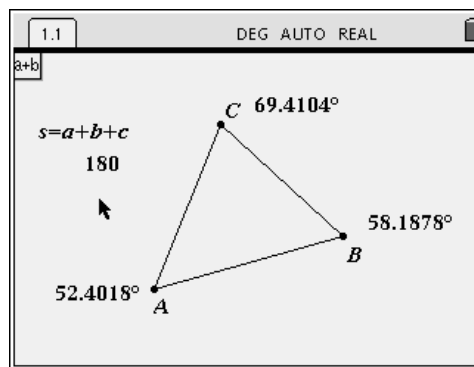


Figure 25






- If desired, use the Text tool to label this numerical result (Figure 26).

Drag one of the vertices, and make a conjecture about the sum of the interior angles of a triangle (Figure 27).

Note: To drag a vertex, follow the steps described earlier for dragging a label.

Does your conjecture hold for any type of triangle? Explain your reasoning.

Increasing or Decreasing the Number of Decimal Places Displayed

1. Press    for Menu 1:Tools, 3:Attributes.
2. Select a number (e.g., an angle measurement) on the screen (Figure 28).
3. Type the number of desired decimal places from the keypad, or use the NavPad right or left cursor controls to increase or decrease the number of decimal places displayed (Figure 29).
4. Press .
5. Press the  key to exit the Attributes tool.

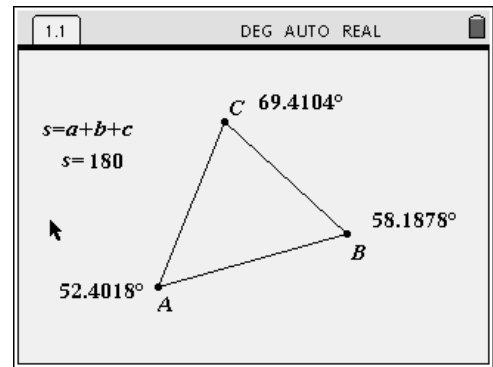


Figure 26

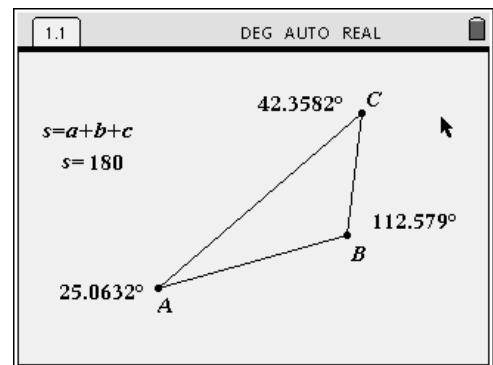


Figure 27

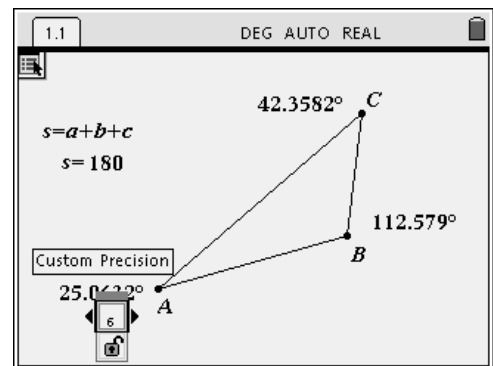


Figure 28

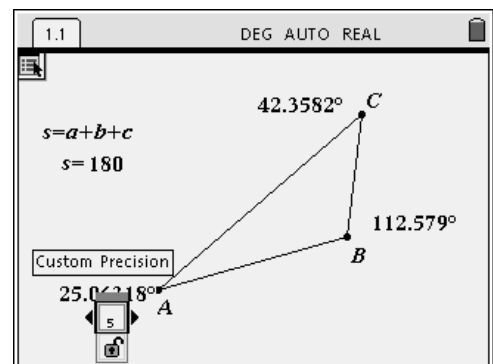


Figure 29

Note: There is an alternate method for increasing or decreasing the number of decimal places.

Press the **esc** key and point to the measurement. When a hand appears, press the plus key (\oplus) to increase the number of decimal places displayed or the minus key (\ominus) to decrease the number of decimal places displayed. If the measurement is exact to a given decimal place, no non-significant digits will appear.

Saving the Document

1. Press **ctrl** **home** **1** **4** for 1:File, 4:Save As (Figure 30).
2. Enter the name 'triangle,' and press **enter** to save the file (Figure 31).

Inserting a New Problem and Constructing a Triangle

1. Open a new problem by pressing **ctrl** **home** **2** **8** for 2:Edit, 8:Insert Problem (Figure 32).
2. Select 2:Add Graphs & Geometry.
3. Press **menu** **2**, and choose 1:Hide Axes.
4. Press **menu** **2** again, and choose 3:Hide Entry Line.
5. Construct a triangle, $\triangle ABC$, near the center of the screen.

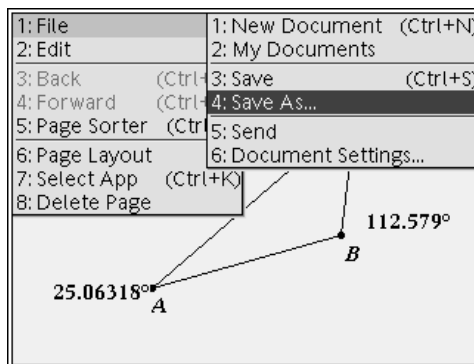


Figure 30

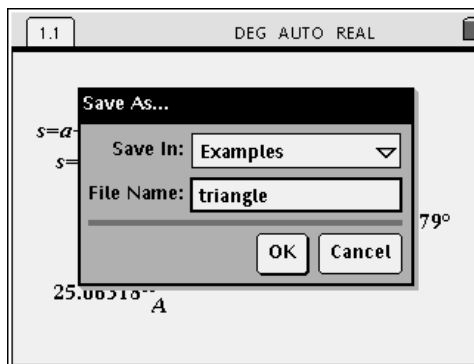


Figure 31

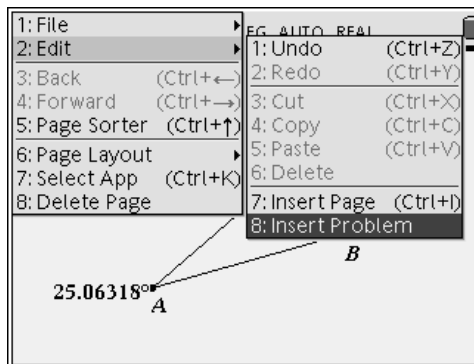


Figure 32

- Choose the Triangle tool by pressing **(menu)** **8** **2** for Menu 8:Shapes, 2:Triangle.
- Label the vertices as you construct the triangle, entering the letter after pressing **(enter)** to construct each vertex (Figure 33).

Measuring Lengths of the Sides and Perimeter of the Triangle

Measure the lengths of the sides of the triangle.

- Press **(menu)** **7** **1** for Menu 7:Measurement, 1:Length.
- Select two vertices of the triangle to measure the length of a side.
 - After two vertices are selected, the measurement will appear with a “hand” attached.
- Drag the measurement to the desired location, and press **(enter)** (Figure 34).
- Repeat to find the length of the other two sides.

Measure the perimeter of the triangle.

- With the Measurement (Length) tool active, select a side of the triangle.
- The entire triangle should blink (Figure 35).
- Drag the perimeter measurement to a desired location, and press **(enter)**.
- Press **(esc)** to exit the Length tool.

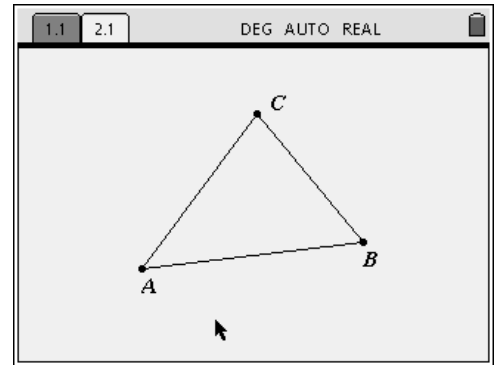


Figure 33

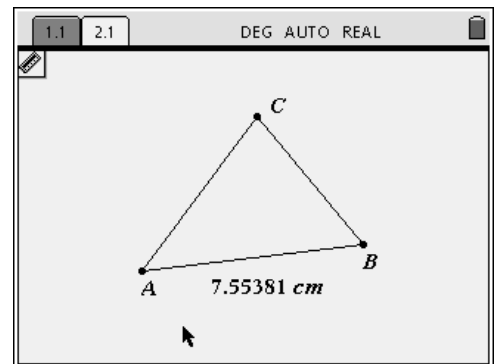


Figure 34

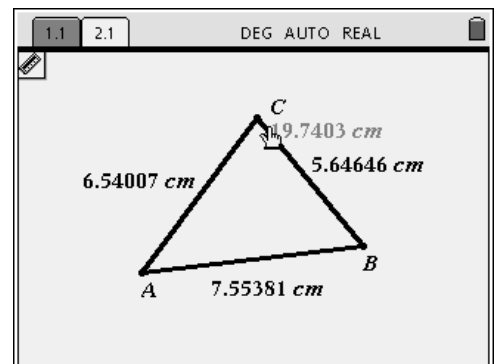


Figure 35

9. Label the perimeter (Figure 36).

Finding the Sum of the Measures of the Three Sides

- Press menu 1 5 for Menu 1:Tools, 5:Text, and press enter to open an edit box.
- Enter the expression $a+b+c$, and press enter (Figure 37).
- Use the Calculation tool (Menu 1:Tools, 7:Calculate) to find the sum of the three sides of the triangle.
- Select the expression and then the three measures of the sides.
- Move the sum next to the expression $a+b+c$, and press enter (Figure 38).
- Press esc to exit the Calculate tool.

Drag a vertex of the triangle, and make a conjecture about the definition of the perimeter of a polygon (Figure 39).

Saving the Document

- Press ctrl home 1 3 for 1:File, 3:Save.
- Note: Pressing ctrl S will also save the document.

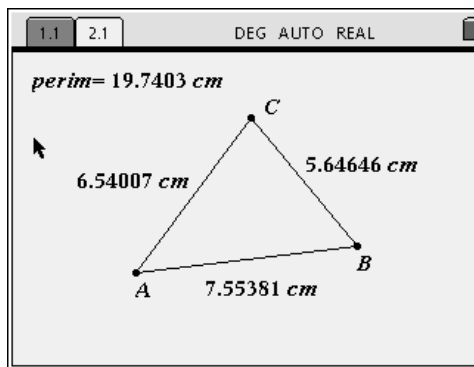


Figure 36

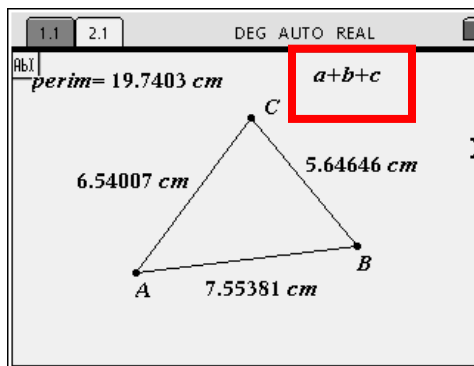


Figure 37

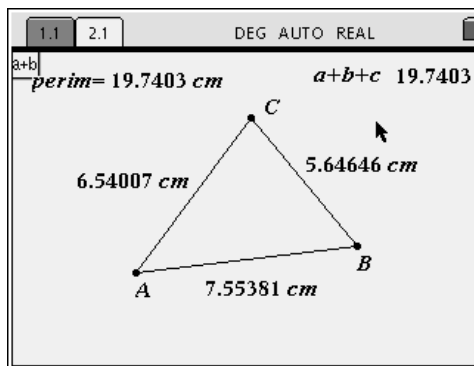


Figure 38

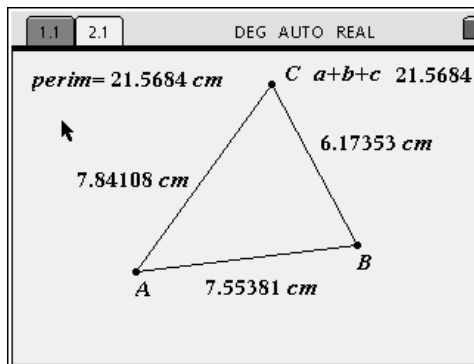


Figure 39

Investigate

Interior Angles of a Triangle

Dragging a vertex of the triangle should suggest that the sum of the angles of a triangle is equal to 180° .

Perimeter of a Triangle

Dragging a vertex of the triangle should suggest that the sum of the sides of a triangle is equal to the perimeter of the triangle.

Measuring the Area of the Triangle

- Press menu 7 2 for Menu 7:Measurement, 2:Area.
- Move the cursor close to a side of the triangle.
 - The entire triangle should blink.
- Press enter or enter .
- Move the measurement to a convenient location on the screen, and press enter .
- Label the area (Figure 40).

Measuring the Altitude of the Triangle

Note: The length tool will measure the perpendicular distance from a point or vertex to a line, line segment, or ray.

Construct a line on one side of the triangle using the line construction tool.

- Press menu 6 4 for Menu 6: Points & Lines, 4: Line (Figure 41).

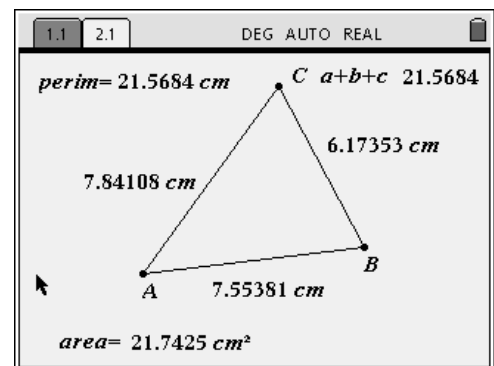


Figure 40

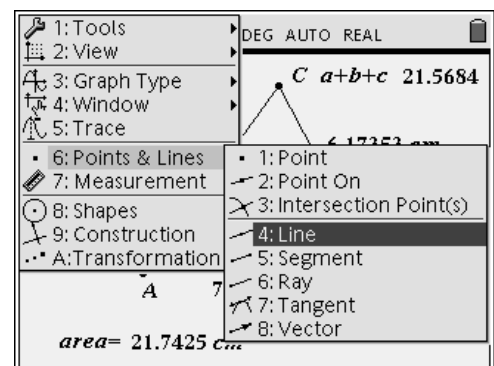


Figure 41

2. Select two vertices.
 - A line on side AB was constructed in this example.
3. Press **menu** **7** **1** for Menu 7:Measurement, 1:Length.
4. Select vertex C and the line constructed on the opposite side of the triangle.
 - In this example, the measurement is the perpendicular distance from vertex C to the line on side AB (Figure 42).
5. Move the measurement to a convenient location on the screen, and label the altitude (Figure 43).
6. To hide the line, press **menu** **1** **2** for Menu 1:Tools, 2:Hide/Show.
7. Move close to the line until an “eye” appears, and press **enter** or **hide**.
8. Press **esc** to exit the Hide/Show tool.

Calculating .5bh

1. Press **menu** **1** **5** for Menu 1:Tools, 5:Text.
2. Move the cursor to an empty space on the screen.
3. Press **enter** or **hide** to open the edit box.
4. Enter $.5b \cdot h$ from the keypad, and press **enter** (Figure 44).
5. Press **menu** **1** **7** for Menu 1:Tools, 7:Calculate.
6. Select the expression $.5bh$.
7. Next, select the base and height measurements.

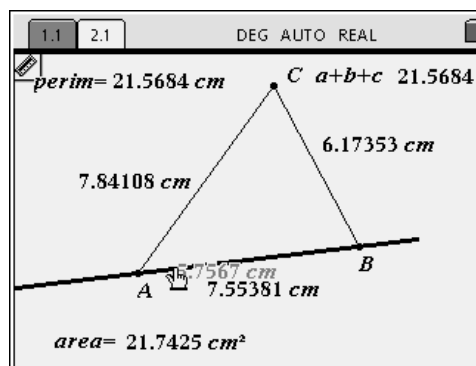


Figure 42

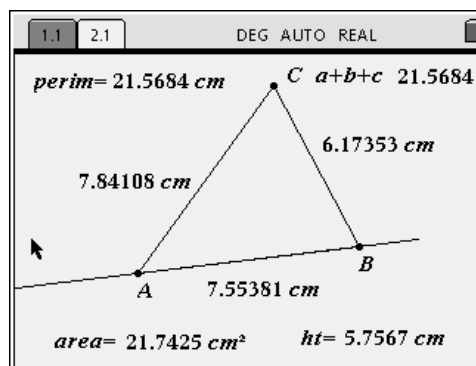


Figure 43

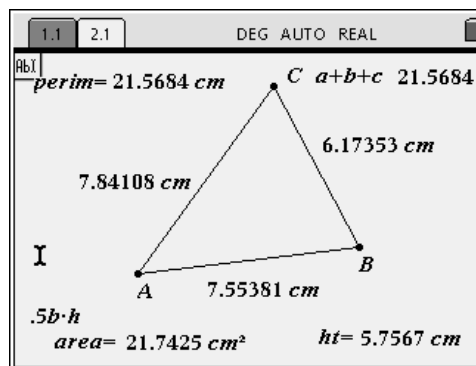






Figure 44

8. Move the result of the calculation next to the expression $.5b \cdot h$ and press  (Figure 45).

Drag a vertex of the triangle, and make a conjecture about the area of a triangle (Figure 46).




Constructing an Altitude

First, construct a line through a vertex perpendicular to the opposite side.

- Press    for Menu 9:Construction, 1:Perpendicular.
- Select a vertex and the opposite side.
 - In this example, vertex C and side AB were selected (Figure 47).

Note: Either the vertex or the side may be selected first.

Overlay the altitude on the perpendicular line using the segment tool.

- Press    for Menu 6:Points & Lines, 5:Segment.
- Select the vertex and then the intersection of the perpendicular line and the base of the triangle (Figure 48).

Note: Both the perpendicular line and the triangle should blink when selecting the intersection of the perpendicular line and the base of the triangle.

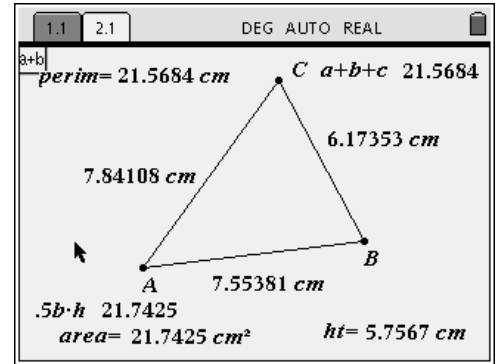


Figure 45

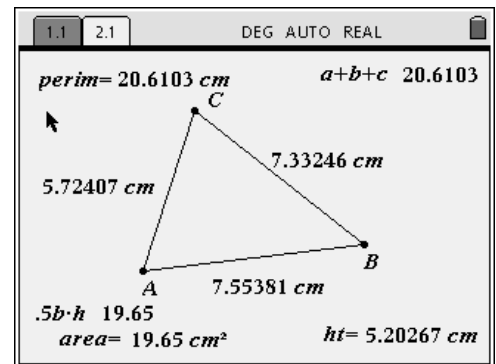


Figure 46

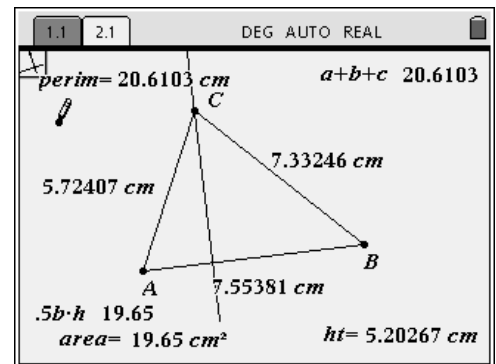


Figure 47

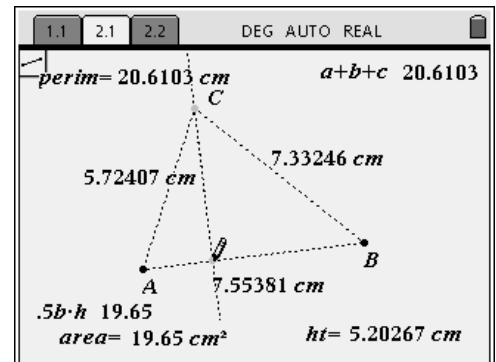


Figure 48

5. Hide the perpendicular line by pressing MENU 1 2 for Menu 1:Tools, 2:Hide/Show.
6. Select the line, press ENTER , and move the cursor away from the line when it appears dashed.
7. Press ESC (Figure 49).

Note: The hidden axes also show when in Hide/Show mode.

8. To Show the line, select the Hide/Show tool again, select the dashed line, and press ENTER .

Storing the Height, Base, and Area Measurements as Variables

Note: To provide additional space, hide the two side measurements.

Select the height measurement.

1. Move the arrow close to the measurement.
2. When the hand appears, press ENTER .
 - The measurement should appear shaded (Figure 50).
3. Press $\text{STO} \rightarrow \text{VAR}$, the var key (Figure 51).
4. Select 1:Store Var by pressing ENTER or 1 .
 - A box with $\text{var} : =$ appears. Var is the default variable name.
5. Type ht to overwrite var .

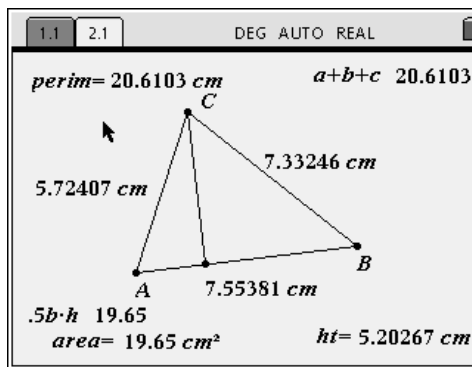


Figure 49

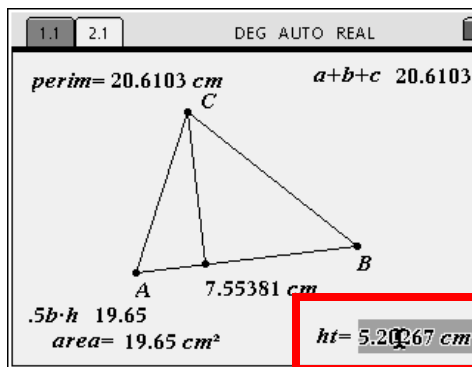


Figure 50

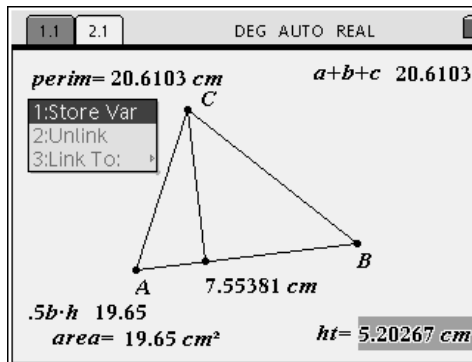


Figure 51

6. Press enter (Figure 52).
- If desired, drag the measurement and variable name to another location on the screen. The variable name **ht** should appear in bold text.

Note: A measurement can also be stored by highlighting the measurement and pressing ctrl sto var . The **var: =** box appears.

7. Press esc before selecting the second measurement.
8. Select the base measurement, and press ctrl sto var .
9. Type the variable name **b** to overwrite **var**.
10. Press enter and then esc (Figure 53).

11. Repeat to assign the variable name **area** to the area value (Figure 54).
12. Hide the original height and area labels, if desired.
13. Hide the altitude and its point of intersection with the base.

Comparing $.5b \cdot ht$ and Area on a Calculator Page

1. Add a new page by pressing home (Figure 55).
- 1:Calculator (Figure 55).

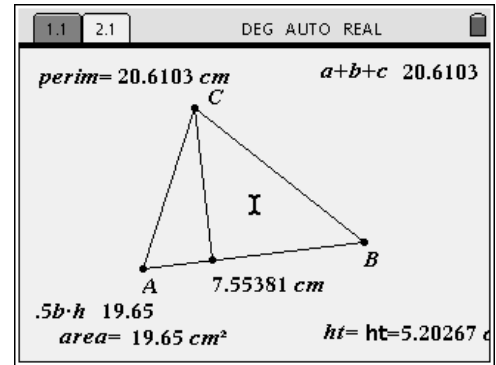


Figure 52

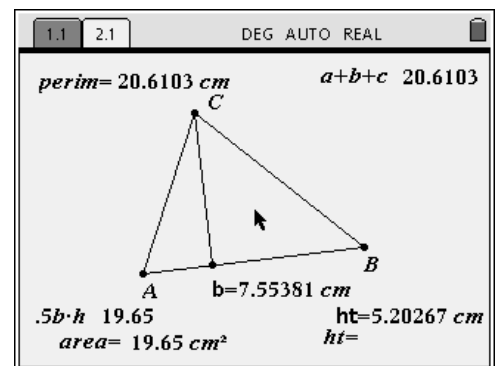


Figure 53

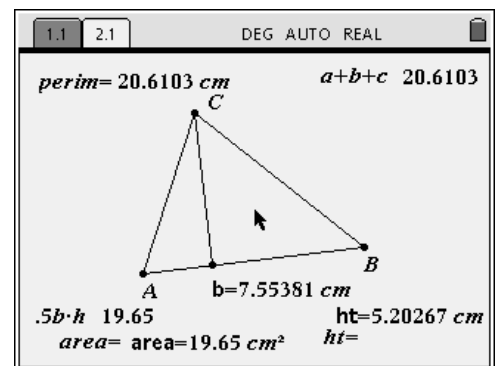


Figure 54

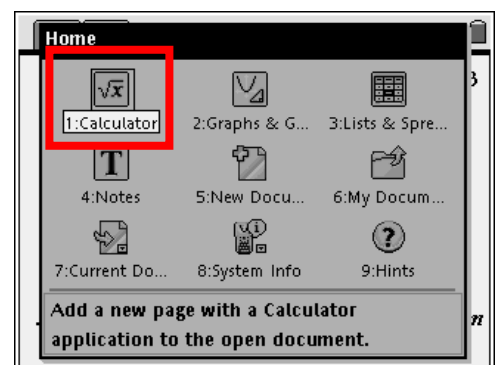


Figure 55

Note: A new page can also be added by pressing ctrl $\left[\right]$ $\left[2 \right]$ $\left[7 \right]$ for 2:Edit, 7:Insert Page.

2. Enter $.5b*ht$, and press enter (Figure 56).

Note: As b and ht are entered, they appear in bold text since they are variable names.

3. Enter $area$, press enter , and compare the values (Figure 57).

Note: If one of the document settings shown at the top of the page is EXACT rather than AUTO, press ctrl enter for an approximate answer.

Note: To change the document setting to AUTO, press $\left[\right]$ $\left[8 \right]$ for System Info. Press $\left[1 \right]$ for Document Settings. Press the tab key until the box to the right of Exact or Approx. is highlighted. Use the NavPad cursor controls to highlight Auto. Press enter to choose Auto and enter again to store the setting. The page should show the word AUTO at the top.

Return to the Graphs & Geometry Page

1. Press ctrl $\left[\right]$ (left of the NavPad) to return to the previous page which was the Graphs & Geometry page.
2. Drag a vertex to change the values on the screen (Figure 58).

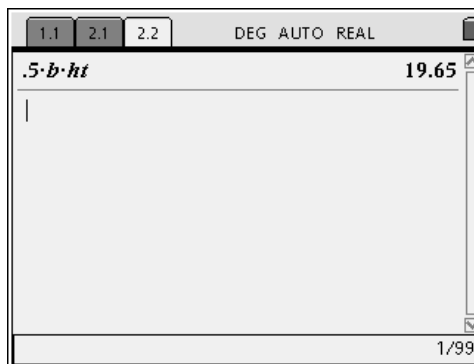


Figure 56

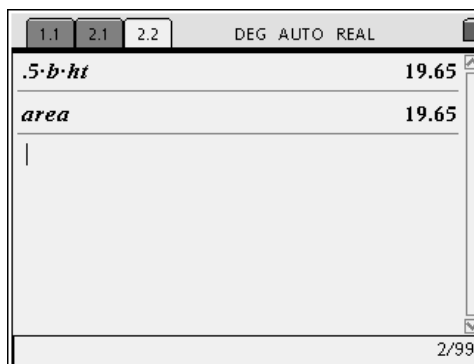


Figure 57

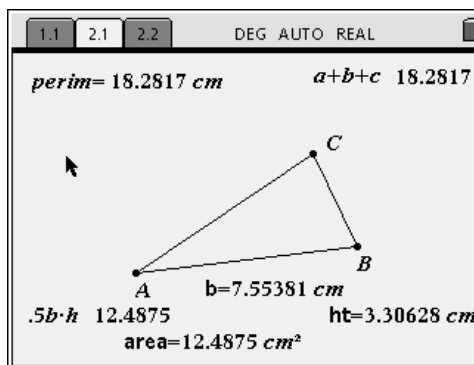


Figure 58

Return to the Calculator Page

1. Press ctrl \rightarrow (right of the NavPad) to return to the Calculator page.
2. Move up using the NavPad to highlight the expression $.5b \cdot ht$ (Figure 59).
3. Press enter to copy the expression to the command line, and press enter again to perform the computation.
4. Move up using the NavPad to highlight the word *area*.
5. Press enter twice. (Figure 60).
6. Compare the values.

Constructing a Line Parallel to Side AB not passing through Point C

1. Press ctrl \leftarrow to return to the Graphs & Geometry page.
 - If necessary, drag the triangle to a lower location on the screen to leave space above the triangle.
2. Show the parallel line constructed on side AB by pressing menu $\left[1\right]$ $\left[2\right]$ for Menu 1:Tools, 2:Hide/Show.
3. Select the line, and press esc to exit the Hide/Show tool.
4. Press menu $\left[9\right]$ $\left[2\right]$ for Menu 9:Construction, 2:Parallel.
5. Select the segment previously constructed on side AB (Figure 61).

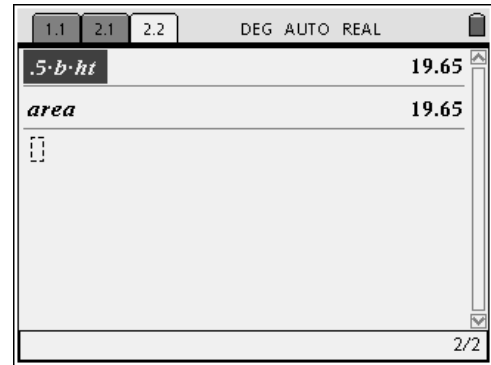


Figure 59

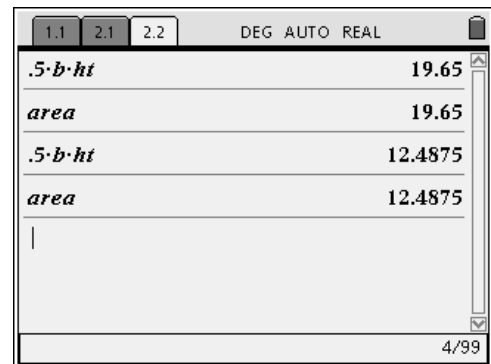


Figure 60

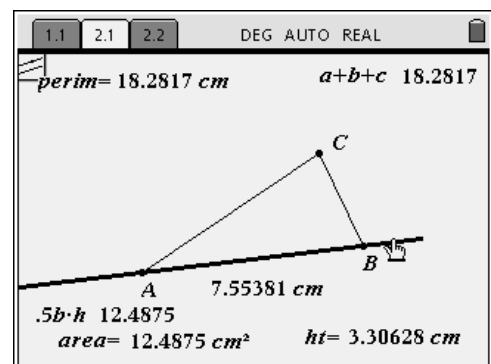





Figure 61

6. Move the parallel line to an open space on the screen, and press  (Figure 62).


Note: Since no specific point on the screen was selected, a point was placed on the screen.

Redefining Point C to be on the Parallel Line

1. Press    for Menu 1:Tools, 8:Redefine.
2. Select point C, and then select the parallel line (Figure 63).

Note: When selecting the parallel line, be sure not to select the defining point of the line.

- The point (vertex C) will be constructed on the line as if it were constructed there initially when the triangle was constructed.

3. Press .
4. Hide the line constructed on side AB.

Drag the point (vertex C) on the parallel line, and make a conjecture about the area of the triangle (Figure 64). Explain the conjecture.

Investigating the Area of a Triangle

Dragging a vertex of the triangle should suggest that the area of a triangle is equal to one-half the base times the height, or $A = .5bh$.

Dragging a vertex of the triangle on a line parallel to the base of the triangle should suggest that the area of the triangle stays the same since there is always a common base and a common height.

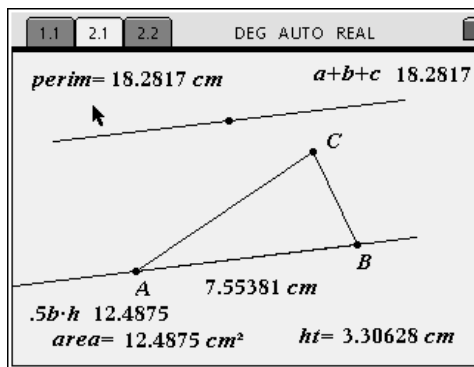


Figure 62

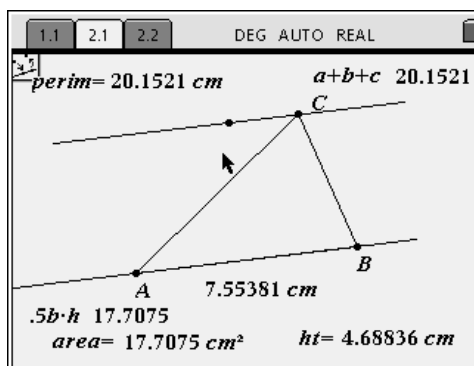


Figure 63

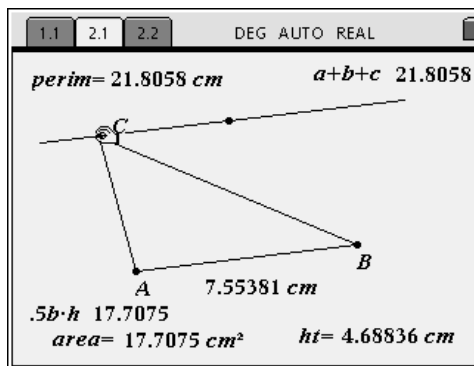


Figure 64

Basic Data Analysis

Materials

- TI-Nspire™ Math and Science Learning Handheld

Overview

This data set is taken from a biology activity and revised to use with the TI-Nspire™ learning handheld.

Introduction



The following table shows the weight/blood volume relationship in humans:

Weight in Pounds	Blood Volume in Pints
60	5
84	7
108	9
132	11
156	13
180	15
204	17
228	19
252	21
276	23
300	25

We will use this data to explore the graphing capabilities of the TI-Nspire™ learning handheld.

Creating a List

In order to enter data and then graph it, you will need to open a New Document.

- Press the Menu key  and  to select New Document. (Figure 1).

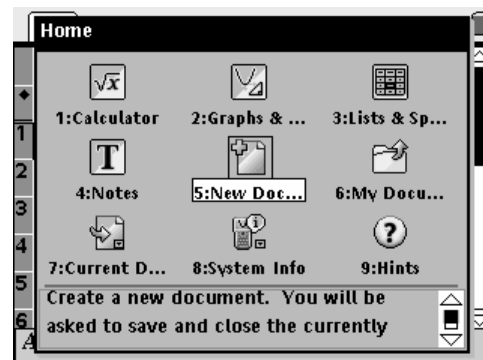


Figure 1

- Press $\boxed{3}$ to choose 3: Add Lists & Spreadsheet (Figures 2 and 3).

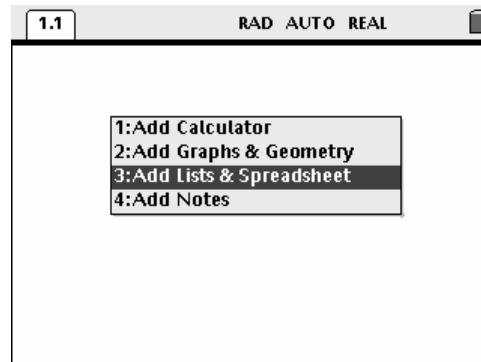


Figure 2

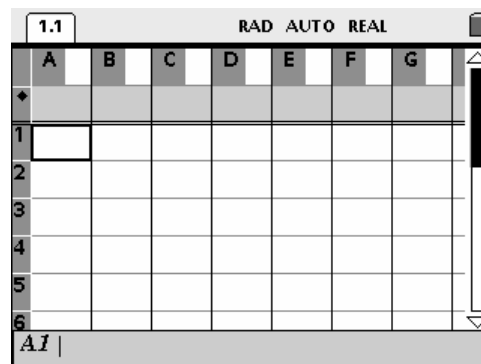


Figure 3

- Enter the weights into column A (Figure 4).
Use the Fill down function for the blood volume.
- Enter 5 and 7 into the first two rows of column B.
- Move up using the NavPad cursor control until 5 is boxed in.
- Press the $\boxed{\text{CAPS} \uparrow}$ key and the down arrow \blacktriangledown at the same time. This will highlight 5 and 7 (Figure 5).

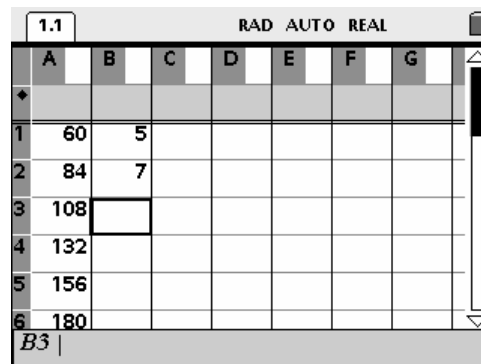


Figure 4

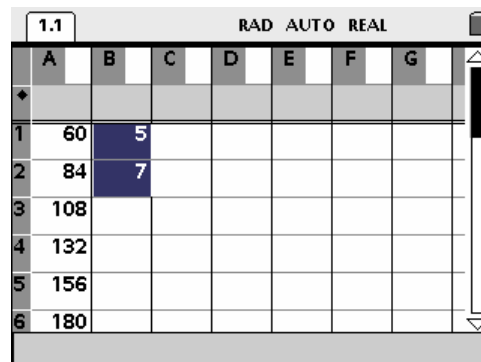





Figure 5

7. Press    to choose Menu
3:Data, 3:Fill Down (Figure 6).

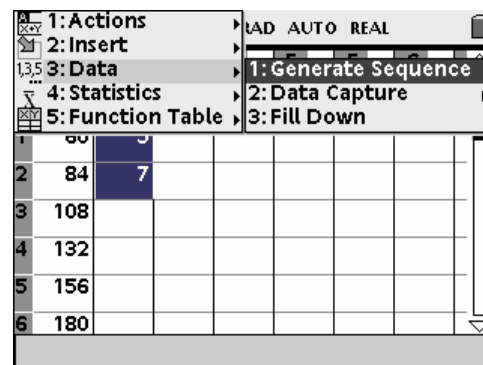



Figure 6

8. Move down using the NavPad until the highlighted border is across from 300.
(Figure 7).

1.1							
A	B	C	D	E	F	G	
7	204						
8	228						
9	252						
10	276						
11	300						

Figure 7

9. Press the  key to fill the column.

1.1							
A	B	C	D	E	F	G	
7	wei						
7	204						
8	228						
9	252						
10	276						
11	300						

Figure 8



11. Do a similar operation for column 'B'
labeling it 'pints' (Figure 9).

1.1							
A	B	C	D	E	F	G	
7	pin						
7	204	17					
8	228	19					
9	252	21					
10	276	23					
11	300	25					




Figure 9



Graphing the Data

In order to graph the data, you will need to add a new page to this problem.

1. Press the  key, and press  to add a Graphs & Geometry page (Figure 10).

2. The default graph screen is shown in Figure 11.

3. Press    to choose Menu 3:Graph Type, 3:Scatter Plot (Figure 12).

4. Press  to open the x-values list, arrow down to 'weight,' and press  to choose weight (Figure 13).

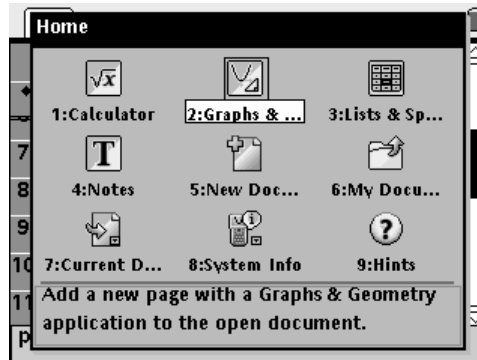


Figure 10

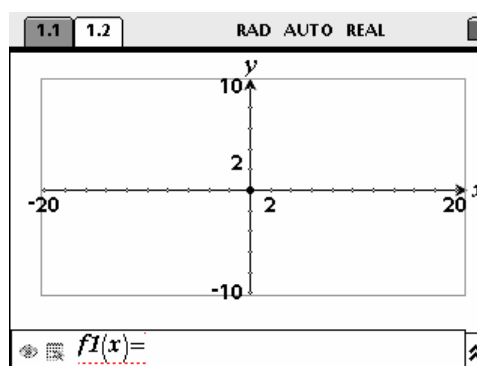


Figure 11

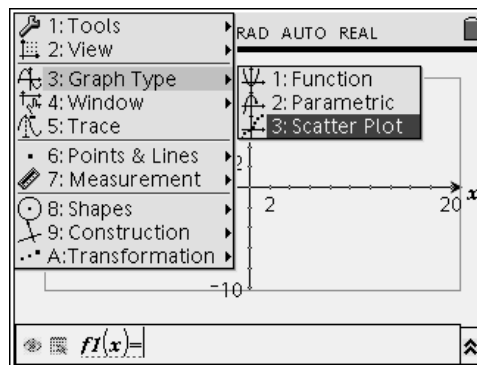


Figure 12

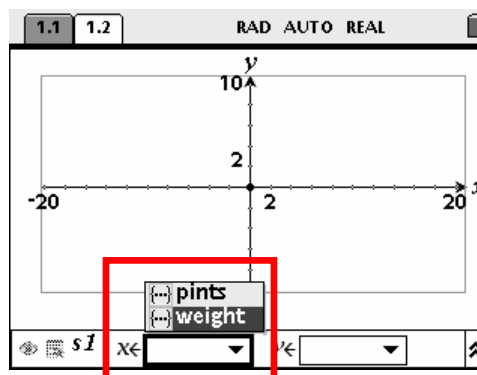


Figure 13

5. Move to the right to highlight the y-values list, and press $\left(\frac{\odot}{*}\right)$.
6. Select 'pints,' and press $\left(\frac{\odot}{*}\right)$. (Figure 14)

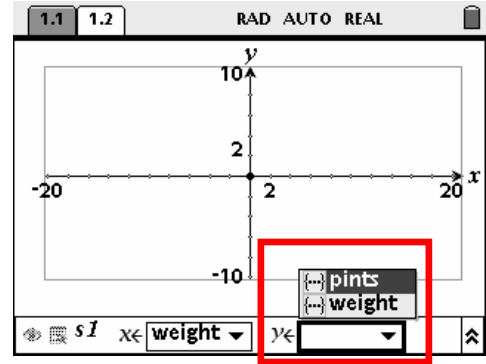


Figure 14

7. Press $\left(\text{menu}\right)$ $\left(4\right)$ $\left(9\right)$ to choose Menu 4:Window, 9:Zoom – Stat (Figures 15 and 16).

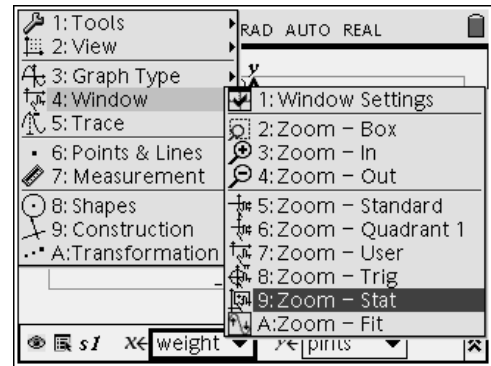


Figure 15

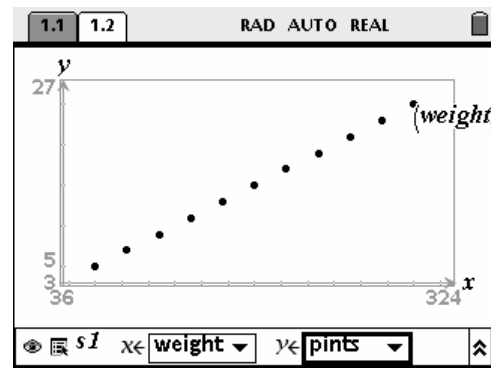


Figure 16

Graphing and Regression Lines

Let's determine the regression equation for the set of data.

1. Press $\left(\text{ctrl}\right)$ $\left(\leftarrow\right)$ (to the left of the NavPad) to return to the spreadsheet. (Figure 17).

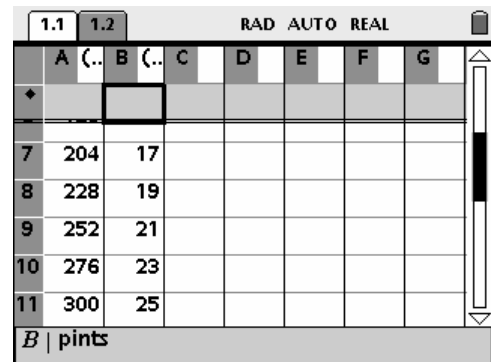


Figure 17

2. Press **menu** **4** **1** to choose Menu 4: Statistics, 1:Stat Calculations, and press **3** to choose 3:Linear Regression (mx+b) (Figures 18 and 19).

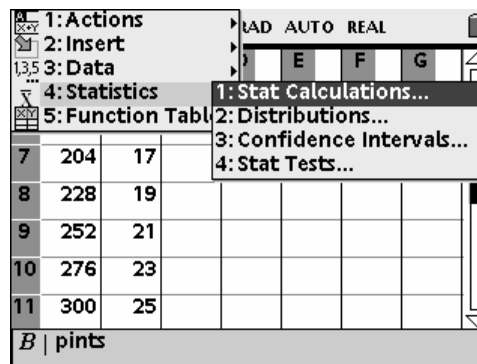


Figure 18

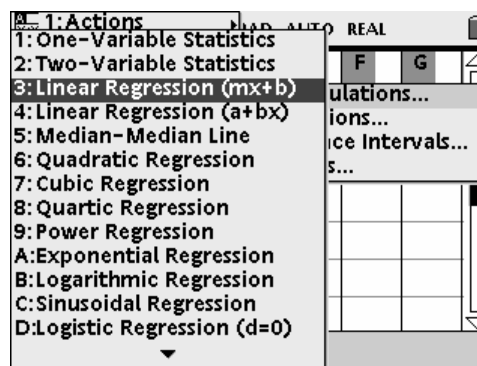


Figure 19

3. The Linear Regression (mx+b) set up box will appear on the screen (Figure 20).



Figure 20

4. Press the down arrow on the NavPad cursor control to choose 'weight,' and press **enter** (Figure 21).

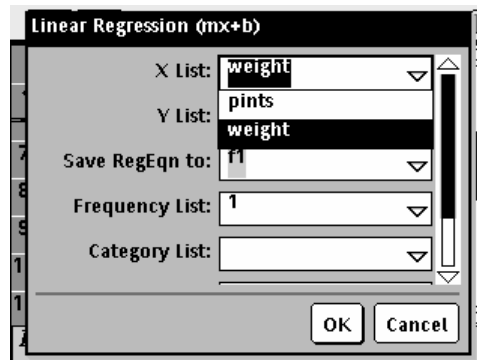


Figure 21

5. Press the **tab** key to change to the Y-List.
Press the down arrow key and choose 'pints'
(Figure 22).

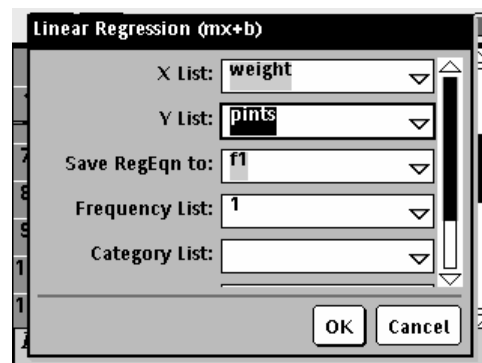


Figure 22

6. Press **tab** until OK is highlighted, and press the **enter** button (Figures 23 and 24)

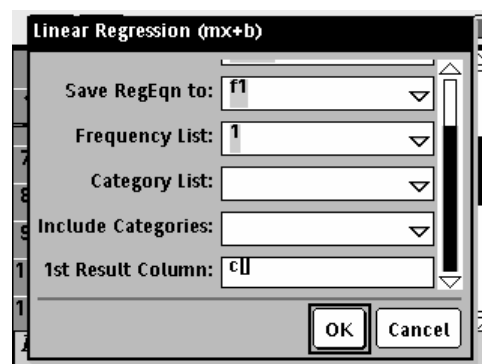


Figure 23

1.1	1.2	RAD AUTO REAL				
A	B	C	D	E	F	G
			=LinR			
1	60	5	Title Lin...			
2	84	7	Re... m*...			
3	108	9	m .08...			
4	132	11	b 1.E...			
5	156	13	r ² 1.			
6	180	15	r 1.			

DI | ="Linear Regression (mx+b)"

Figure 24

7. Press **ctrl** **graph** to return to the Graphs & Geometry page (Figure 25).

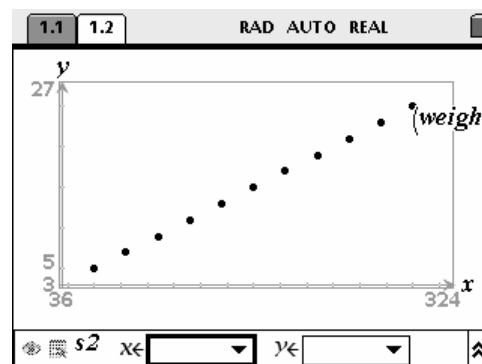


Figure 25

8. Press MENU 3 1 to choose Menu 3:Graph Type, 1:Function (Figures 26 and 27).

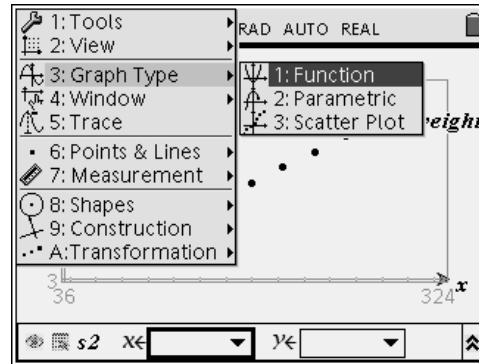


Figure 26

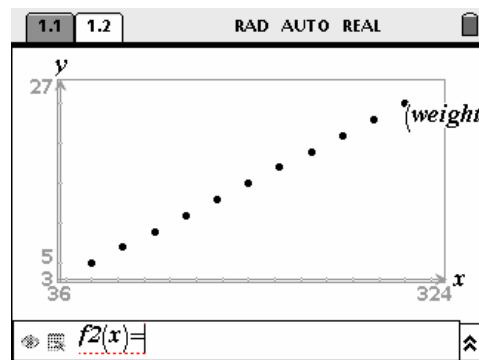


Figure 27

9. Press the \blacktriangle key so that $f1(x)$ appears in the Entry Line, and then press the ENTER key. (Figures 28 and 29).

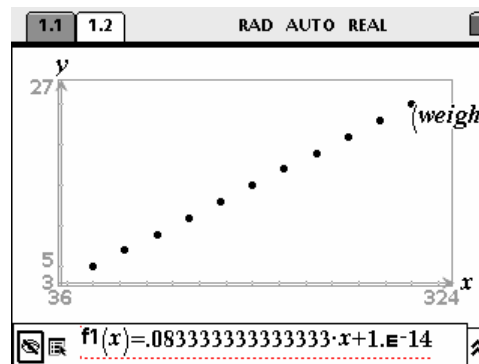


Figure 28

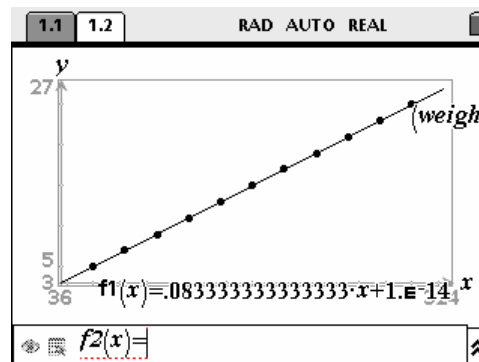


Figure 29

Hiding Equations and Moving Labels

1. Press the **esc** key to move up from the entry line and get an arrow cursor (Figure 30).
2. Press **menu** **1** **2** to choose Menu 1:Tools, 2:Hide/Show (Figure 31).
3. Move the cursor over the equation until the hide cursor appears, and press the **h** key (Figures 32 and 33).

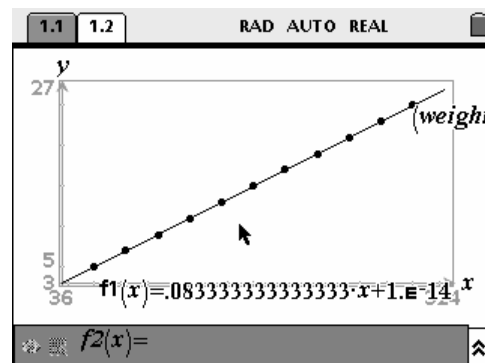


Figure 30

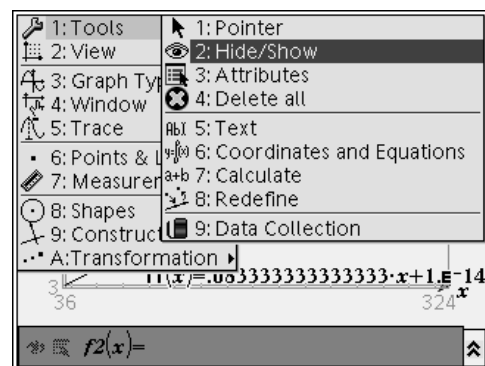


Figure 31

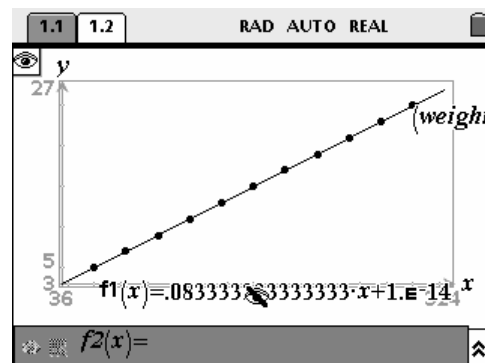


Figure 32

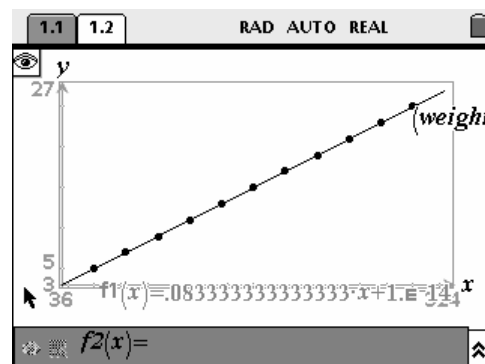


Figure 33

4. Press the **esc** key to exit Hide/Show.
5. Move the cursor over the label 'weight' and press the **ctrl** and **Ⓜ** keys to grab the label on the screen (Figures 34 and 35).
6. While the hand is closed, drag the label to a location where it can be viewed, and press the **Ⓜ** key to release the label and affix it to the new location on the screen (Figure 35).

Explore Various Weights with the Regression Equation

1. Press **menu** **5** to choose Menu 5:Trace (Figure 36).
2. Press the **▲** key on the NavPad to change from the plot to the equation (Figure 37).

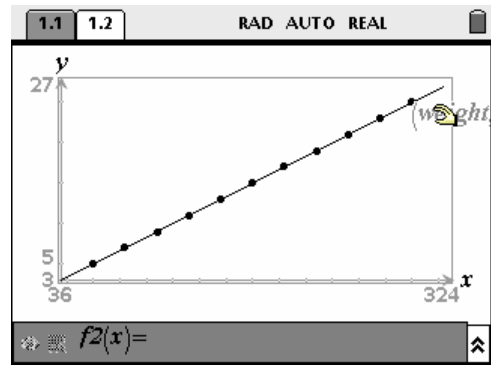


Figure 34

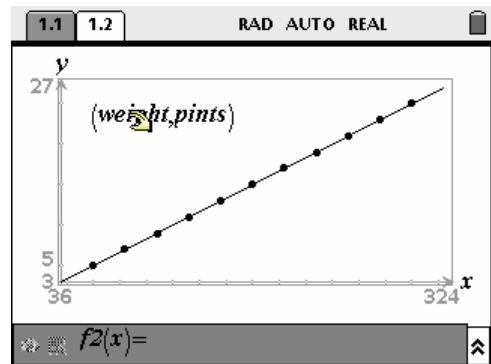


Figure 35

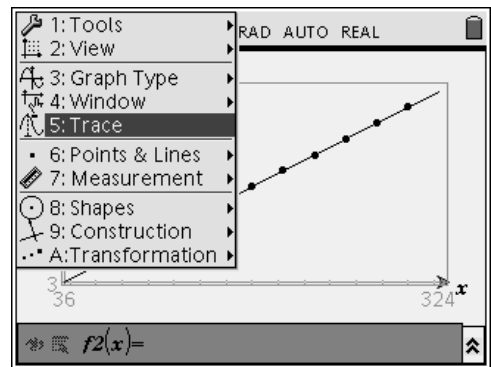


Figure 36

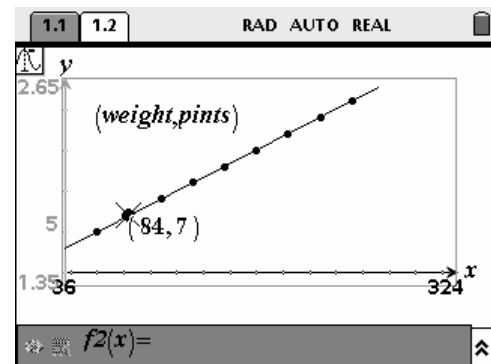


Figure 37

- Press the \blacktriangleleft keys on the NavPad to move left and right on the graph of the regression equation until you reach your weight (Figure 38).
- Type 200, and the value will appear on the screen. (Figure 39).
- After typing the number, press \approx to find the number of pints of blood at 200 pounds.

Other Ways to Analyze the Data

Place a point on the line, and manipulate the x-value to determine y or manipulate the y-value to determine the x.

- Press the esc key to exit trace mode.
- Press menu $\{6\}$ $\{2\}$ to choose Menu 6:Points &Lines, 2:Point On (Figure 40).
- Move the cursor pencil onto the line until an ordered pair appears, and press the enter key to drop the point on the line. (Figure 41).
- Press the esc key to exit the Point On mode.

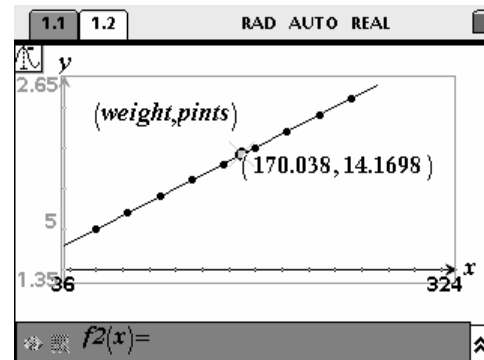


Figure 38

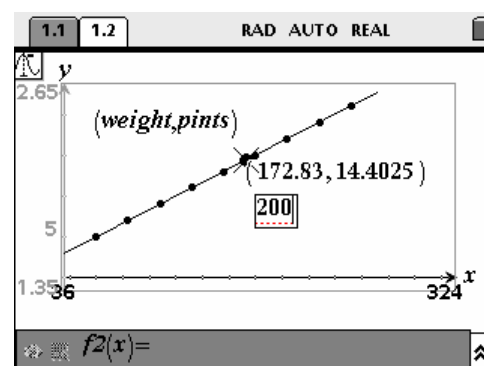


Figure 39

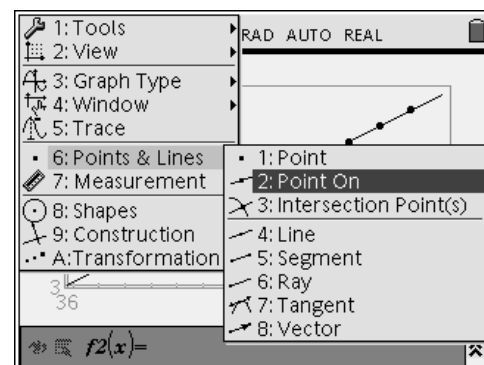


Figure 40

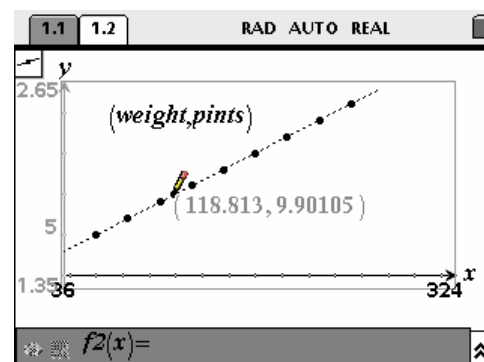


Figure 41

5. Place the cursor over the x -coordinate, and press the $\left(\frac{\text{data}}{\text{stat}}\right)$ key twice (Figure 42).
6. Enter a new value for weight, and press the $\left(\frac{\text{data}}{\text{stat}}\right)$ key to see the number of pints.

Change the pints to determine the weight.

7. Place the cursor over the y -coordinate and press the $\left(\frac{\text{data}}{\text{stat}}\right)$ key twice (Figure 43).

8. Enter the new number of pints (y), and press the $\left(\frac{\text{data}}{\text{stat}}\right)$ key to see the new weight (x) (Figure 44).

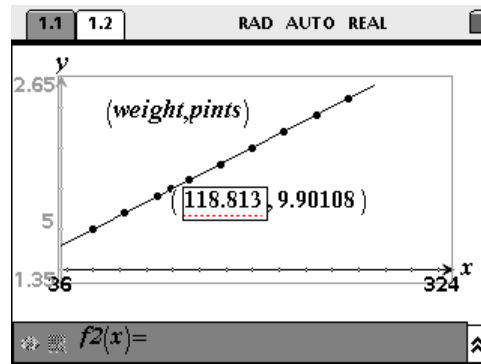


Figure 42

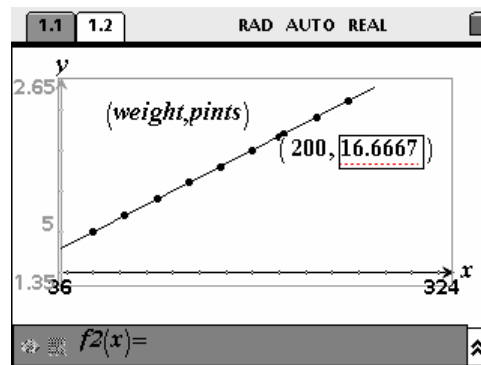


Figure 43

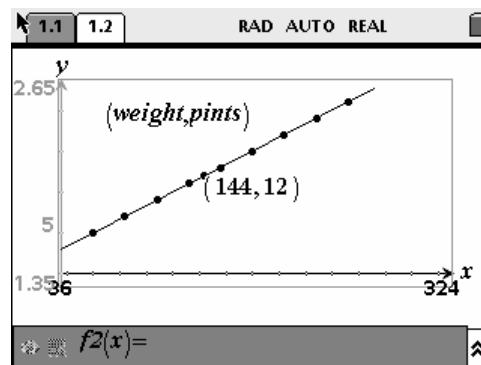


Figure 44

Differences between TI-Nspire™ Handheld and Computer Software

Materials

- TI-Nspire™ Math and Science Learning Handheld
- TI-Nspire™ Computer Software for Math and Science

Overview

This document provides a quick reference guide explaining the minor differences between using the TI-Nspire™ math and science learning handheld versus the TI-Nspire™ computer software for math and science.

Introduction

In designing the TI-Nspire™ math and science learning technology, Texas Instruments has taken into consideration the innate differences between handheld technology and computer software.

When using the TI-Nspire™ handheld technology, users access menus and operations through keys and the multi-directional NavPad cursor control.

Computer software users rely on the computer keyboard and mouse when using the TI-Nspire™ computer software.

These native differences necessitate differences in navigation and access between TI-Nspire™ handhelds and computer software. By taking advantage of these native differences, TI is able to provide improved usability for both the handheld and computer software user while maintaining a parallel learning and teaching experience.

This document provides an overview of the differences users will find in tasks and menus:

Task	Handheld	Desktop Software
Browse and organize documents	My Documents	MS Windows® file system
Organize problems and pages within a document	Page Sorter (must be accessed)	Page Sorter (optional side pane)
Access applications, documents, system information, and hints/help	Home Screen	Desktop menus: <ul style="list-style-type: none"> • File • Insert
Manage/edit documents, manage pages, and select applications	Page Tools Menu	Desktop menus: <ul style="list-style-type: none"> • File • Edit • Insert
Change views	Fonts: <ul style="list-style-type: none"> • small • medium • large 	Views: <ul style="list-style-type: none"> • Standard/Default • Presentation • TI-Nspire™ Handheld • Keypad
Move between applications on a handheld	CTRL+Tab Key	n/a
Move between the page sorter, application menu bar, and page workspace in the computer software	n/a	CTRL+Tab Key
Navigate the screen	NavPad cursor controls Tab and Arrow Keys	Mouse Keyboard Tab and Arrow Keys

Except for the HOME screen and Page Tools menu of the TI-Nspire™ handheld, the underlying structure of the menus in each application is the same for both the TI-Nspire™ handheld and computer software. However, users will find differences between the handhelds and the computer software in the ways in which they access menus.

Tasks

Task: Browse and organize documents

TI-Nspire™ handheld:

The handheld has its own document browser called My Documents.

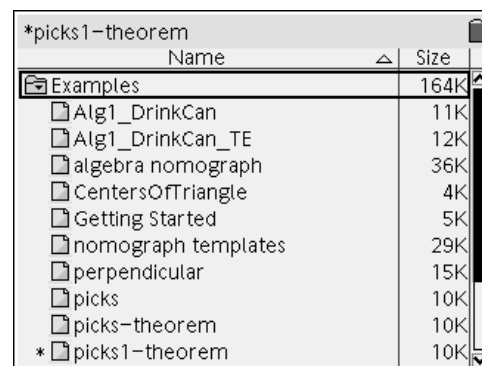
With this document browser, users can organize all the documents stored on a handheld, creating multiple folders and storing documents in the order that works best for them (Figure 1).

To go to My Documents:

1. From the Home Screen, select 6: My Documents.

TI-Nspire™ computer software:

The computer software uses the File menu to open TI-Nspire™ documents and the Microsoft Windows® file system to organize documents and folders (Figure 2).



Name	Size
Examples	164K
Alg1_DrinkCan	11K
Alg1_DrinkCan_TE	12K
algebra nomograph	36K
CentersOfTriangle	4K
Getting Started	5K
nomograph templates	29K
perpendicular	15K
picks	10K
picks-theorem	10K
* picks1-theorem	10K

Figure 1

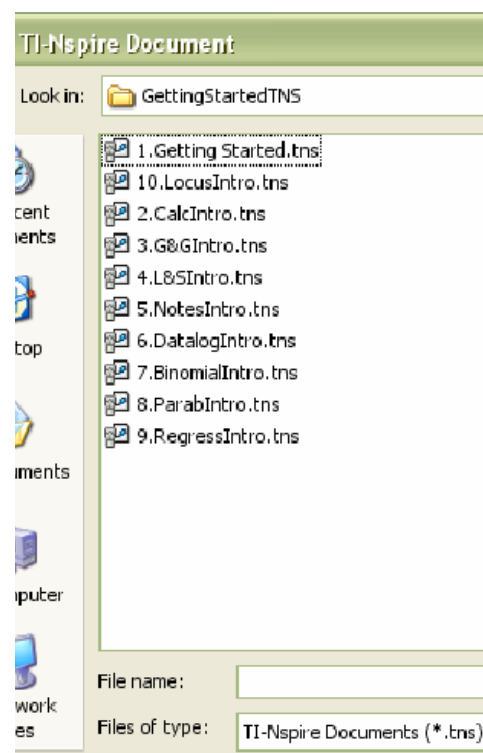




Figure 2

Task: Organize problems and pages within a document

TI-Nspire™ handhelds:

The handheld Page Sorter allows users to see and sort all the pages in a document in one place (Figure 3).

To go to the Page Sorter:

1. Press  and  (Up Arrow key on the NavPad) from within a document.

TI-Nspire™ computer software:


The computer software has a vertical page sorter pane on the left, visible at all times except when in Presentation View (See Figure 4).

Task: Access applications, documents, system information, and hints/help

TI-Nspire™ handheld:

Only the handheld has the Home Screen that allows handheld users to quickly launch an application, create new documents, open existing documents, return to an open document, access system information, and refer to handheld hints (Figure 5).

To go to the Home Screen:

1. Press  (Home) from within a document.

TI-Nspire™ computer software:

The computer software uses three menus to execute actions similar to those found on the handheld’s Home Screen. They are: Insert, File, and Help.

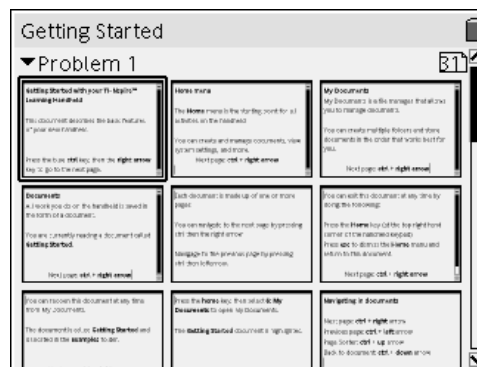


Figure 3

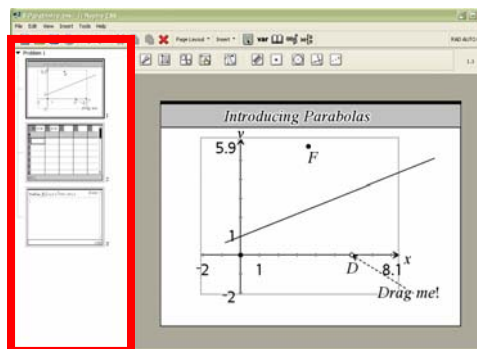


Figure 4

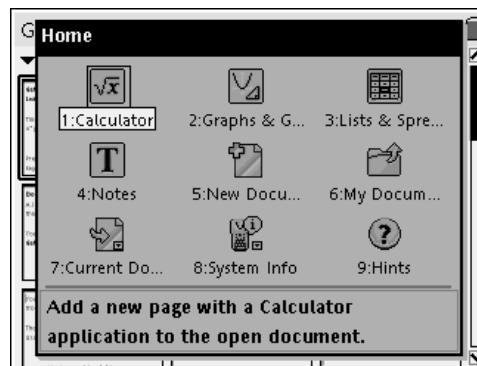


Figure 5

Here is a detailed comparison of the TI-Nspire™ handheld's Home Screen and the similar functionality in the TI-Nspire™ computer software.



Handheld	Desktop
1:Calculator	Insert>Calculator
2:Graphs & Geometry	Insert>Graphs & Geometry
3:Lists & Spreadsheets	Insert>Lists & Spreadsheets
4:Notes	Insert>Notes
5:New Document	File>New Document
6:My Docs	n/a, or File>Open Document...
7:Current Document	n/a
1:Full Page View	n/a – just click on page
2:Page Sorter	On the left-hand side when not in Presentation View
8:System Info	
1:Document Settings...	File>Document Settings...
2:System Settings...	n/a
3:Handheld Status	n/a
4>About	Help>About
9:Hints	Help>TI-Nspire Help (different content)

Task: Manage/edit documents, manage pages, and select applications

TI-Nspire™ handheld:

Only the handheld has the Page Tools menu (Figure 6).

To go to the Page Tools menu:

1. Press  and  (Home) from within a document.

TI-Nspire™ computer software:

The computer software uses the File, Edit, and Insert menus to execute actions similar to those found on the handheld's Page Tools menu.

The next page shows a detailed comparison of the TI-Nspire™ handheld's Page Tools menu and the similar functionality in the TI-Nspire™ computer software.

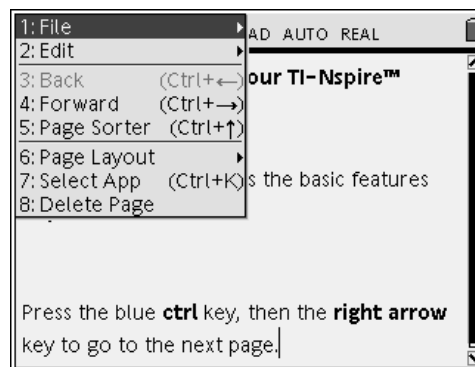


Figure 6

Handheld	Software
1:File	File
1:New Document	File>New Document
2:My Documents	File>Open Documents...
3:Save Document	File>Save Document
4:Save As...	File>Save As...
5:Send	n/a
6:Document Settings...	File>Document Settings...
2:Edit	Edit
1:Undo	Edit>Undo
2:Redo	Edit>Redo
3:Cut	Edit>Cut
4:Copy	Edit>Copy
5:Paste	Edit>Paste
6>Delete	Edit>Delete
7:Insert Page	Insert>Page
8:Insert Problem	Insert>Problem
3:Back	n/a, use page sorter or Ctrl+right/left arrow (same as on handheld)
4:Forward	n/a
5:Page Sorter	On the left-hand side, when not in Presentation View
6:Page Layout	View>Layout
1:Custom Split	n/a – click and drag separator bar to get custom layout
2:Select Layout	View>Layout>
3:Swap Applications	Edit>Swap Applications
7:Select App	Edit>Select Application
8>Delete Page	n/a – select page in page sorter pane and Delete (using keyboard, toolbar, menu)

Task: Change views*TI-Nspire™ handhelds:*

On the handheld, users can change the font size using the System Settings dialog box (Figure 7).

To change the font size:

1. From the Home Screen, select 8: System Information and 2: System Settings.

TI-Nspire™ computer software:

In the computer software, the default view is sized in proportion to the computer window setting. The page sorter pane is on the left, and the font is set to regular size (Figure 8).

Changing views:

1. From the View menu, users can toggle into and out of the following views:
 - TI-Nspire™ Handheld View – increases image size (text, objects, icons), but everything is in proportion to the screen size/ratio of the handheld (i.e., 320x240) (Figure 9).
 - Presentation View – removes page sorter pane and increases image size (text, objects, icons) while page is still sized in proportion to the screen resolution setting. (How does this relate to the 320x240 mentioned in the previous bullet?)
 - Keypad – displays the handheld keypad for users to input information (Figure 9).

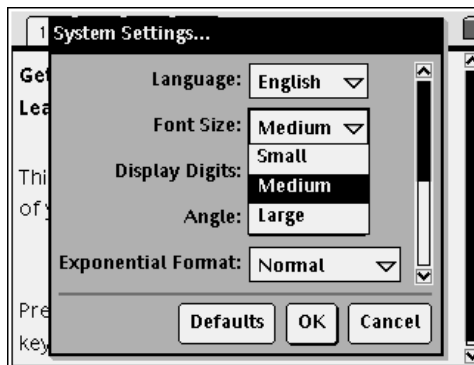


Figure 7

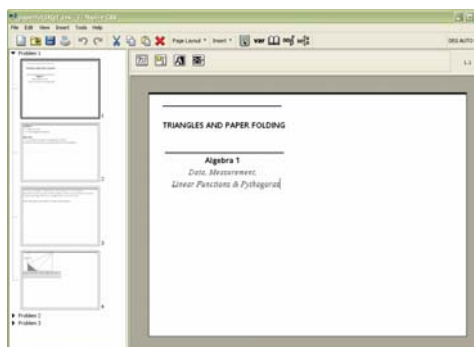


Figure 8

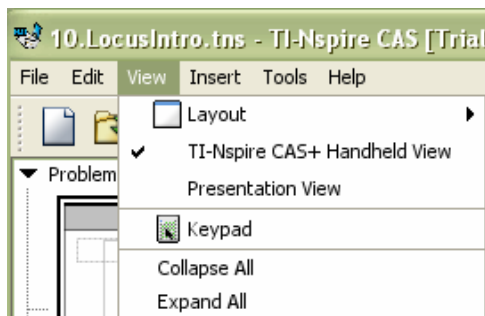




Figure 9

Task: Move between applications on a page

TI-Nspire™ handhelds:

On the handheld, users can move between applications on a page by pressing  .






TI-Nspire™ computer software:

In the computer software, users click on the applications in a split layout to move between applications.

Task: Navigate the screen

TI-Nspire™ handheld:

Users use the multi-directional NavPad cursor controls to navigate around the handheld screen.

- Graphs & Geometry: Users can move around the screen using the NavPad.
- Other applications: Users use a combination of the  and     Arrow keys on the NavPad (north, south, east and west directions) to move around the screen.



TI-Nspire™ computer software:

Use either a mouse or the keyboard's Tab and arrow keys to navigate within the computer software.

Menus

Except for the Home Screen and Page Tools menu on the TI-Nspire™ handheld, the underlying structure of the menus in each application is the same for both the TI-Nspire™ handheld and computer software.

However, there are some differences in the way in which users access the menus due to the innate differences between using handhelds and software.

Menus	Handheld	Computer Software
Application menus	Select by pressing number or alpha keys (e.g., 1:Pointer) or by using arrow keys.	Select by clicking on an icon.
Context menus	Use  and  keys for access.	Right-click mouse button for access.

Lesson Plan Notes for Factoring

Materials

- TI-Nspire™ CAS Math and Science Learning Handheld
- *Factoring* student worksheet
- Access to the Internet for the Extension

Overview

In this activity, students will use inductive methods to discover rules for factoring. The activity consists of three parts and an extension. In Part 1, students discover how to factor the difference of squares. In Parts 2 and 3, they discover how to factor the difference and sum of two cubes. The extension discusses applications of factoring in cryptography and computer security.

NCTM Standards

All students should....

Grades 9-12 Algebra Standards:

1. understand the meaning of equivalent forms of expressions, equations, inequalities, and relations
2. use symbolic algebra to represent and explain mathematical relationships
3. judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology
4. draw reasonable conclusions about a situation being modeled

Prerequisite Knowledge:



Students should be able to multiply two binomials and multiply a binomial by a trinomial.

Pedagogy:

This activity uses the TI-Nspire™ CAS to allow students to discover rules and patterns. When students discover a mathematical concept, they have a sense of ownership that promotes interest and retention. Their experience is closer to that of a real mathematician.

As students look at the results of several examples on their calculators, they will begin to notice patterns. When asked to describe the patterns, they may use somewhat vague descriptions and nonmathematical terms. Encourage them to use concise symbolic descriptions. Help them appreciate the power of algebraic representations.

Activity Notes:

1. In Part 1, step 10, you should note that **factor** ($x^2 - a^2, x$) produces a result that looks different from **factor** ($x^2 - a^2, a$).
 - **factor** ($x^2 - a^2, x$) factors the expression with respect to x .
 - **factor** ($x^2 - a^2, a$) factors the expression with respect to a .
 - This will also be an issue in Part 1, step 11.
2. In the Extension of Part 1, you might want to note the difference between **factor** ($x^2 - 5, x$) and **factor** ($x^2 - 5$).
 - When giving the command “factor the expression with respect to x ,” the **factor** command factors over the real numbers.
 - Without the command “factor the expression with respect to x ,” the **factor** command factors over the integers.
3. In Parts 2 and 3, students will use the exponent key  to write expressions such as $x^3 - 1$.
 - They must press the right arrow  on the NavPad cursor control to exit the exponent mode or the rest of the expression they type will also appear in the exponent.

Factoring

Student Worksheet

Name _____

Class _____

In this activity, you are going to use Computer Algebra to discover several factoring patterns.

When instructed to use the factor command, use your TI-Nspire™ CAS handheld.

When asked to make a prediction, you should use only paper and pencil.

After you have written your prediction, you may then use your calculator to check your work. Look for patterns in both your calculator and paper and pencil results.

Part 1 – Factoring the difference of two squares

1. Open a new document on the TI-Nspire-CAS handheld. Add a Calculator application page. Press **(menu)**, and then select 4:Algebra to open the Algebra menu. See Figure 1.

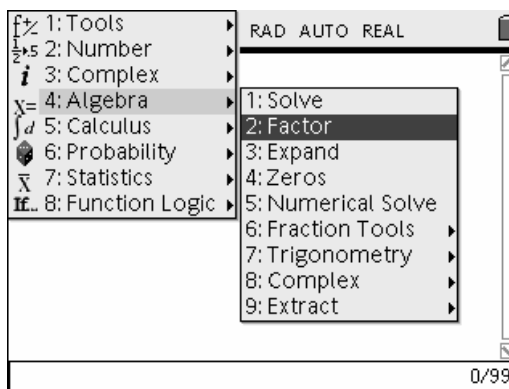


Figure 1

2. Select 2:Factor to copy the **factor** command to the Calculator work area.
3. Next, complete the command: **factor** (x^2-1, x) by pressing the following keys:

(X) **($\sqrt{\quad}$)** **(\ominus)** **(1)** **(,)** **(X)** **(\approx)**. See Figure 2.

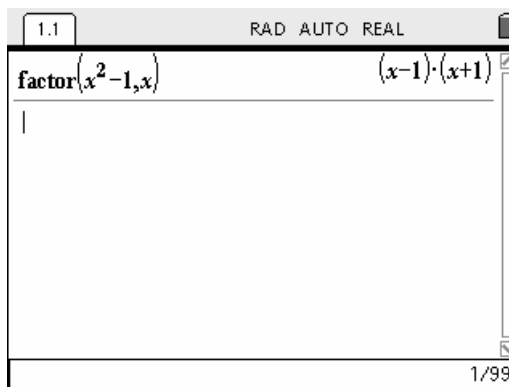


Figure 2

- The result indicates that $x^2 - 1 = (x - 1)(x + 1)$.

- Check this result by multiplying the two terms on the right side of the equation.
- Now use the **factor** command to factor the following expressions. Write the result below.

$x^2 - 4$ _____

$x^2 - 9$ _____

- Predict the results of factoring the following expressions. Write your predictions in the spaces provided, and then check your prediction with the **factor** command.

$x^2 - 16$ prediction: _____

factor command result: _____

$x^2 - 49$ prediction: _____

factor command result: _____

- Describe the pattern you have noticed.
- Predict the result of factoring the following expression. Write your prediction in the space provided, and then check your prediction with the **factor** command.

$x^2 - a^2$ prediction: _____

factor command result: _____

- Was your prediction correct? What is the pattern for factoring the difference of two squares?

- Factor the following expressions with paper and pencil only, and then check your results with your TI-Nspire™ CAS handheld.

$b^2 - a^2$ _____

$4x^2 - 9y^2$ _____

$16p^2 - 25q^2$ _____

Extension: Predict the result of factoring $x^2 - 5$, and then check with your calculator.

Part 2 – Factoring the difference of two cubes

10. Open a new document on your TI-Nspire™ CAS handheld.
11. Add a Calculator application page.
12. Enter the following command: $\text{factor}(x^3 - 1, x)$.
13. In order to enter the expression $x^3 - 1$, press X $\sqrt[n]{x}$ 3 \blacktriangleright $-$ 1 See Figure 3.

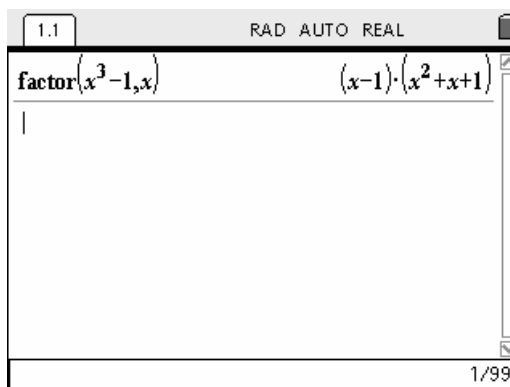


Figure 3

Note: The \blacktriangleright key is the right arrow key on the NavPad cursor control. Pressing this key exits the exponent mode. Otherwise, the rest of the expression is included in the exponent.

- The result indicates that $x^3 - 1 = (x - 1)(x^2 + x + 1)$.

14. Multiply the two terms on the right side of this expression to verify the result.
15. Use the **factor** command to factor the following expressions. Write the results in the space provided. Look for patterns.

$$x^3 - 8 \quad \underline{\hspace{10em}}$$

$$x^3 - 27 \quad \underline{\hspace{10em}}$$

$$x^3 - 64 \quad \underline{\hspace{10em}}$$

16. Predict the result of factoring the following expressions. Write your predictions and then compare with the results of the **factor** command.

$$x^3 - 125 \quad \text{prediction:} \quad \underline{\hspace{10em}}$$

$$\text{factor command result:} \quad \underline{\hspace{10em}}$$

$$x^3 - 343 \quad \text{prediction:} \quad \underline{\hspace{10em}}$$

$$\text{factor command result:} \quad \underline{\hspace{10em}}$$

17. Can you describe the pattern in these examples?

18. Predict the result of factoring $x^3 - a^3$ then check your prediction.

prediction: _____

factor command result: _____

19. Factor the following expressions with paper and pencil, and then check your results with your TI-Nspire™ CAS handheld.

$$8x^3 - y^3 \quad \underline{\hspace{10em}}$$

$$p^3 - 27q^3 \quad \underline{\hspace{10em}}$$

Part 3 – Factoring the sum of two cubes

20. Use the **factor** command to factor the following expressions. Record the results.

$$x^3 + 1 \quad \underline{\hspace{10em}}$$

$$x^3 + 8 \quad \underline{\hspace{10em}}$$

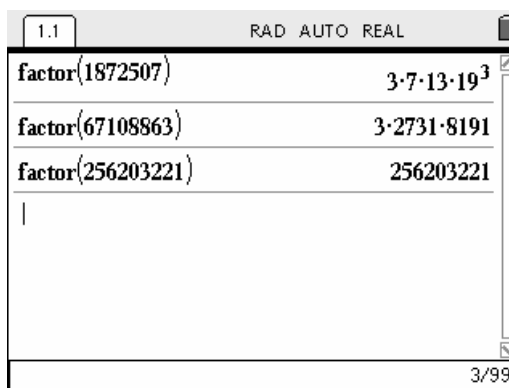
$$x^3 + 27 \quad \underline{\hspace{10em}}$$

21. Based on these results, can you write a conjecture about factoring the sum of two cubes?

$$x^3 + a^3 \quad \underline{\hspace{10em}}$$

Extension: Numeric Factoring

The **factor** command can also be used to factor composite numbers. See Figure 4.



Command	Result
<code>factor(1872507)</code>	$3 \cdot 7 \cdot 13 \cdot 19^3$
<code>factor(67108863)</code>	$3 \cdot 2731 \cdot 8191$
<code>factor(256203221)</code>	256203221

Figure 4

Note: The third **factor** in Figure 4 did not produce factors because 256,203,221 is prime.

Although the TI-Nspire™ CAS handheld **factor** command is powerful, there is no known formula to factor all composite numbers. This fact is the basis for many computer security methods. One computer security company offers substantial rewards to anyone who can factor certain large composite numbers. You can see these challenge numbers at

<http://www.rsa.com/rsalabs/node.asp?id=2093#RSA640>

RSA Challenge number 704: Factor this number, and win \$30,000

740375634795617128280467960974295731425931888892312890849362326389727650340
282662768919964196251178439958943305021275853701189680982867331732731089309
00552505116877063299072396380786710086096962537934650563796359

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Relationship between Circumference and Diameter of a Circle

Concepts

- Circumference of a circle
- Diameter of a circle
- Scatter plots
- Equations of line
- Slope
- Area
- Simple Quadratics
- Transformations

Overview

Students will explore the ratio of the circumference to the diameter of a circle. After investigating many circles and looking at relationships, students will use Lists and Spreadsheets along with a scatter plot to explore the algebraic relationship between diameter and circumference. Part 2 will investigate the relationship of radius to Area.

Materials

- TI-Nspire™ Computer Software for Math and Science

Background

Depending on the background of the students, this activity could be done with measuring radius, diameter, and circumference of many circular objects prior to using the technology.

Another possibility is to count and compare the number of candies in the radius, diameter, circumference, and area of circles.

NCTM Standards

All students should....

Grades 6-8 Geometry Standards:

1. precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties
2. understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects

Grades 9-12 Geometry Standards:

1. analyze properties and determine attributes of two- and three-dimensional objects
2. draw and construct representations of two- and three-dimensional geometric objects using a variety of tools

3. use geometric models to gain insights into, and answer questions in, other areas of mathematics

Grades 6-8 Algebra Standards:

1. represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules
2. relate and compare different forms of representation for a relationship
3. model and solve contextualized problems using various representations, such as graphs, tables, and equations

Grades 9-12 Algebra Standards:

1. generalize patterns using explicitly defined and recursively defined functions
2. identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships

Teacher Preparation

The teacher may opt to lead the students through the investigation using the prepared TI-Nspire document file. The student activity sheet is a companion to the TI-Nspire document file.

Alternatively the teacher could have the students create the document on the handheld or in a lab.

This document shows the general outline for creating the TI-Nspire™ document file on the computer. Similar steps would be used if creating the document on the handheld.

Please note that the page and problem numbers in this document do not match exactly the numbers in the pre-made TI-Nspire document.

Construction

1. Under the File menu, choose New Document, and use the mouse to select a Graphs & Geometry application (Figure 1).

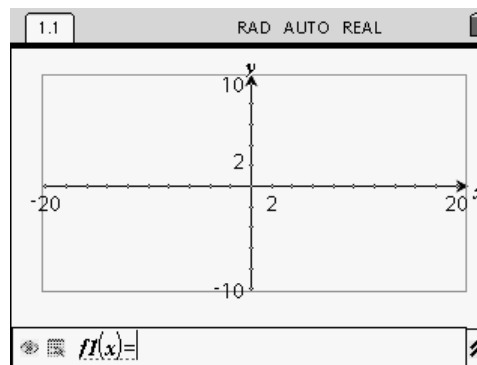






Figure 1


Under View on the Toolbar, select TI-Nspire Handheld View.

2. From the View menu , select Hide the Entry Line , and Hide Axis .
3. Construct a segment by selecting Segment from the Points and Lines menu (Figure 2).
 - Press Enter or click for one endpoint of the segment, move to another location and click a second time to complete the segment.
 - Labeling points is not really necessary for this construction but can be done if desired.
 - Label points immediately after pressing Enter or click.

Note: Selection in the software is normally done with the mouse click.

4. Construct a circle by selecting Circle from the Shapes  icon. Click on one endpoint of the segment to place the center of the circle at this endpoint. Then move and click on the segment to attach the radius point to the segment. See Figure 3.

Measurement and Variables

1. Measure the circumference and the radius by selecting Length from Measurement menu .
2. Move the cursor over the circle, and when circle is selected (it will be blinking), left-click for the circumference. Move the measurement to the top left of the screen. Press Enter or Click.

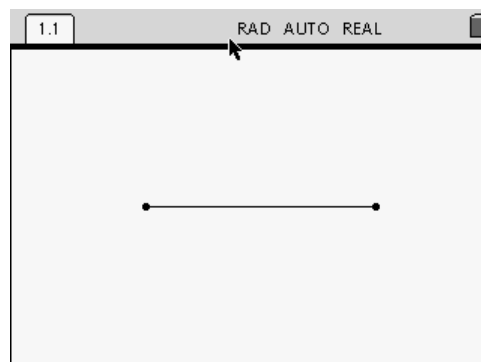


Figure 2

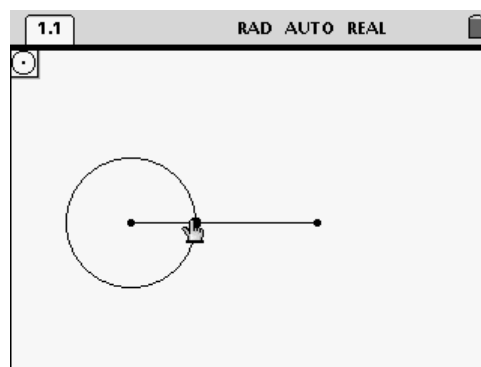




Figure 3

3. With the measurement tool still active, move the cursor to the center point, click, move the cursor to other radius point, and click again.
4. Again, drag the radius value to the top left corner (Figure 4) and press Enter or Click. Press Esc to exit the Measurement tool.
5. To label and make each measurement a variable to be used later, select the measurement to be labeled, and select VAR on the top menu bar.
6. After selecting STORE VAR, in place of the word VAR, type `c i r c u m` for the circumference. Press Enter.
7. Press **(esc)** to deselect the circumference label and repeat for the radius measurement, labeling it `r a d i u s` (Figure 5).
 - Each measurement should now become bold.
8. Select Text from the Tools  menu, move the mouse to an empty location on the screen, and press Enter.
9. At the text insertion point, type `2*r`, and press Enter (Figure 6).
10. Next select Calculate from the Tools  menu. Select the text just entered (`2*r`), and then the radius measurement.
11. Move this measurement closer to the `2*r` text and press Enter or Click.
12. Store this measurement as variable `d i a m`, as done earlier (Figure 7).

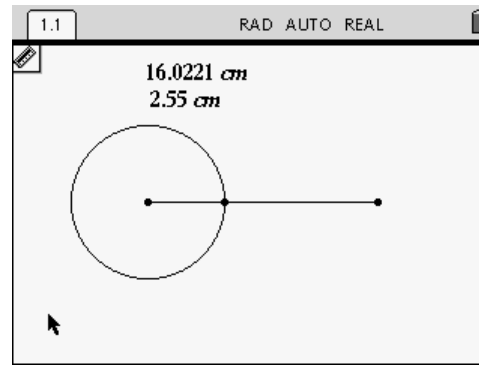


Figure 4

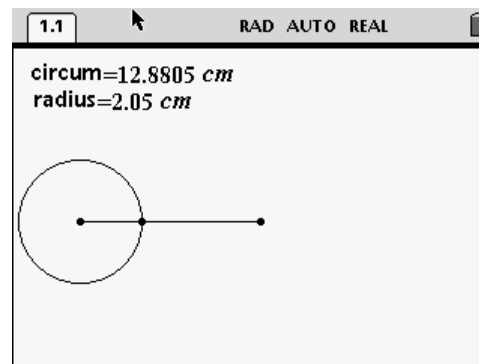


Figure 5

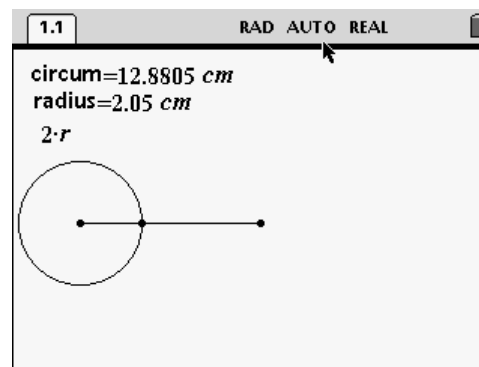


Figure 6

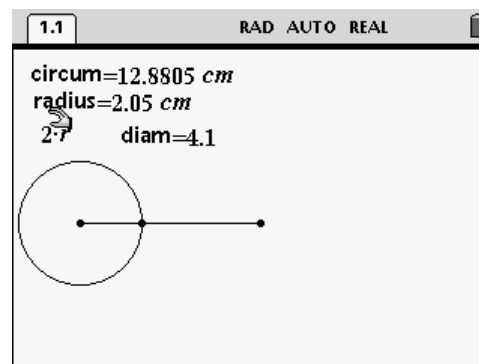



Figure 7

13. To compute the ratio of circumference to diameter, open the Text box and type c/d (Figure 8).
 - Other letters may be used—there is nothing necessarily unique about c and d .
14. Select Calculate from the Tools  icon.
15. First, select the expression c/d , then the values associated with `circum` and `diam`.
16. Move the new measurement close to the c/d text added earlier, as shown in Figure 9.
17. Drag the radius point back and forth along the segment to see which measurements change and which remain the same (See Figure 10).
 - Dragging is accomplished by clicking on the point, holding the left mouse button down and moving.

Lists & Spreadsheet

To store the data, a Lists & Spreadsheet application will be used to save the data as the radius changes.

1. First, split the screen by pressing Page Layout and moving the selection down to the second (vertical split) layout.
2. Add a Lists & Spreadsheet Application to the right side of the page.
3. Place the cursor in the white space to the right of A in column A to name the list.

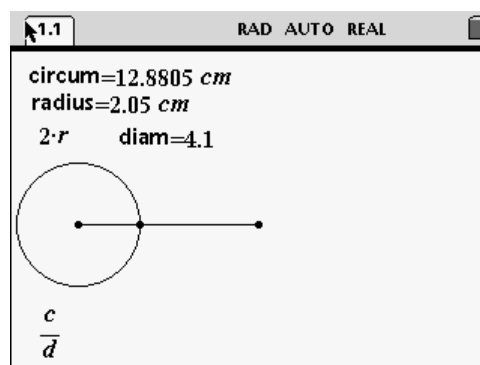


Figure 8

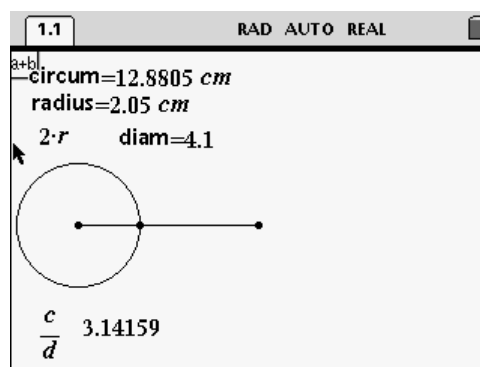


Figure 9

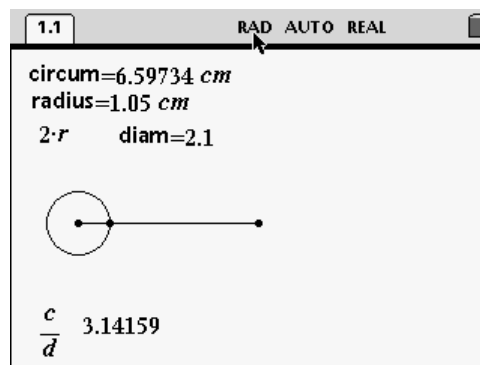


Figure 10

4. Enter d .
5. Move to the white space in column B, and enter c (Figure 11).
6. Move the cursor to the formula cell just below the column name.
7. Select Data Capture > Manual Data Capture from the Data menu.
8. Overstrike VAR by pressing the Var (computer icon) key in the menu bar. Select Link to:, and select d . Press Enter.
9. Move to the formula cell just below column B, and repeat the process, selecting c .

 - When Enter is pressed the first row is populated by the current values of the variables. (Figure 12).

10. In Column C, in the formula cell, enter a formula by following these steps:
 - Press the = key, and then VAR and then Link to:.
 - Select c , then press /.
 - Select VAR and then Link to:, and select d .
 - Press Enter (See Figure 13).
11. To collect more data, click the mouse in the Graphs & Geometry work area.
12. Drag the radius point. To collect data, hold down Ctrl and then press the [Period] key. Repeat this several more times to collect additional data. See Figure 14.

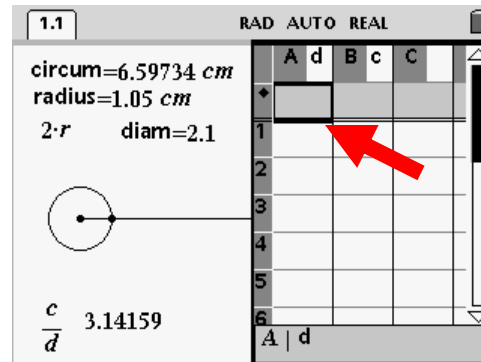


Figure 11

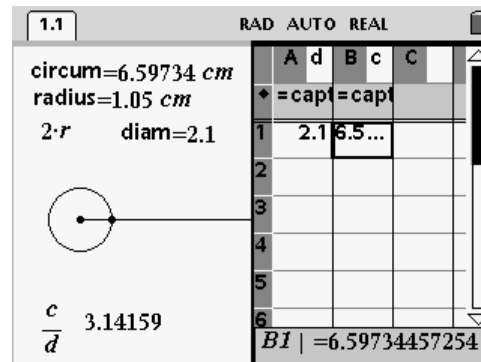


Figure 12

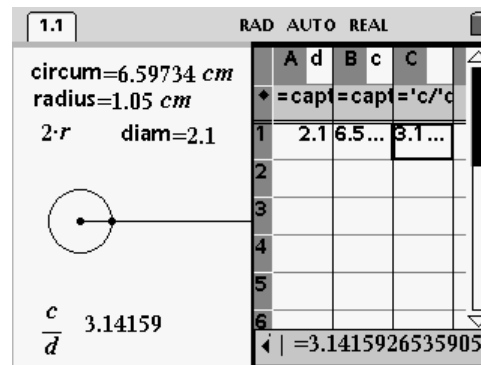


Figure 13

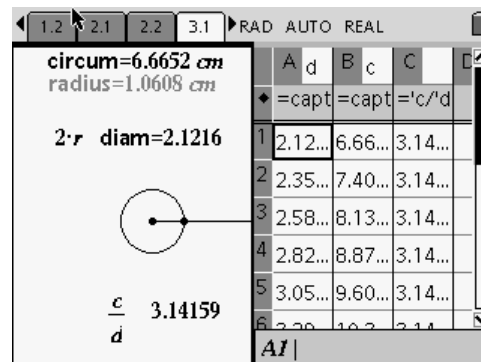


Figure 14

Graphical Representation

- To create a scatter plot of the diameter and circumference data, first insert a new page by selecting Insert > Graphs & Geometry from the menu bar.
- From the Graph Type icon, select Scatter Plot, and press Enter.
- While the x -entry is highlighted, press Enter to display the lists, select the list d , and press Enter.
- Press Tab to move to the y -entry and repeat for the list c (See Figure 15).
 - A scatter plot will be graphed immediately.
- Select Zoom Stat from the window icon to see all the data (Figure 16).

*What type of curve is the relationship of c to d ?
What is true about the plotted points? In which quadrants do the plotted points lie? Why?*

- To fit a line to the data, select Line from the Points and Lines menu.
- Using the mouse, click on one point of the data set, and then click another point.
- From the Measurement menu, select Slope, move to the line, and press Enter on your keyboard.
- Find the equation of the line by selecting "Coordinates and Equations" from the Tools menu (Figure 17).
 - What ratio does this slope represent?

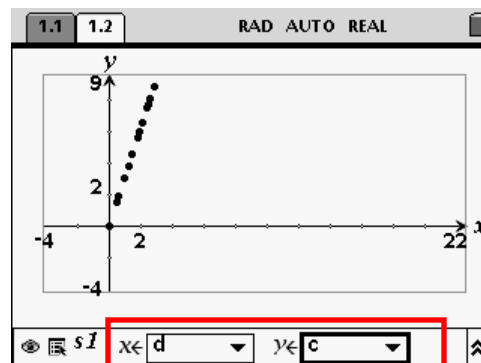


Figure 15

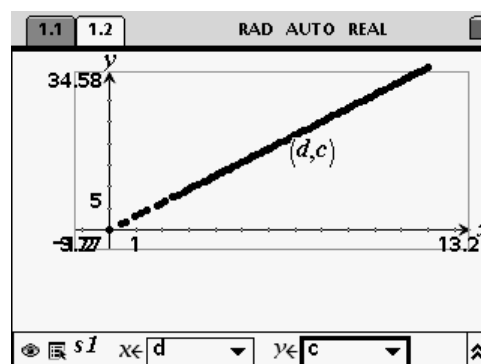


Figure 16

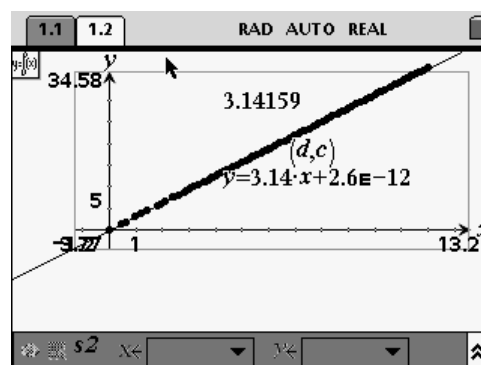



Figure 17

Part 2: Radius and Area

1. Mouse click on the Graphs & Geometry side of page 1.1. Press Ctrl K.
2. Next press Ctrl C to copy the Graphs & Geometry page.
3. Select Insert Problem from the menu bar and click on the new page.
4. Press Ctrl V to paste the Graphs & Geometry page (Figure 18).
5. Choose Hide/Show from the Tools  icon, and hide all objects except the radius measurement (Figure 19).
6. From the Measurement menu, choose Area. Move the cursor to the circle and select it. Drag the measurement to the left corner.
7. Press Esc. Select the measurement by pressing Enter, click on VAR, choose Store VAR, and type Area. Press Enter.
 - The word Area should become bold.
8. With the mouse, move the radius point along the segment to see the radius and area change (Figure 20).

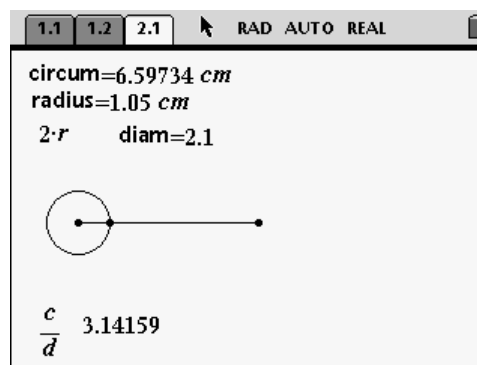


Figure 18

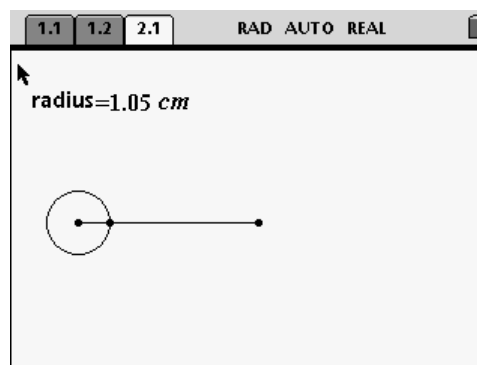


Figure 19

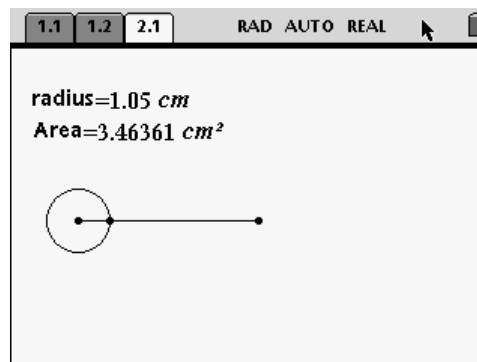


Figure 20

Lists & Spreadsheet

To store the data, a Lists & Spreadsheet application will be used to save the data as the radius changes.

1. First, split the screen by pressing Page Layout and moving the selection down to the second (vertical split) layout.
2. Add the Lists & Spreadsheet application.
3. Place the cursor in the white space in column A to name the list (Figure 21).
4. Enter r .
5. Move to the white space in column B, and enter a .
6. Move the cursor to the formula cell just below the column name.
7. Select Data Capture > Automated Data Capture from the Data icon.
8. Overstrike the VAR by clicking on Var, selecting Link to:, selecting $radius$, and pressing Enter.
9. Move to the formula cell just below column B, and repeat the process, selecting $area$.
 - When Enter is pressed the first row is populated by the current values of the variables (Figure 22).
10. Click the mouse in the Graphs & Geometry work area. Drag the radius point. (Figure 23).

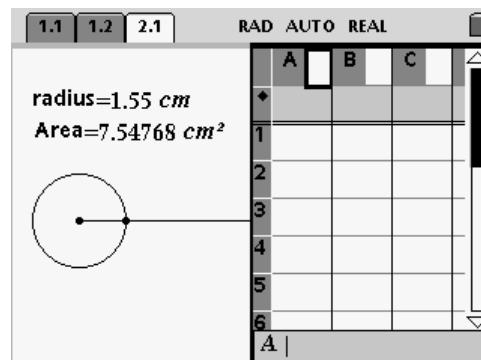


Figure 21

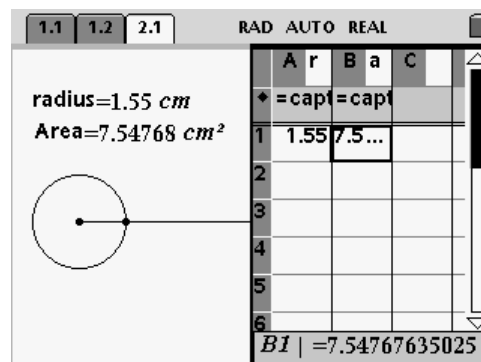


Figure 22

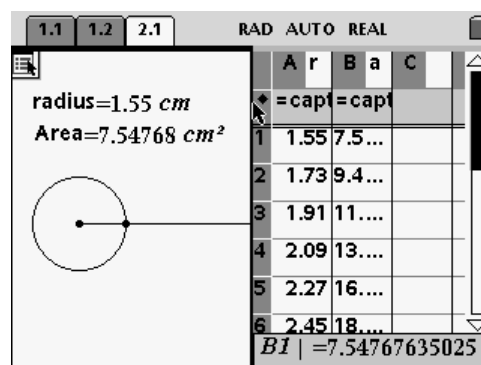


Figure 23

Graphical Representation

1. To see the graphical representation, insert a new page by pressing Insert > Graphs & Geometry from the menu bar.
2. From the Graph Type menu, select Scatter Plot, and press Enter.
3. While the x -entry is highlighted, press Enter to display the lists, select the list r , and press Enter.
4. Press Tab to move to the y -entry and repeat to choose list a (Figure 24).
5. Select Zoom Stat from the Window menu to display all the data (Figure 25).
 - What type of curve does this appear to be?
6. From the Graph Type menu, select Function and enter the function $f1(x) = x^2$. Press Enter (Figure 26).
7. Press Esc to move the cursor back to the Graphs & Geometry screen.
8. Transform the function by grabbing the curve with the mouse and moving the curve.
 - If the four-directional cursor appears, the curve will translate, and if the two-directional cursor appears, the curve will stretch.

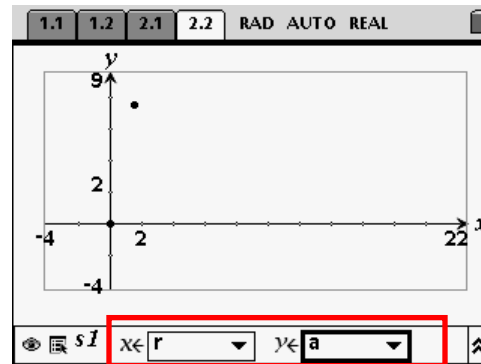


Figure 24

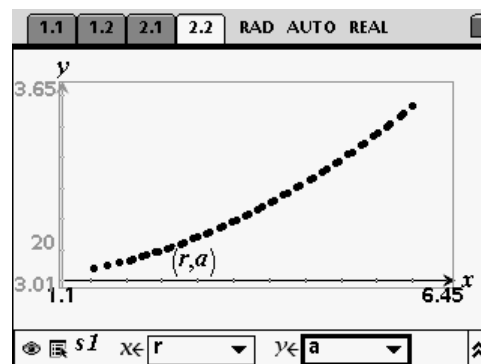


Figure 25

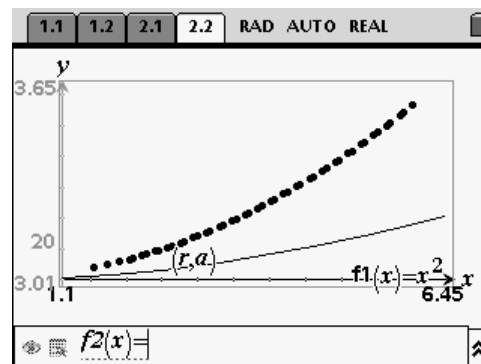


Figure 26

9. Manipulate the curve to get the best fit (Figure 27).
- What does the value of the vertical stretch of the function represent?

Extension possibilities

1. Students can look at other relationships of radius, diameter, circumference, and area. Try creating scatter plots of these relationships.
2. Students can be shown the technique of “animation.”

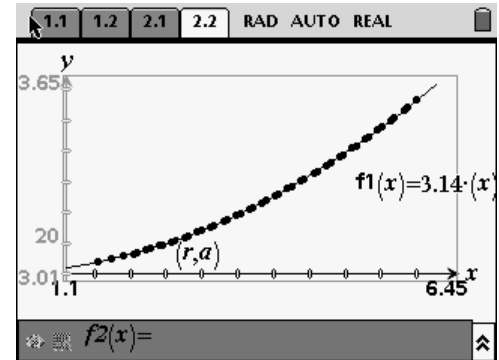


Figure 27



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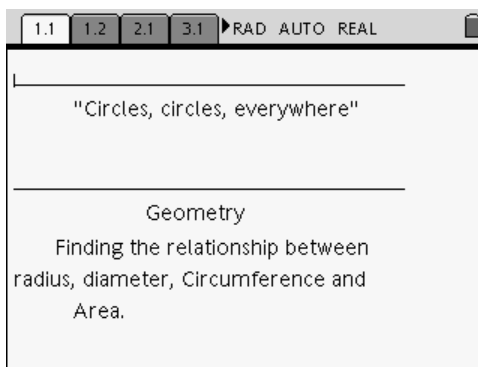
Relationship between
Circumference and
Diameter of a Circle
Student Worksheet




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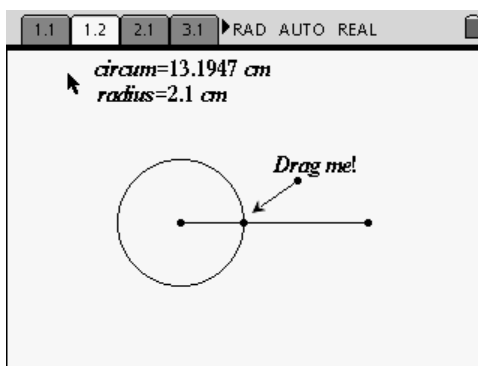
Class _____

Problem 1: 'Circle, circles, everywhere'





1. Open up the **D2_1DiaCir.tns** file on your TI-Nspire™ handheld by pressing . Select 6: My Documents, scroll to the file, and press . Figure 1 shows what the **D1_6DiaCir.tns** file looks like when it is open on your TI-Nspire™ handheld.

**Figure 1**

- On page 1.2 of Problem 1 (press  ) , you will find a circle with a measured radius and circumference (Figure 2).
2. Drag the radius point to see the values change.
 - To drag, move to the point, press and hold the  key until the “hand” closes, and use the NavPad cursor controls to move the point around the screen.

**Figure 2****Measurement**

We will now determine the diameter. First we need to enter some text.

3. Press    for Menu 1:Tools, 5:Text.
4. Move the pointer to an empty location, and press .

5. At the text insertion point, type $2 \cdot r$, and press [enter] (Figure 3).

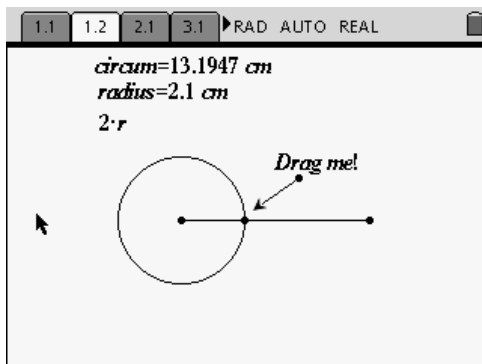


Figure 3

6. Next, press [menu] [1] [7] for Menu 1: Tools, 7:Calculate, move the cursor to select the text just entered ($2 \cdot r$), and then select the radius measurement
7. Move this calculation closer to the $2 \cdot r$ text on the screen (Figure 4).

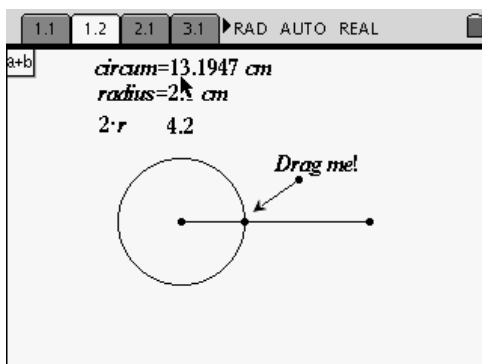


Figure 4

8. Once again, drag the radius point to see values change.

Ratio of Circumference to Diameter

9. To explore the ratio of circumference and diameter, first select the text tool and enter the text c / d on the screen.
10. Press [menu] [1] [7] to select 7:Calculate from the 1:Tools menu.
11. First, select the expression c / d , then the circumference measurement, and finally the diameter measurement. Move the calculation close to the c / d text (Figure 5).

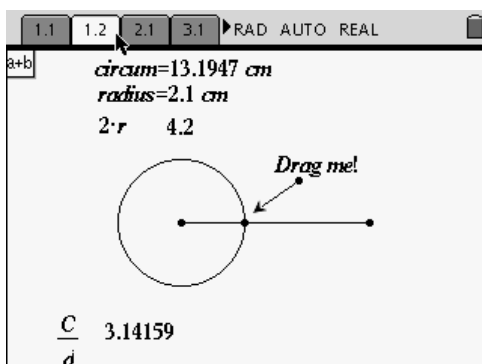


Figure 5

12. Drag the radius point back and forth along the segment to see which measurements change and which remain the same (as in Figure 6).

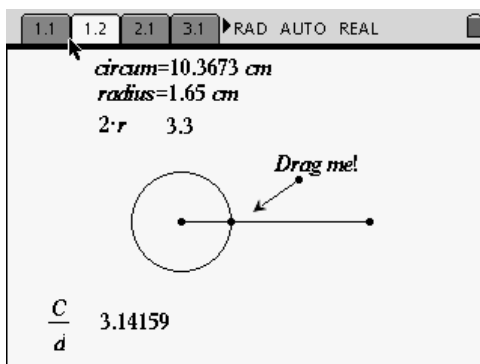


Figure 6

13. Answer question 1 at the end of this worksheet, and then proceed to problem 2.

Problem 2: Lists & Spreadsheet

A Lists & Spreadsheet application will be used to save the data as the radius point moves. In the Lists and Spreadsheet application, there are 3 lists: One tracks the diameter (list d), another tracks the circumference (list c), and the third the circumference/diameter relationship. One set of data is given in Figure 7 below.

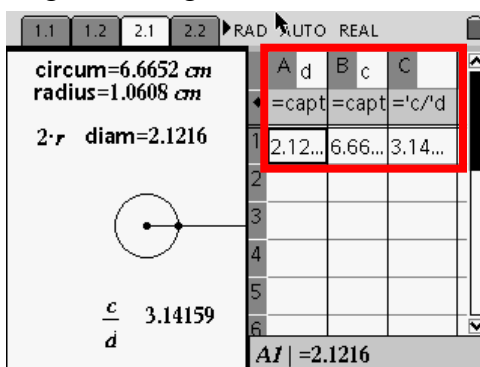


Figure 7

14. To collect data, press **ctrl** **tab** until you are in the Graphs & Geometry work area.
15. Drag the radius to a new position. Press **ctrl** **.** to capture the data and record it in the Lists and Spreadsheet work area. Repeat with five different radii values.
16. Answer questions 2 and 3 at the end of this worksheet.

Graphical Representation

17. To see the graphical representation, **ctrl**  to advance to the next page. A scatter plot has been set up to show the data (See Figure 8).

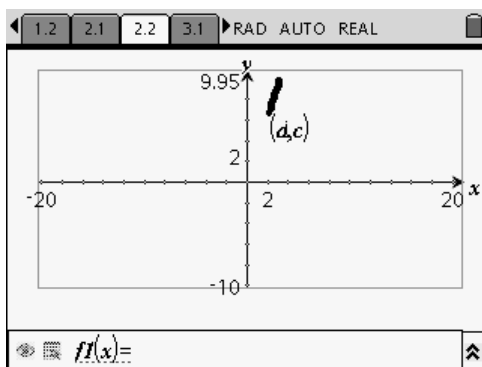


Figure 8

18. Press **menu** **4** **9** for Menu 4:Window, 9:Zoom-Stat to see a more appropriate window (See Figure 9).

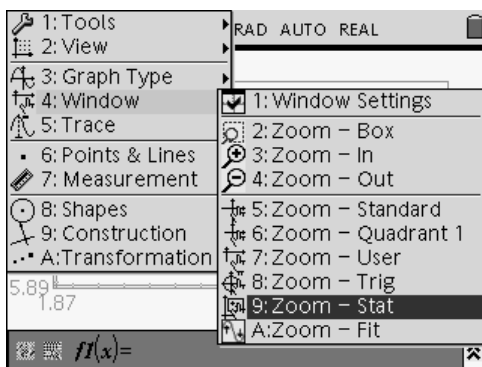


Figure 9

19. Answer questions 4 to 6 at the end of this worksheet.
 20. To fit a line to the data, press Menu, 6:Points & Lines menu, 4:Line (See Figure 10).

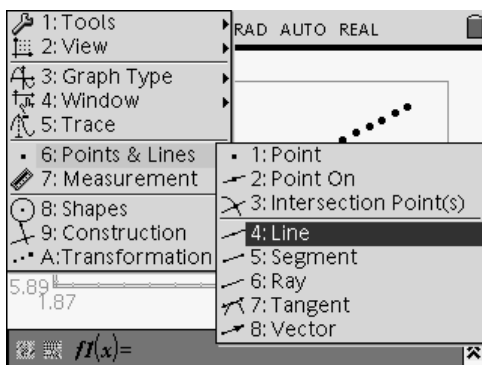


Figure 10

21. Move to one data point and press \approx (enter), and then press \approx (enter) again on another point.
22. Press Menu, 7: Measurement, 3:Slope, then move to the line, and press \approx (enter).
23. Find the equation of the line by selecting Menu, 1:Tools, 6:Coordinates and Equations.

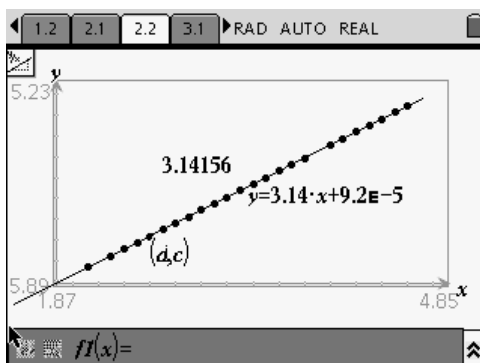


Figure 11

24. Answer question 7, and then proceed to Problem 3.

Problem 3: Radius and Area

In problem 3, you are given the same circle with only the radius measured.

25. Press Menu, 7:Measurement, 2:Area. Find the area of the circle by moving the cursor to the circle, and press Enter or click to select it. Move the measurement to the left corner (See Figure 12).

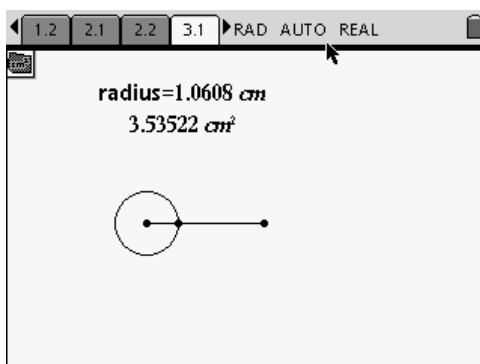


Figure 12

26. Drag the radius point to see the values change. Proceed to Problem 4.

Problem 4: Lists & Spreadsheet

27. The Lists & Spreadsheet application will track the data for the radius and the area (Figure 13).

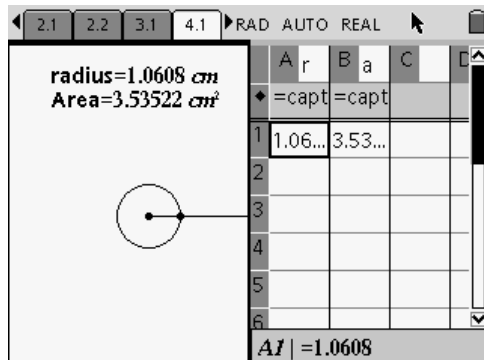


Figure 13

28. To collect more data, press **ctrl** **tab** to move into the Graphs & Geometry work area.

29. This time, we will use automated data capture. Drag the radius point to get more data which populates the lists automatically (Figure 14).

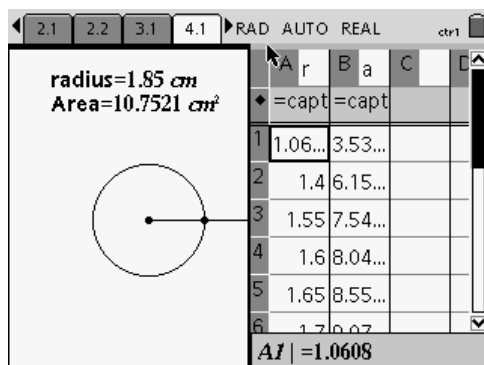


Figure 14

30. To see the graphical representation on your TI-Nspire™ handheld, move to page 4.2 which shows the scatter plot of area versus radius.

31. Press MENU 4 9 for Menu 4:Window, 9:Zoom-Stat to see a more appropriate window (Figure 15).

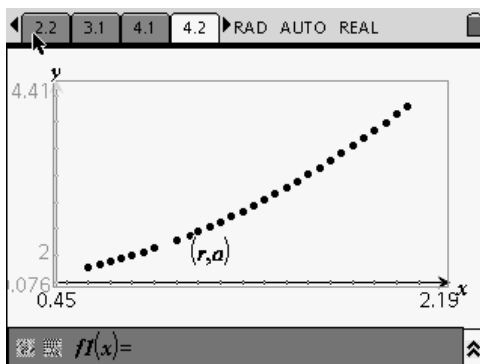


Figure 15

32. Answer question 8 at the end of this worksheet.

Graph the Function

33. Press Menu, 3:Graph Type, 1:Function. Enter the function $f1(x) = x^2$ and press Enter (Figure 16).

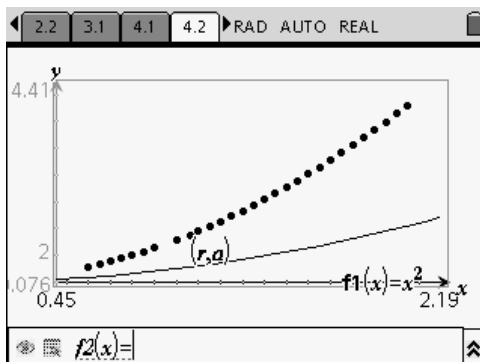


Figure 16

34. Press ESC to move out of the entry line and see a cursor on the graph.
35. Transform the function by grabbing the curve with the CROSSHAIR and moving the curve.
- If the four-directional cursor appears, the curve will translate, and if the two-directional cursor appears the curve will stretch.

36. Manipulate the curve to get the best fit (Figure 17).

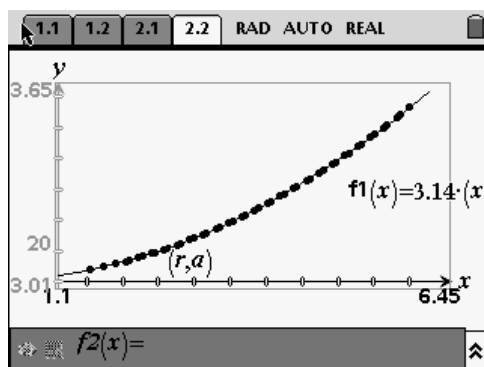


Figure 17

37. Answer question 9 at the end of this worksheet.

Answer the Following Questions

1. Which measurements are changing? Which measurements are staying the same?
2. Make a conjecture about Column C. Use the words of circumference and diameter in your conjecture.
3. Long ago, mathematicians named the circumference/diameter ratio with the Greek letter π (pi).

Complete the conjecture: If C is the circumference and d is the diameter of a circle, then there is a number Π such that $C = \underline{\hspace{2cm}}$.

Because $d = 2r$, where r is the radius, then $C = \underline{\hspace{2cm}}$.

4. What pattern are the plotted points?
5. What type of curve would be a good fit for the plotted points?

Points, Lines, Slopes
(Oh, My!)
Student Worksheet

Name _____

Class _____

In this activity, you will explore:

- the relationship between coordinates of points and locations on the coordinate plane
- the relationships of lines with their equations, slopes and y intercepts
- the relationships of slopes of parallel and perpendicular lines

1. Open the file *D2_2PointsandLines.tns* on your TI-Nspire™ Math and Science learning handheld (Figure 1).

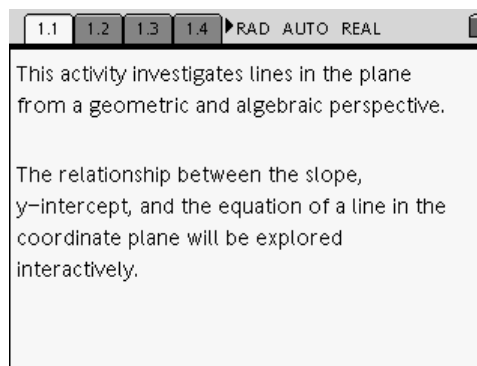


Figure 1

- Follow along with this document and your teacher.

2. Drag the y-intercept around, and notice the coordinates (Figure 2).

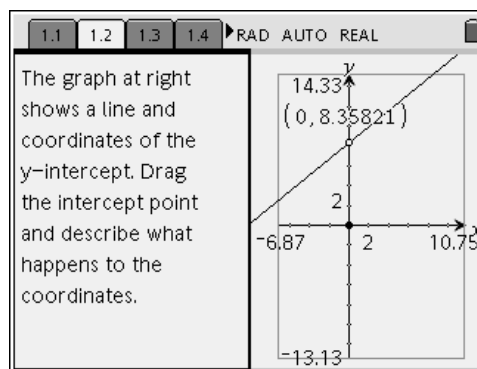


Figure 2

3. Explain any patterns that you observe about y-intercepts.

4. Move to page 1.3. Predict what will happen to the line and the slope if you move the y-intercept. Predict what will change as you drag the line (Figure 3).

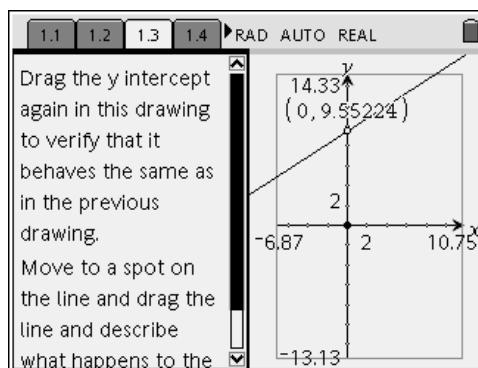


Figure 3

5. Describe what happens to the slope as the line or the y-intercept moves. Enter your responses on page 1.4.
6. Move to page 1.5, and read the instructions.

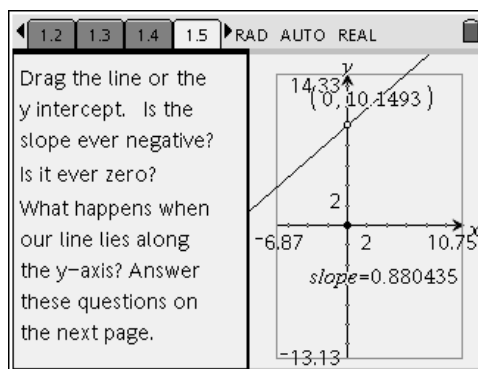


Figure 4

7. When is the slope positive? When is it zero? When is it ∞ (or undefined)? Describe a line with negative slope. Enter your responses on page 1.6.
8. Move to page 1.7 (Figure 5). This page describes what will take place on page 1.8.

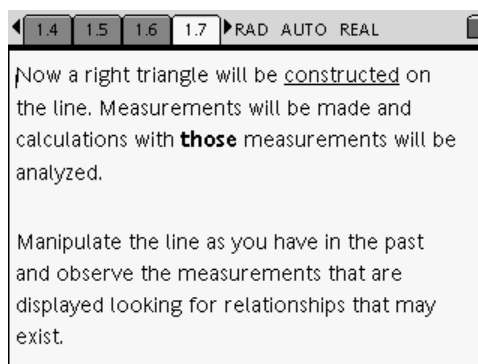


Figure 5

- Manipulate the line as before by dragging the y-intercept, and then by dragging points A and/or B. How does this impact the slope and/or ratios?
- Conjecture about slope and measurements in the right triangle.
- Which ratio represents the slope? (Figure 6).

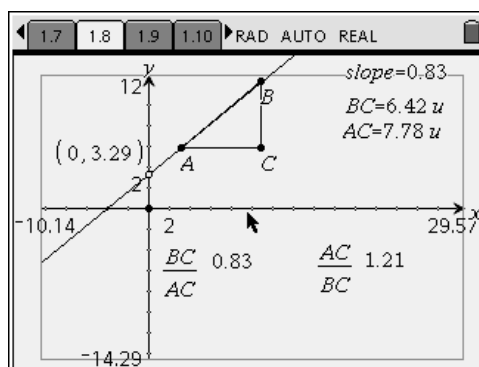


Figure 6

9. Move to page 1.9 and read the instructions. (Figure 7).

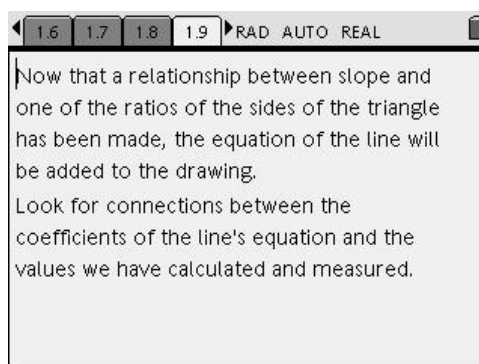


Figure 7

10. Move to page 1.10 and drag the y-intercept. Then drag points A and/or B.
- What is the relationship of the coefficients in the equation of the line and the measured and calculated values? Enter your answer on page 1.11.

11. Read the instructions on page 1.12. Predict the relationship between the slopes of parallel lines. Check your prediction by finding the slope of each line (Figure 8).

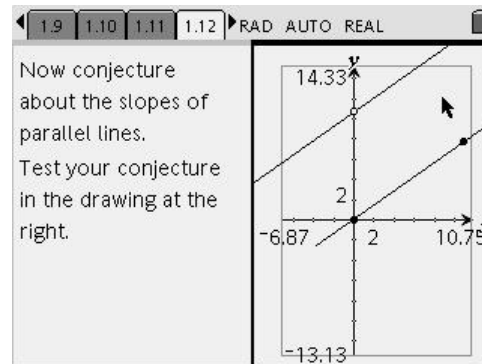


Figure 8

12. Move to page 1.13. Perpendicular lines and their slopes are shown in Figure 9.

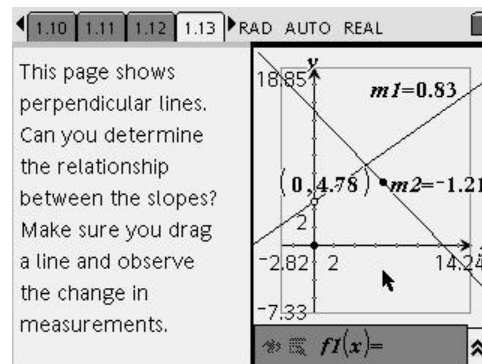


Figure 9

- Predict the relationship between the slopes of perpendicular lines.
- Drag the lines in Figure 9 to test your prediction.

13. Move to page 1.14 (Figure 10).

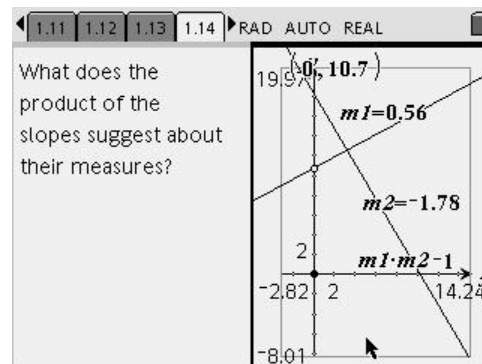


Figure 10

- What appears to be true about the product of the slopes of perpendicular lines?
- Discuss with your group members.

Points, Lines, Slopes (Oh, My!)

Concepts

- Slope
- y-intercept

Materials

- TI-Nspire™ Math and Science Learning Handheld
- TI-Nspire™ Computer Software for Math and Science
- D2_2PointsandLines.tns
- Points and Lines Student worksheet

Overview

In this activity, students will investigate the relationships of lines, slopes and equations - geometrically and algebraically. The investigation continues with parallel and perpendicular lines and their slope relationships.

NCTM Standards

All students should...

Grades 6-8 Algebra Standards:

1. represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules
2. relate and compare different forms of representation for a relationship
3. develop an initial conceptual understanding of different uses of variables
4. explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope
5. use graphs to analyze the nature of changes in quantities in linear relationships

Grades 9-12 Algebra Standards:

1. generalize patterns using explicitly defined and recursively defined functions
2. analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior
3. understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions
4. interpret representations of functions of two variables
5. approximate and interpret rates of change from graphical and numerical data

Prerequisite knowledge:

Students should know vocabulary terms to include points, equations, parallel and perpendicular.

Activity Overview:

1. This investigation offers many possible extensions depending on the level of the student.
2. At the Pre-Algebra level, this activity can be used to introduce or review properties of points in the coordinate plane.
3. At the Algebra I level, this activity can review coordinates and then go on to introduce or review the relationships of points, lines, slopes and equations.
 - The screen could also be split into different zones by graphing $f(x) = x$ and $f(x) = -x$. Ask students to generalize the slopes of lines for each of these zones.
4. At the Algebra 1 level, after graphing lines and looking at relationships, another point could be placed on the plane and students could look at the coordinates of the point with respect to the line. This could be used to introduce inequalities.
5. In Geometry, the angles formed by horizontal lines and parallel lines could be measured to begin the discussion of proving lines parallel by using measurement of angles.
6. This activity could also provide the opportunity to prove algebraically as well as geometrically that the slopes of perpendicular lines are negative reciprocals.

Classroom Management:

1. This activity can be done by the students using the pre-made TI-Nspire document file or by them creating the file along with the teacher. The following pages illustrate how to create the file.
2. The student worksheet is intended to guide students through the main ideas of the

activity, while providing more detailed instruction on how they are to perform specific actions using the tools of TI-Nspire™.

Creating the TI-Nspire Document File

- On the computer, open a New document under the File Menu. Under View select TI-Nspire Handheld View.
 - Insert a Notes page and type the text as shown in Figure 1.
- Insert another Notes page and choose page layout.
 - Select the second option for a vertical split, and select a Graphs & Geometry application on the right side of the screen.
 - Mouse click back in the Notes page on the left and type the text shown in Figure 2.
- Mouse click on the right side in the Graphs & Geometry page.
 - Drag the origin so that the 1st and 4th quadrants are displayed (Figure 3).
 - Extend each axis by dragging the arrows as well.
 - Under the View menu, choose Hide Entry Line. Select the Line tool under the Points & Lines menu and click on the y-axis to attach the line to the y-axis (but not on a grid point).
 - Move the mouse to the 1st quadrant and click anywhere to finish the line graph.
 - Drag the end of the line to extend it.
- Show the coordinates of the y-intercept by selecting “Coordinates and Equations” from the Tools menu and clicking on the point. Drag the coordinates to the desired location and press Enter.
 - The appearance of the y-intercept can be changed, if desired.

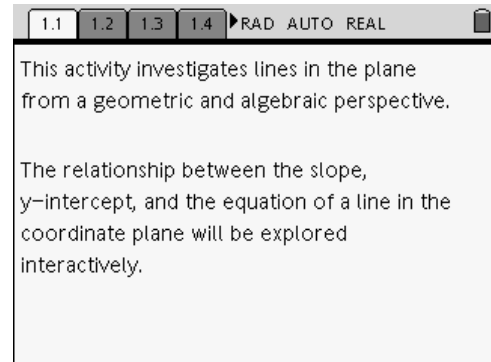


Figure 1

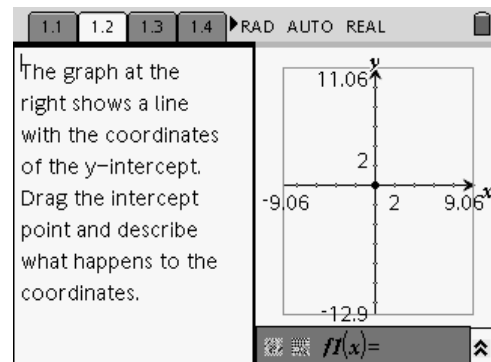


Figure 2

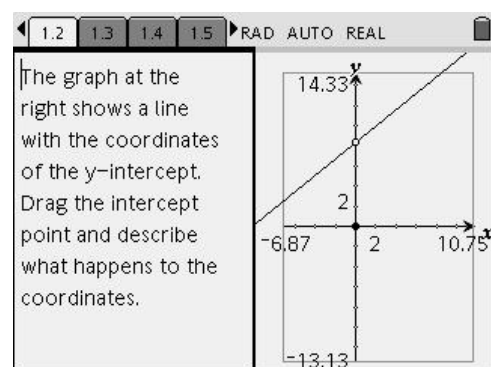


Figure 3

- Right click on the point, select Attributes, and use the arrow keys to select the desired attribute.
 - Press Enter to exit Attributes. (Figure 4).
5. Click on Insert and select New Page. Do not choose an Application.
 6. Click on page 1.2 in the page sorter. Press Ctrl K to select the page and then press Ctrl C to copy the page. Click on the new page and press Ctrl V to paste page 1.2 to the new page, page 1.3.
 - On the copied page (page 1.3), delete the current notes and change the Notes page to read as shown in Figure 5.
 7. Insert another Notes page and choose Q & A from the Templates menu. Type the sentences shown in Figure 6.
 8. Click on Insert and select New Page. Do not choose an Application.
 9. Click on page 1.2 in the page sorter. Press Ctrl K to select the page and then press Ctrl C to copy the page. Click on the new page and press Ctrl V to paste page 1.2 to the new page, page 1.5.
 - In the Notes section, type the text shown in Figure 7.
 - In the Graphs & Geometry section, find the slope of the line by selecting Slope from the Measurement menu.
 - To label the slope, double click on the slope measurement, and arrow to the beginning of the measurement. Type “slope =” and press enter or mouse click. Move the measurement to a convenient location on the screen.

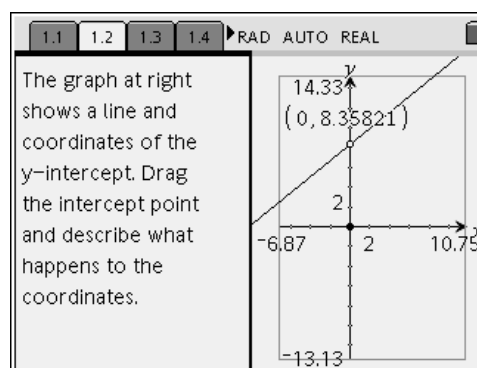


Figure 4

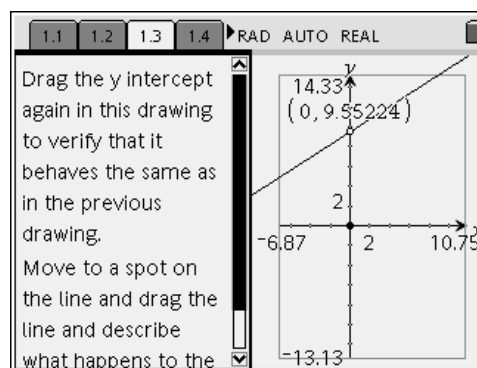


Figure 5

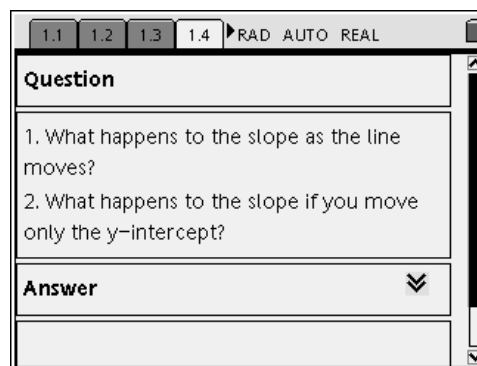


Figure 6

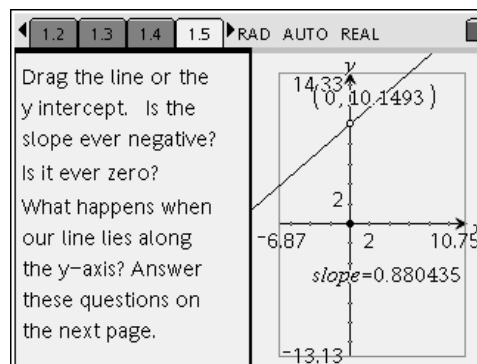


Figure 7

10. Insert two new Notes pages and type the text as shown in Figures 8 and 9.
11. Click on Insert and select New Page. Do not choose an Application.
12. To copy only the graph shown in Figure 7, click on the Graphs & Geometry portion of page 1.5. Press Ctrl K to select the application (the Graphs & Geometry window should be flashing). Press Ctrl C to copy the window.
13. Click on the new page (page 1.8). Press Ctrl V to paste the Graphs & Geometry window to this page.
14. Select the Point tool from the Points & Lines menu and place 2 points on the line. Label each point immediately after clicking the point (Figure 10).
 - You can also use the text tool to label the points.
 - Drag the arrows of each axis to extend the graphing window.
 - Delete the Drag Me text and the vector pointing to the y intercept.
15. To construct the right triangle, select the Perpendicular tool in the Construction menu.
 - Click on point A and then on the y-axis to construct the line through point A perpendicular to the y-axis.
 - For the second perpendicular line, select point B and the x- axis (Figure 11).
 - After constructing both perpendicular lines, select Intersection Point(s) from the Points & Lines tool and click on each line.
 - Immediately label the point C.
16. From the Tools menu, select Hide/Show and hide each perpendicular line.

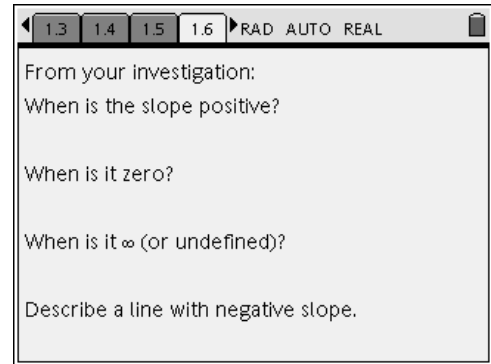


Figure 8

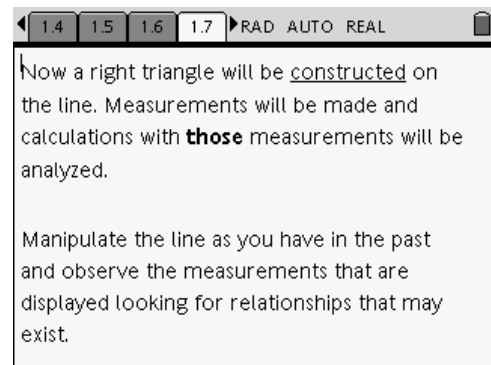


Figure 9

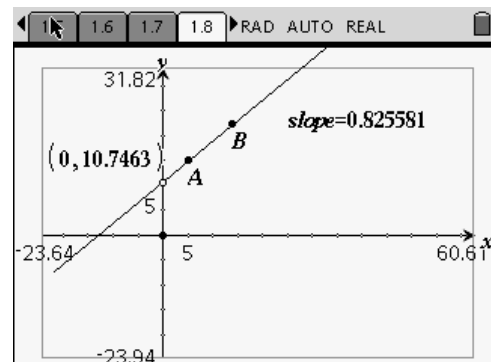


Figure 10

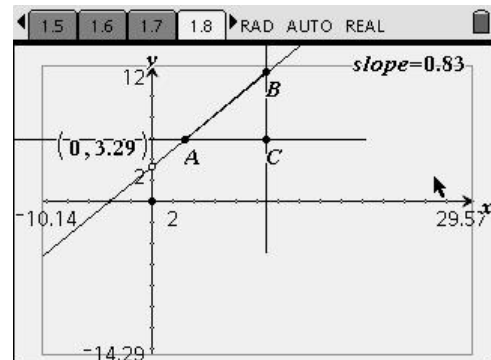


Figure 11

17. Select the Triangle tool From the Shapes menu.

- Construct a triangle with vertices A, B and C.
- From the Measurement menu, select the Length tool and click on the points B and C to measure the length of BC.
- Repeat to find length AC.
- Label the two measurements (Figure 12).

18. Using the Text tool from the Tools menu, type the expression BC/AC and the expression AC/BC .

- From the Tools menu, select Calculate and first select the text by clicking on it. Then, click on each variable needed for the computation.
- Move the measurements close to the appropriate text (Figure 13).
- To shorten the number of decimal places displayed for any measurement, first hover the mouse over the measurement until a hand appears.
- Press the \ominus key to decrease or the \oplus key to increase the number of decimal places displayed.

19. Insert a Notes page and enter the text as shown in Figure 14.

20. Add a new page and copy page 1.8 to this page, (page 1.10).

21. Add the equation of the line by choosing Coordinates and Equations from the Tools menu. Select the line.

- Drag the equation to a location so that is easy to see all of the relationships (Figure 15).

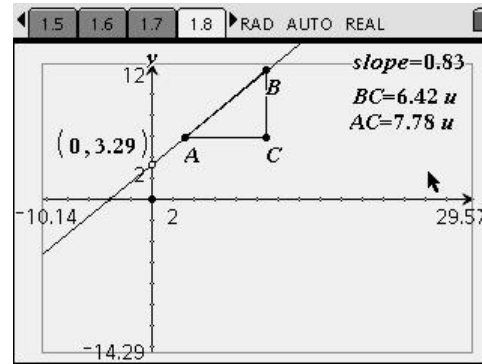


Figure 12

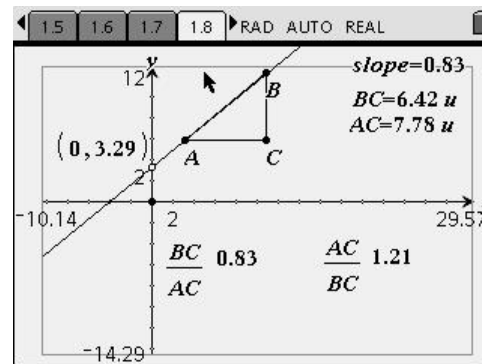


Figure 13

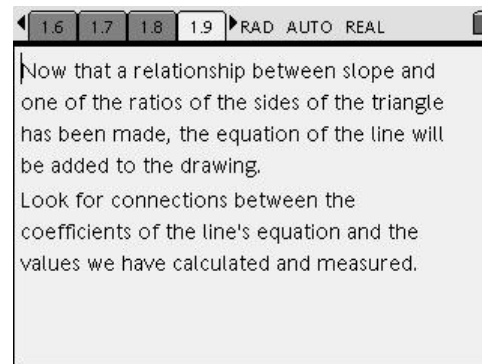


Figure 14

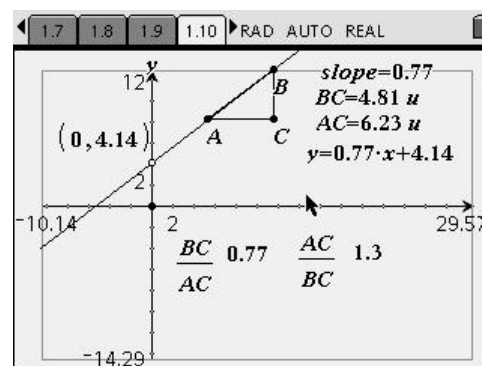


Figure 15

22. Insert a Notes page, choose the Q & A template, and add the question shown in Figure 16.
23. To explore the relationship of parallel lines and their slopes, add a new page after the Notes page shown in Figure 16. Copy page 1.3 and paste it to the new page, (page 1.12).
- Revise the Notes pages information as shown in Figure 17.
 - To construct a parallel line, select Parallel from the Construction menu and click on the given line and then anywhere in the coordinate plane.
24. To explore the relationship of perpendicular lines and their slopes, add a new page. Copy page 1.3 and paste it to the new page, (page 1.13).
- Revise the Notes pages information as shown in Figure 18.
 - On the Graphs & Geometry page, construct a perpendicular line by selecting Perpendicular from the Construction menu.
 - Click on the original line and then anywhere in the coordinate plane.
 - Find the slope of each line and label the slopes as before.
25. Copy and paste page 1.13 to a new page.
- Revise the Notes page as shown in Figure 19.
 - On the Graphs & Geometry page, select the text tool from the Tools menu and press enter to open a text box.
 - Type “ $m_1 \cdot m_2$ ” and press Enter to close the text box.
 - Select Calculate from the Tools menu. Click on the text and on each slope measurement. Place the result next to the text.

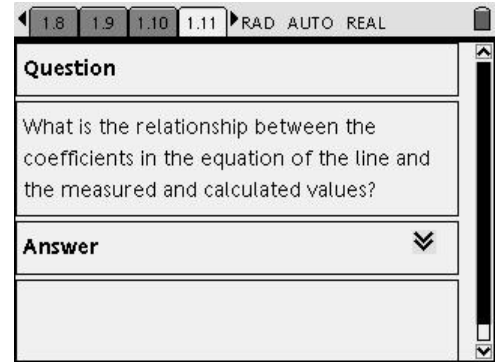


Figure 16

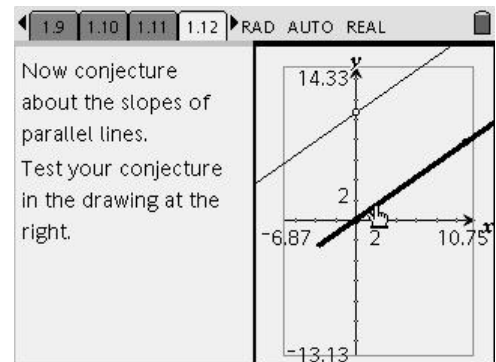


Figure 17

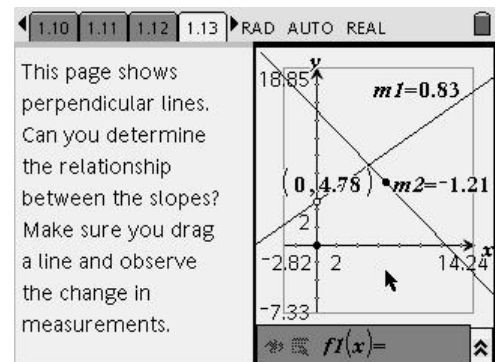


Figure 18

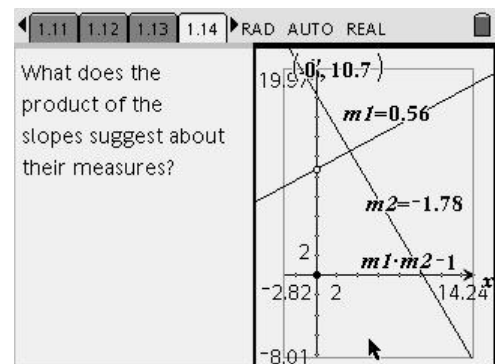


Figure 19

Reflections:

26. Reflect on how this activity, built in the document model, could enhance student's understanding of slope, y-intercepts, parallel lines and perpendicular lines.

27. What advantages are there to investigations in this format?

28. Share your thoughts with a neighbor.

The Open Topped Box

Student Worksheet

Name _____

Class _____

Introduction

You will investigate this problem in five ways.

- You will use pieces of paper to model the problem and construct several physical models.
- Using the Graphs & Geometry application, you will manipulate a geometric model.
- Using the geometric model, you will create a function that you can graph and find the maximum function value.
- Using the Lists & Spreadsheet application, you will create a numeric model.
- Finally, using the CAS tool, you will find the optimum algebraically.

You should find that the solutions are close regardless of the approach used. Part of your task will be to determine the advantages and disadvantages of each approach.

For the physical model, work in groups of three students.

- Each student will use three sheets of 8.5 by 11 inch paper.
1. The first member of the group will cut a 0.5" square out of each corner, and fold up the flaps that were created to form a box.
 - Tape the vertical edges together.
 - Measure the length, width and height of the box, and record those measurements along with the volume of your box.
 2. The first member should also cut out a corner of 1.25" from a second sheet and 2" from a third sheet.
 3. The second member should cut out corner of 0.75" from their first sheet, 1.5" from their second sheet and 2.25" from their third sheet.
 4. The third member should cut out corner of 1" from their first sheet, 1.75" from their second sheet and 2.5" from their third sheet.

Analyzing Data

5. Complete the following table as a group with all the data you have collected:

Cut Size	Height of Box	Length of Box	Width of Box	Volume of Box
0.50"				
0.75"				
1.00"				
1.25"				
1.5"				
1.75"				
2.00"				
2.25"				
2.50"				
x	h=	l=	w=	V=

6. Examine the table and determine which of these cut sizes produces the maximum volume?
7. Examine the patterns and determine the mathematical formulae that can be used to find the height, length, width and volume as functions of the size of the square cut out. (Cut size is represented by x .) Place those formulae in the last row.
8. To graph a relationship, we will let the size of the corner be the independent variable and let x represent this quantity. The dependent variable will be the volume of the box.
- Construct a scatter plot of the data.
 - Draw a curve through the points.
 - How would you describe the shape of the curve?
 - On your graph, circle the point which indicates the greatest volume.
 - Interpret the meaning of the coordinates of this point with respect to the paper and box.

TI-Nspire™ Math and Science Learning Technology

Locate the TI-Nspire™ learning technology file entitled “D2_3BoxProblem.tns”. Open this file and read the first two pages of the TI-Nspire document and follow the instructions.

Note:

This file has some of the basic setups created for you for each type of solution, along with Notes pages with questions, comments and instructions. Be sure to read each page.

When you are finished, your TI-Nspire™ learning technology file will be collected. Be sure to use appropriate units in your work.

9. Throughout the file, the terms “cut size” and the “height” are used interchangeably. Is this appropriate? Explain.

10. On page 1.3 of the file, you dragged point P up and down line segment AD. The point is only allowed to go to the midpoint of AD.
 - Explain why the geometric model was set up in this way.

11. What value of h gives you the maximum volume?
 - Cut size _____

12. What is the volume at this point?
 - Volume _____

13. Is this the optimum volume? Explain.

Return to the TI-Nspire™ document file and go to page 1.4 and follow the instructions. After completing page 1.9, answer question 14.

14. Referring to the scatter plot on page 1.7, what type of function did you choose to model the relationship between cut size and volume when you did the regression on page 1.9?
 - Explain your choice of model—what lead you to choose this type of function rather than others?

- After answering question 14, return to the TI-Nspire™ file and go to page 1.10.

15. On page 1.11 you graphed the function on a clean Graphs & Geometry page.

- What problems did you have in finding the maximum volume?

16. What value did you get for the cut size and maximum volume using this model?

- Cut size _____
- Volume _____

After answering questions 15-16, return to the TI-Nspire™ file and go to page 1.12.

17. On page 1.12, you are asked to do a numerical analysis of the problem using the Lists & Spreadsheet on page 1.13. Pages 1.14 and 1.15 provide more information about the iteration process.

- The first iteration is done for you.
- In each table, you are required to list three rows from your spreadsheet.
 - The middle row should be the maximum volume for that iteration.
 - The row above should be the row preceding the maximum volume from your table.
 - The row below should be the row following the maximum volume from your table.
- You will keep repeating these steps.
- Each step is called an iteration. You may want to look up the meaning of the word.
- You may not need four iterations, so explain why you stopped when you did.

First iteration

- Lower bound: 0
- Upper bound: 4.25
- Increment: 0.25

	Height	Volume
	1.25	63.75
Maximum	1.5	66
	1.75	65.625

Second iteration

- New lower bound: 1.25
- New upper bound: 1.75
- New increment: 0.1

Revise the sequence command in the List and Spreadsheet to reflect the new lower and upper bounds and increment. You will need to select “OK” to overwrite the original data.

Complete the table below and repeat the process for subsequent iterations.

	Height	Volume
Maximum		

Third iteration

- New lower bound _____
- New upper bound _____
- New increment _____

	Height	Volume
Maximum		

Fourth iteration

- New lower bound _____
- New upper bound _____
- New increment _____

	Height	Volume
Maximum		

What value did you obtain for the cut size and maximum volume using this model?

- Cut size _____
- Volume _____

18. Explain why you stopped at the iteration you selected.

- Return to the TI-Nspire™ file and go to page 1.16.

19. For the CAS solution, record the function $f_1(x)$.

20. Use CAS to determine the first derivative function $f'(x)$
What polynomial do you get for the first derivative function?
21. Continue to use CAS to determine what value(s) of cut size produces the maximum volume?
22. If there is more than one solution, explain why you can eliminate the other solution(s).
23. What is the maximum volume?
24. How did you get that value?

Summary

25. Which of the four solutions gives the most accurate answer? Explain your choice.
26. Which type of solution did you prefer? Explain your choice?
27. What would you do if the problem started with a sheet of metal that was 50cm by 30cm?
- Would all of the steps be the same or would you need to change any of the processes that were used in the solution above?

Rubric for the Open Topped Box Problem using
TI-Nspire™ Math and Science Learning Technology

Knowledge/Skills	Level 1	Level 2	Level 3	Level 4
Thinking/Inquiry/ Problem Solving <ul style="list-style-type: none"> Prepares a plan to solve the problem Carries out the plan 	Needs extensive assistance to begin organizing a plan and needs clearly laid out steps to follow.	Needs some assistance to begin organizing a plan and needs some steps to follow.	Needs minimal assistance to organize and implement an effective strategy.	Needs no assistance to organize and implement an effective strategy.
Knowledge/Understanding <ul style="list-style-type: none"> Models the problem with paper Creates a scatter plot and regression function Inputs values into the spreadsheet to simulate the problem Uses CAS tools to find the derivative function and its roots 	Demonstrates limited understanding of using a physical, geometric, and/or numeric model. Needs extensive assistance to use CAS tools.	Demonstrates some understanding of using a physical, geometric, or numeric model. Needs some assistance to use CAS tools.	Demonstrates considerable understanding of using a physical, geometric, and numeric model. Needs minimal assistance to use CAS tools.	Demonstrates thorough understanding of using a physical, geometric, and numeric model. Needs no assistance to use CAS tools.
Application <ul style="list-style-type: none"> Uses several physical models to approach the optimal solution Manipulates the geometric model to approach the optimal solution Finds the maximum value of the regression function in an appropriate domain Manipulates the intervals between input values in a spreadsheet to approach the optimal solution Interprets the information from the CAS solution to find the optimal solution 	Interprets the information ineffectually. Has difficulty manipulating and applying data from a physical, geometric, and/or numeric model. Needs extensive assistance to interpret the information from using the CAS tools. Solutions include major errors.	Interprets the information somewhat effectually. Has some difficulty manipulating and applying data from a physical, geometric, or numeric model. Needs some assistance to interpret the information from using the CAS tools. Solutions include minor errors.	Interprets the information with considerable effectiveness. Has little difficulty manipulating and applying data from a physical, geometric, and numeric model. Needs minimal assistance to interpret the information from using the CAS tools. Solutions are accurate.	Interprets the information with a high degree of effectiveness. Has no difficulty manipulating and applying data from a physical, geometric, and numeric model. Needs no assistance to interpret the information from using the CAS tools. Solutions are accurate and include an explanation for disallowed negative answers.
Communication <ul style="list-style-type: none"> Includes clear explanations and full justifications Correctly uses mathematical language, mathematical form and measurement units 	Does not clearly explain or justify solutions. Maintains the correct units and good mathematical form in some of the solutions.	Explains and justifies solutions somewhat. Maintains the correct units and good mathematical form throughout most of the solutions.	Explains and justifies solutions fully. Maintains the correct units and good mathematical form throughout the solutions.	Explains, justifies and shows insight into the complexities of the solutions. Maintains the correct units and good mathematical form throughout the solutions.

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Lesson Plan Notes for the Open Topped Box

Course(s): Algebra 1, Geometry, Pre-Calculus and Calculus

Topic(s): Scatter plots, functions, graphing, geometric modeling, equations, recursive sequences, rotations, reflections, nets, volume, derivatives, and local maximums.

Related NCTM Standards:

1. Number and Operations Standards

- Students should judge the reasonableness of numerical computations and their results.

2. Algebra Standards:

- Students should generalize patterns using explicitly defined and recursively defined functions.
- Students should use symbolic algebra to represent and explain mathematical relationships.
- Students should identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.
- Students should draw reasonable conclusions about a situation being modeled.
- Students should approximate and interpret rates of change from graphical and numerical data.
- Students should analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.
- Students should judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.
- Students should draw reasonable conclusions about a situation being modeled.

3. Geometry Standards

- Students should use Cartesian coordinate and other coordinate systems, such as navigational, polar or spherical systems to analyze geometric situations.
- Students should understand and represent translations, reflections, rotations and dilations of objects in the plane by using sketches, coordinates, vectors, function notation and matrices.
- Students should investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates.
- Students should construct representations of two- and three-dimensional objects using a variety of tools.

4. Measurement Standards

- Students should make decisions about units and scales that are appropriate for problem solving situations involving measurement.
- Students should analyze precision, accuracy, and approximate error in measurement situation.

Suggested Total Time: Three hours.

Specific Pre-requisite knowledge: Students should be familiar with the following:

- Finding volume of a rectangular solid
- Manipulating a geometric model by dragging points
- Using spreadsheet formulae
- Recursive sequences
- Constructing a scatter plot
- Finding the function of best fit
- Derivatives of functions

Materials Required/Classroom

Set-up/Preparation: The following equipment will be needed:

- Paper, scissors and tape.
- Each student should have access to a TI-Nspire™ math and science learning handheld with the file “D2_3BoxProblem.tns” pre-loaded.
- If the CAS solution is to be done, then students should have access to a TI-Nspire™ CAS math and science learning handheld.

Lesson Notes:

1. The step-by-step solutions present four models for solving this problem but each solution fits a different course. This is a problem that you can re-visit from year to year.
 - In Algebra 1, you could use the physical model to generate data that gives a scatter plot and function found by using an appropriate regression. The “function maximum” feature could be used to find the optimal point.
 - In Geometry, the scale model can be used as well as one or both of the approaches.
 - In Pre-Calculus, the cubic function can be developed and a discussion of the nature of the function is applicable in that course.
 - In Calculus, the derivative function can be generated through the CAS tool to find the optimal solution.
2. The rubric was set up in order to address the entire activity. You may want to consider the following guidelines when using the rubric.
 - The Thinking/Inquiry/Problem Solving portion is generic and addresses the manner in which the student addresses the question.
 - The Knowledge/Understanding category is

more specific and you will see bullets that address each of the models in the problem, where appropriate.

- The Application category is also more specific to each of the five models.
- Finally, the Communication category is more generic addressing how well the student justifies and explains as well as the use of proper mathematical form.

3. As students are working through the solutions, you may want to keep the following issues in mind.

- It is likely that the students will do quite well on the first two models (physical and geometric).
- Anticipate some problems with the graphical model as some students will resist the instruction to create a clean graph with just the function and attempt to find the maximum on the graph with the scatter plot.
- Even those students who use a new graph may find it difficult to locate the maximum as they tend to drag points quickly and a slower approach works better here.
- In the numeric model, students may experience some problems in identifying the smaller domain and then more trouble when trying to re-define the sequence.
- The CAS model is the most straightforward, although younger students would have no appreciation for the mathematics involved in this solution.

4. Students will need to be kept on task. Each model may take some time and they may become discouraged.

The Open Topped Box

Concepts

- Nets and Scatter plots
- Functions and Graphing
- Sequences
- CAS (optional)

Materials

- TI-Nspire™ Math and Science Learning Handheld
- (CAS model is needed for the optional section)

Overview

In the student activity, students explore solving the problem of maximizing volume by various approaches. In this portion, the teacher notes, we will cover how to construct the models for the student document.

Constructing the Models

In this activity we will provide instructions on the creation of the various document models, Geometric, Scatter Plots, Numeric, Algebraic.

Geometric Model

1. Open a Graphs and Geometry page.
 - To hide the axes press **(menu)**, select the View menu by pressing **2** and choose Hide Axes, option **1**.
2. Construct a rectangle to model the problem. Press **(menu)**, choose **8**: Shapes, and then **3**: Rectangle.
 - Move your cursor out into the open area of the screen and press **enter** on three of the corners of your rectangle (Figure 1).
3. Label the corners A, B, C and D.
 - Construct line segments AB and BC. Measure the lengths of these segments (Figure 2).
4. In order to make the measurements realistic, we will use the Scale feature.
 - Press **(menu)**, select **2**: View, and then **4**: Show Scale (Figure 3).

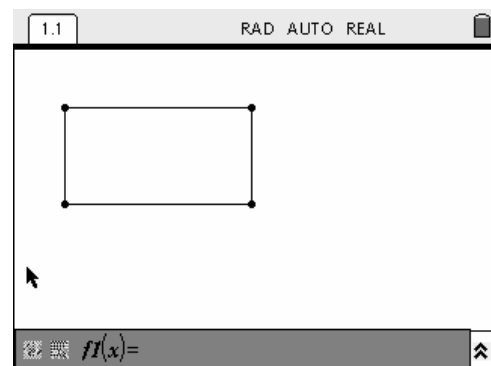


Figure 1

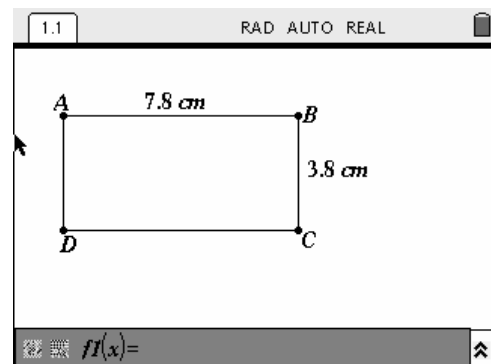


Figure 2

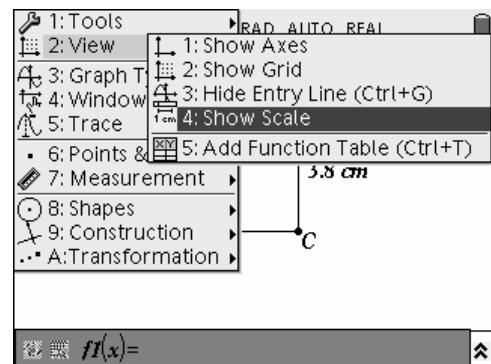


Figure 3

5. The default unit is centimeters.
 - Click twice on the scale and change the units from “cm” to “in”.
6. To make the model more like the real sheet of paper, follow these steps:
 - Press $\left(\frac{\text{cm}}{\text{in}}\right)$ twice on the measurement to edit it. Hold the $\left(\text{CAPS}\uparrow\right)$ key and use the $\left(\leftarrow\right)$ and $\left(\rightarrow\right)$ keys to highlight the measurement for the length. Press $\left(\text{ctrl}\right)$ and $\left(\text{C}\right)$ (to copy).
 - Move over to the scale, press $\left(\frac{\text{cm}}{\text{in}}\right)$ twice to edit the scale. Use $\left(\text{clear}\leftarrow\right)$ to remove the digit.
 - In its place, type in “11” press $\left(\frac{\text{in}}{\text{cm}}\right)$, and press $\left(\text{ctrl}\right)$ and $\left(\text{V}\right)$ (to paste).
 - This will paste the measurement from the diagram into the scale calculation.
 - Press $\left(\text{enter}\right)$. The unit of measurement will change and the length of your rectangle will adjust to 11 inches.
 - To set the width to 8.5 inches, click on the measurement, press $\left(\text{clear}\leftarrow\right)$ to remove the existing value and type in 8.5.
 - Your rectangle will immediately adjust to this length. You now have a “sheet of paper” on your screen (Figure 4).
 - You may need to move the figure back into the work space.

7. Press $\left(\text{menu}\right)$. Choose $\left(9\right)$: Construction and then $\left(5\right)$: Midpoint.
 - Click on each of the four sides of the rectangle (Figure 5).

8. Construct a line segment connecting A to the midpoint of AD.

9. Press $\left(\text{menu}\right)$ $\left(6\right)$: Points & Lines and then $\left(2\right)$: Point On to construct point P on this line segment. See Figure 6.

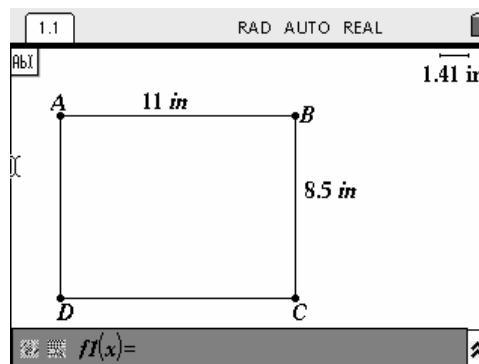


Figure 4

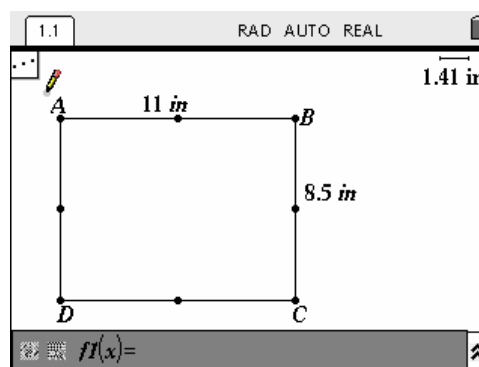


Figure 5

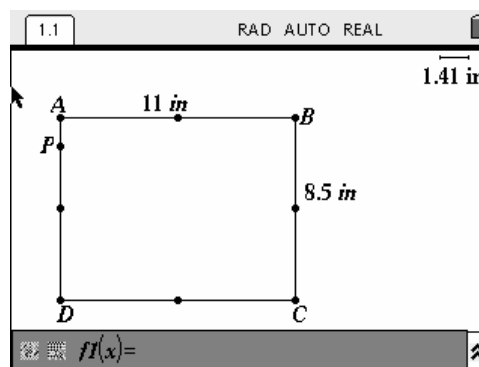


Figure 6

10. In the next step you need to rotate point P through an angle of 90° to get an image point on AB. In order to accomplish this, you need to ensure that the settings for angle measurements have been changed to degrees. If you look at the top of Figure 6 above, you will see that the default is radians.

- Press **ctrl** followed by **h**.
- Choose option **1**: File and then **6**: Document Settings.
- Press **tab** to move to the Angle window and press **▼** to open the window.
- Scroll down to Degree and press **enter** to change the setting. See Figure 7.
- Press **tab** to move to the OK button at the bottom of the window.
- Press **enter** to return to the Graphs and Geometry window.
- You should now see DEG at the top.

11. In an open area, enter the text “90”. This will serve as the angle of rotation. See Figure 8.

12. Press **menu** followed by **A**: Transformation and **4**: Rotation.

- Click on Point A then click on the text “90”, and finally, click on point P. This will create a new point on AB that is the same distance from A as point P is from A (Figure 8).

13. Label the new point as Q.

- Construct line segments connecting the vertical and horizontal midpoints (Figure 9).

14. Press **menu** followed by **A**: Transformation and choose option **2**: Reflection.

- Using the vertical line segment as the line of reflection, reflect points P and Q in this line (Figure 10).

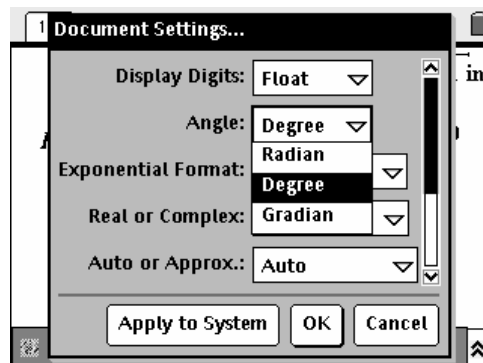


Figure 7

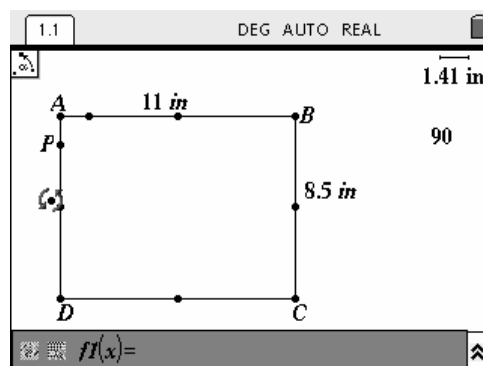


Figure 8

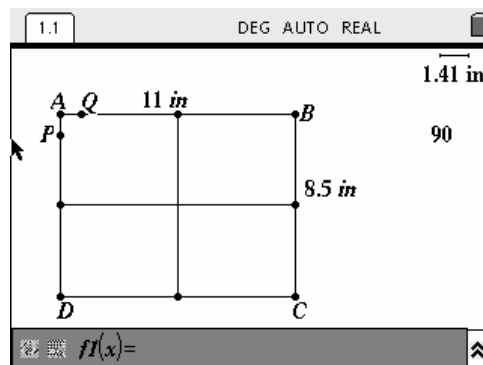


Figure 9

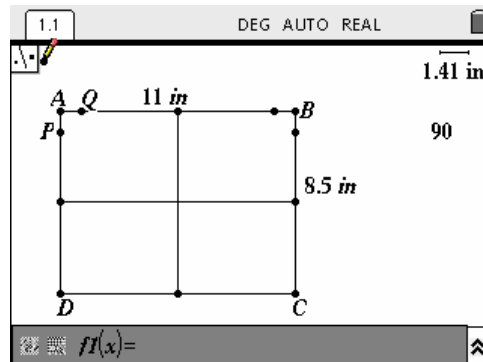


Figure 10

15. Using the horizontal line segment as the line of reflection, reflect points P, Q and the two new points in this line (Figure 11).

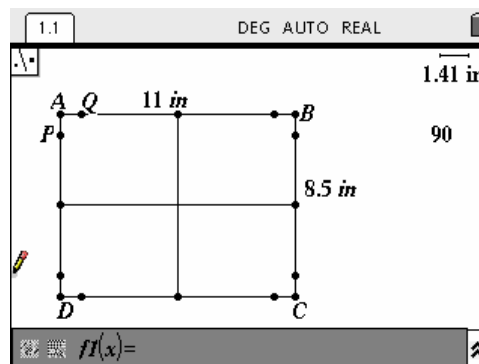


Figure 11

16. Hide the two line segments and the four midpoints and construct line segments as shown in Figure 12.

- Describe what happens as you drag point P.

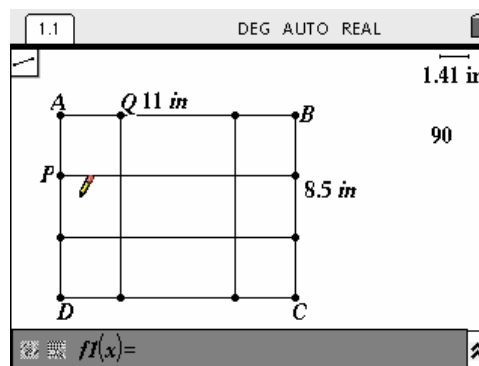


Figure 12

17. Press MENU $\{6\}$: Points & Lines and $\{3\}$: Intersection Point(s) to construct points of intersection where the vertical and horizontal segments intersect (Figure 13).

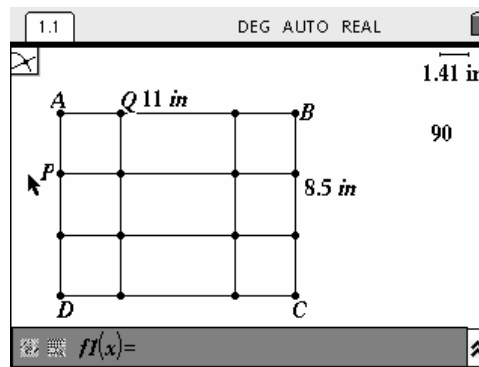


Figure 13

18. Hide all of the line segments, leaving just the four points of intersection (Figure 14).

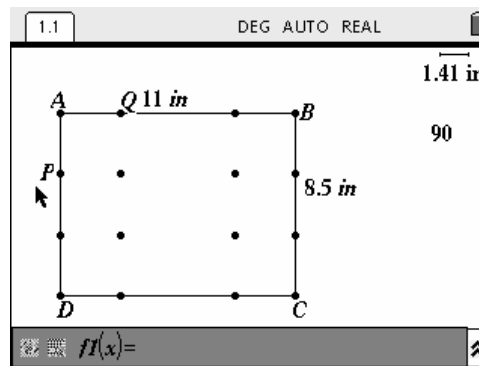


Figure 14

19. Press **(menu)** followed by **(8)**: Shapes and **(4)**: Polygon.

- Click on all four points in the center of the screen, and click a second time on the first point selected. This will construct a rectangle. This will serve as the base of the open topped box (Figure 15).

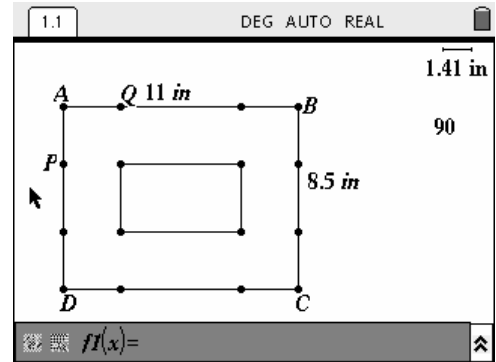


Figure 15

20. Use the polygon tool to construct a square in each corner of the original rectangle (Figure 16).

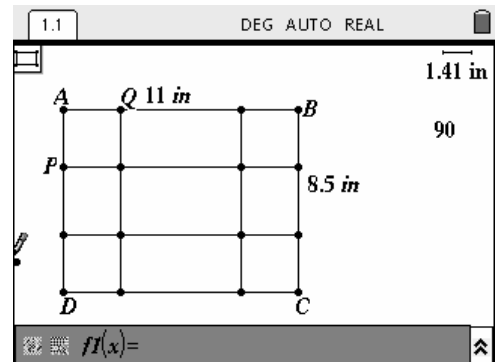


Figure 16

21. Press **(menu)** **(1)** **(3)** for Menu, Tools, Attributes. Click on each square and a dialog box will open.

- Press **▼** to the first section and then press **►** to select a shading for the square (Figure 17).

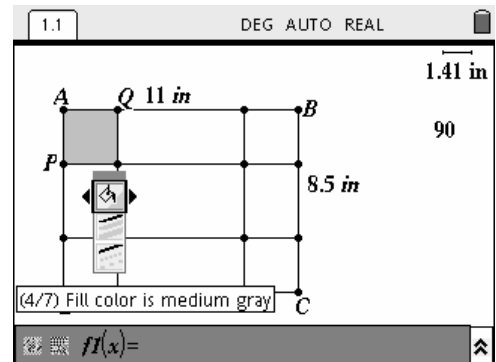


Figure 17

22. For each square press **▼** twice to move to the last section in the window.

- Press **►** to change the line style to either dotted or dashed (Figure 18).

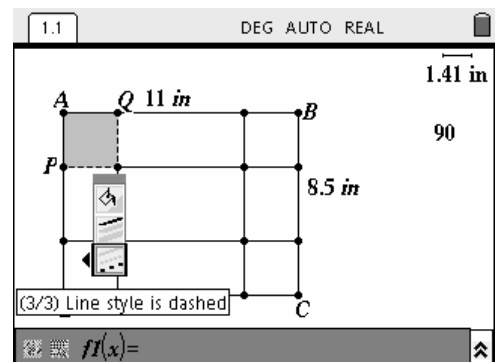


Figure 18

23. When you are finished all four squares, you should have a sketch that resembles Figure 19.
- The shaded corners are the areas to be cut out along the dashed lines.

24. Drag point P, and describe what happens to the different sections in the original rectangle.
- Hide the labels for points P and Q and add the measurements shown in Figure 20.
 - For the open topped box, which measurement is the height?
 - Which measurement is the length?
 - Which measurement is the width?

25. To assign variables to the measurements in the construction, click on the variable and press **ctrl** followed by **sto>var** (Figure 21).
- Type in the name of the variable, "h".
 - Assign variables (l and w) to the measurements for length and width.

26. Press **menu** followed by **1**: Tools and **5**: Text to add the expression $l \cdot w \cdot h$ in an open space. This expression will be used to calculate the volume of the open topped box. (Figure 22).

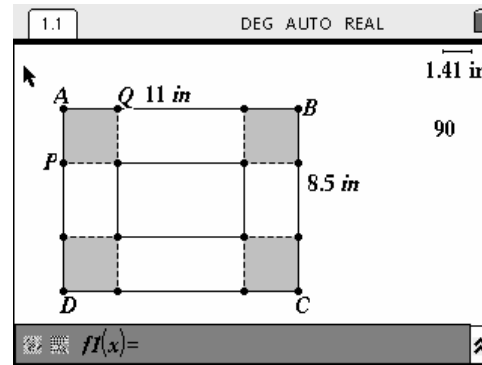


Figure 19

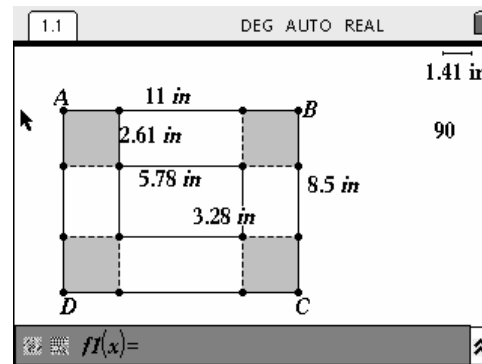


Figure 20

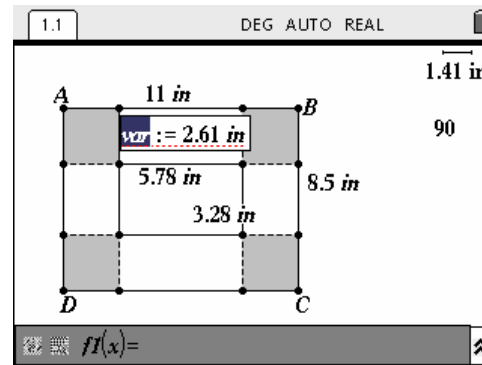


Figure 21

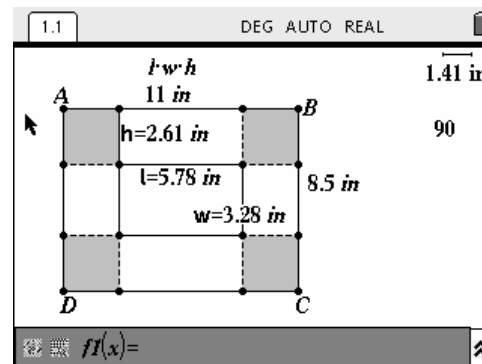


Figure 22

27. Press $\text{\textcircled{MENU}}$ followed by $\text{\textcircled{1}}$: Tools and $\text{\textcircled{7}}$: Calculate. Click on the expression for volume.

- As you move away from the expression, a caption appears asking you to identify a variable.
- The volume expression is highlighted and a caption asking for variable h is shown in Figure 23.
- Move the variable and click on it.

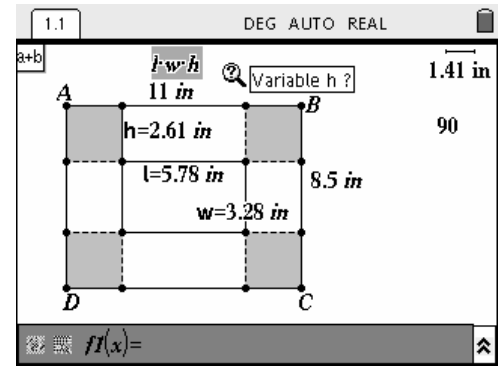


Figure 23

28. Continue selecting variables.

- When you are finished, the value of the expression will appear.
- Move this to the right of the volume expression (Figure 24).

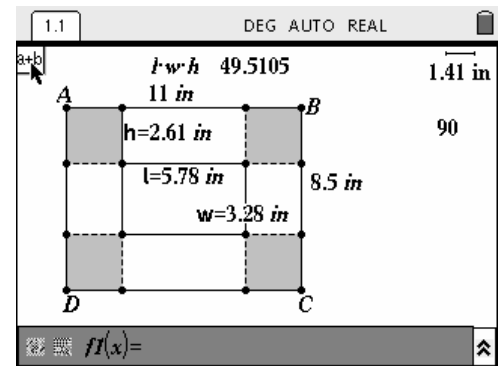


Figure 24

29. Drag point P until the value of volume expression is a maximum. This value for volume should be linked to a variable as you did with h, l and w. Use “v” for this result. See Figure 25.

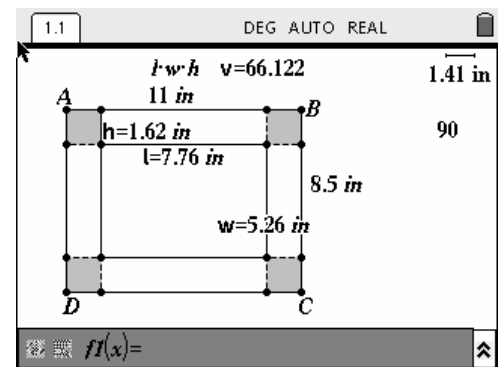


Figure 25

30. Alternate Construction of Model:

In the previous steps 9 to 13 point P is rotated to obtain point Q.

- These two points were reflected in the vertical line segment connecting the midpoints of AB and CD to obtain points about point B. Finally these four points were reflected across the line segment joining the midpoints of AD and BC to obtain points about C and D.
- An alternate approach would be to construct a line segment AP, and use the Compass tool (press $\text{\textcircled{MENU}}$ followed by $\text{\textcircled{9}}$: Constructions and $\text{\textcircled{7}}$: Compass).
- You would then need to find points of intersection of each of these circles with AB, BC, CD and AD.
- In the end, the amount of work is about the same, but you might prefer to emphasize the compass tool over rotations and reflections. See Figure 26.

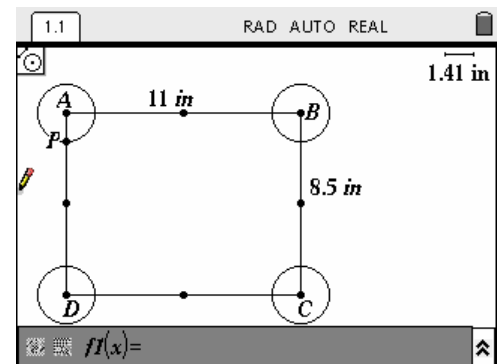









Figure 26

Constructing a Scatter Plot





1. The data from this geometric model will be captured in the Lists & Spreadsheet tool.

- Press  and select option  3: Lists & Spreadsheet (Figure 27).

2. Type the headings “cut” in column A and “volume” in column B.

- Press  followed by : Actions and : Resize. At this point, the entire column will be highlighted. (Figure 28).
- Pressing  will widen the column.
- Press  to save that width.
- Repeat for column B.

3. In order to capture data from the geometric model, move to the row where you see the ♦ Symbol (between the column header and row 1).

- Press  followed by : Data, : Data Capture, and : Automated Data Capture (Figure 29).

4. When this option is selected, the formula *capture(var,1)* will appear in the cell with the word “var” highlighted (Figure 30).

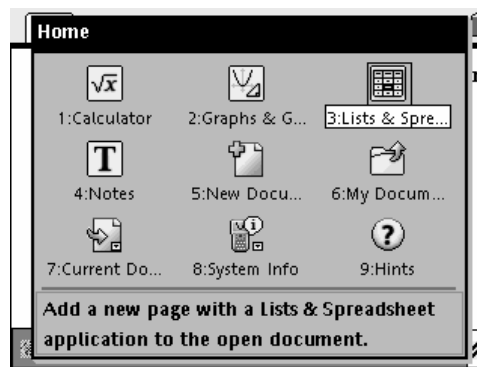


Figure 27

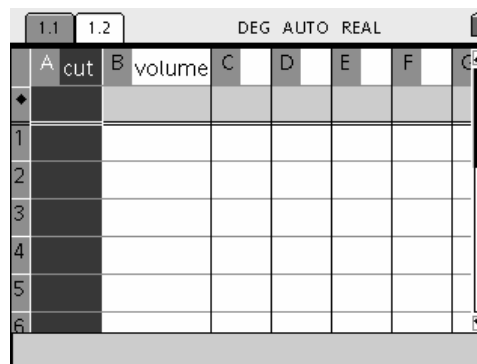


Figure 28

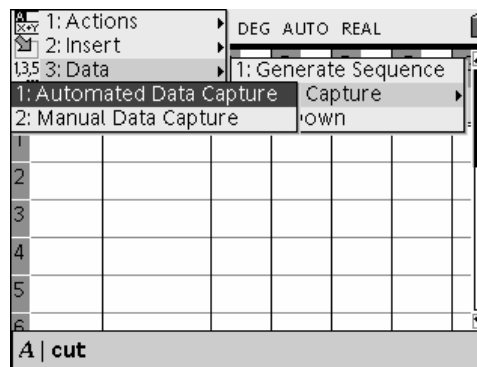


Figure 29

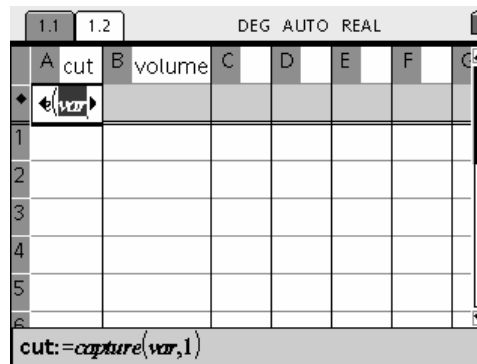


Figure 30

5. Referring back to the geometric model, the cut refers to the height of the box, so we will replace “var” with “h”.
 - When you press [enter] , you may get a Conflict Detected dialog box. The software wants you to choose between the variable h from the Graphs & Geometry page or column H in the Lists & Spreadsheets page.
 - Press \blacktriangledown to open the options box and scroll down to Variable Reference. See Figure 31.

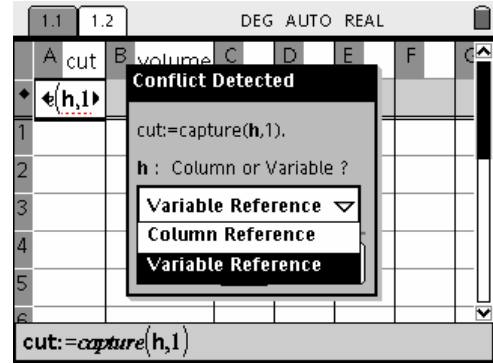


Figure 31

6. Press [enter] and [tab] to move to the OK box.
 - Press [enter] to close the dialog box.
 - Repeat for column B by capturing data from variable v. See Figure 32.

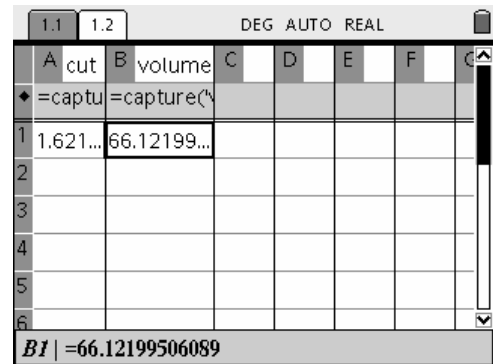


Figure 32

7. Note in Figure 32 that the first row now contains values. These are the current values from the Graphs & Geometry page.
 - Press [ctrl] followed by \blacktriangleleft to go back to the Graphs & Geometry page.
 - Drag point P through its range and return to the Lists & Spreadsheet page (Figure 33).

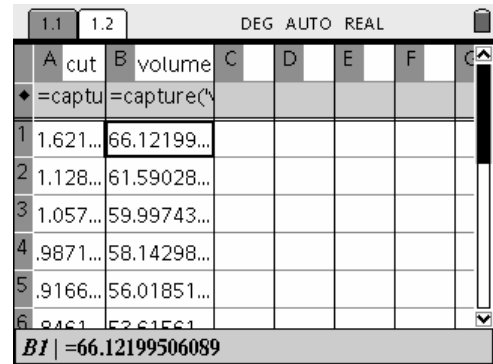


Figure 33

8. As point P was dragged, each location of the point generated another row of data in the Lists & Spreadsheets page.
 - You can use these lists to generate a scatterplot in a new Graphs & Geometry page, page 1.3 (Figure 34).

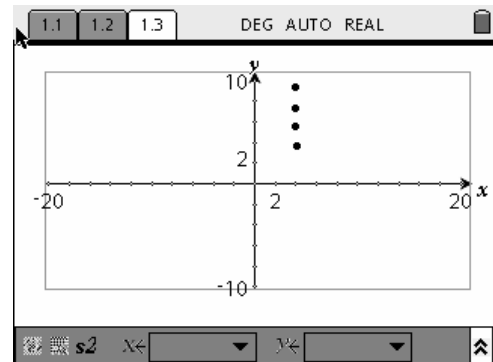


Figure 34

9. Select Zoom–Stat to see the entire scatter plot.

- Press **(menu)** followed by **(4)**: Window and **(9)**: Zoom – Stat. See Figure 35.

10. Since this relationship is the product of three variables we could safely conclude that this might be a cubic relationship.

- Open a Calculator page (Page 1.4).
- Press **(menu)** followed by either **(6)**: Statistics in TI-Nspire or **(7)**: Statistics in TI-Nspire CAS. Next, press **(1)**: Stat Calculations and **(7)**: Cubic Regression.
- Complete the dialog box as shown in Figure 36.
- Press **(tab)** to select OK and press **(enter)**.

11. The results of the linear regression calculation are shown in Figure 37.

- Note that some of the coefficients are “messy”.

12. Move back to the Graphs & Geometry page.

- Press **(menu)** followed by **(3)**: Graph Type and **(1)**: Function.
- Scroll up to f1(x) and press **(enter)** (Figure 38).

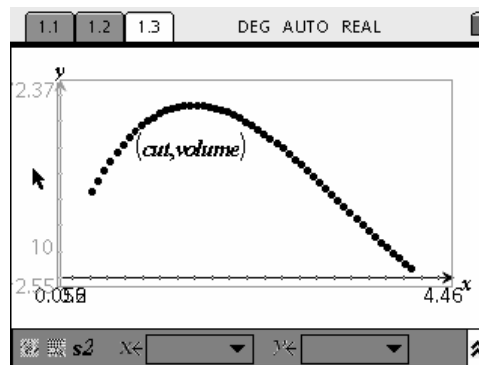


Figure 35

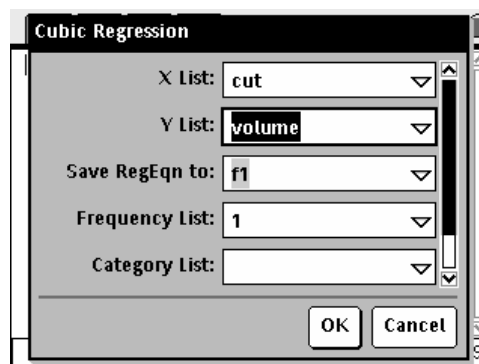


Figure 36

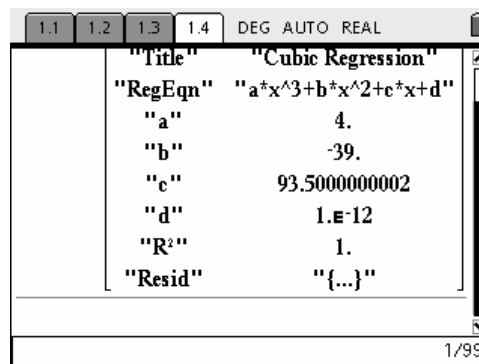


Figure 37

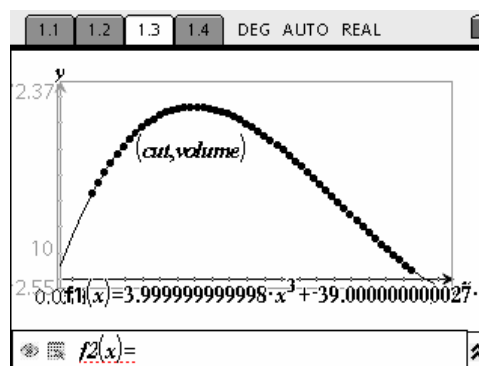


Figure 38

13. With the scatter plot on top of the function, it is difficult to use a Trace Point to determine the optimal value.

- Insert a new Graphs & Geometry page (Page 1.5) in your document and plot the function $f_1(x)$ in this window (Figure 39).

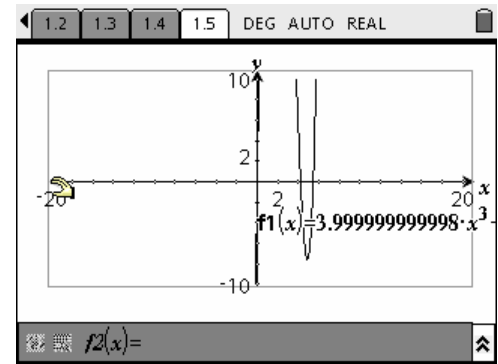


Figure 39

14. For this graph, you can alter the window settings yourself.

- Press MENU followed by 4 : Window and 1 : Window Settings.
- Change the values as shown in Figure 40.

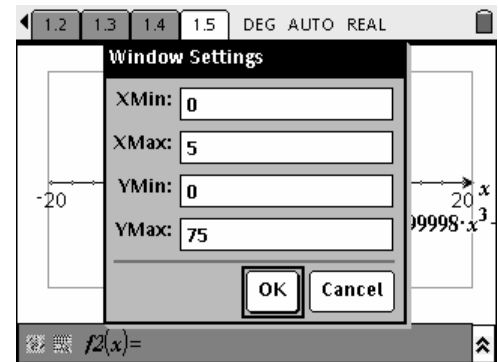


Figure 40

15. The graph will appear without the scatter plot. See Figure 41.

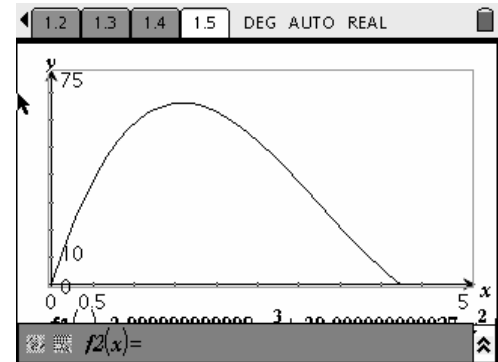


Figure 41

16. You can add a Trace Point that will allow you to find the maximum value.

- Press MENU followed by 5 : Trace. A point and its coordinates will appear near the origin.
- Pressing LEFT or RIGHT will move this point along the curve. See Figure 42.

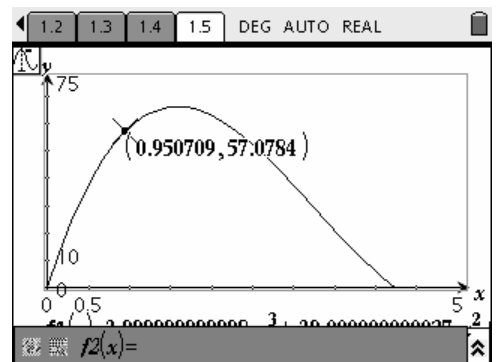



Figure 42

17. Press  to anchor the point on the curve.
 - Drag the point until you see a caption with a capital letter M.
 - This point is a local maximum (Figure 43).

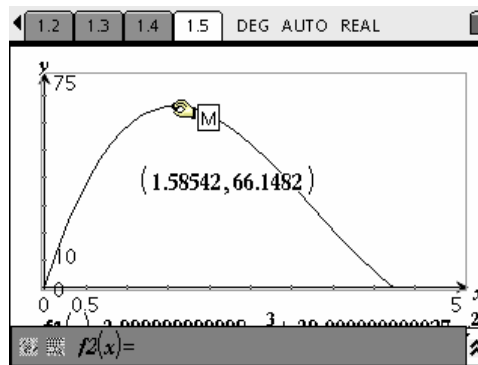







Figure 43

Exploring A Numeric Solution

1. Create a new Lists and Spreadsheet page.
 - On this page, you will look for a numeric solution.
 - Label the first two columns as ht and vol (Figure 44).
 - It will be helpful to make the two columns as wide as possible.
 - Press  followed by : Actions and : Resize. At this point, the entire column will be highlighted.
 - Pressing  will widen the column.
 - Press  to save that width.
 - Repeat for column B. See Figure 45.

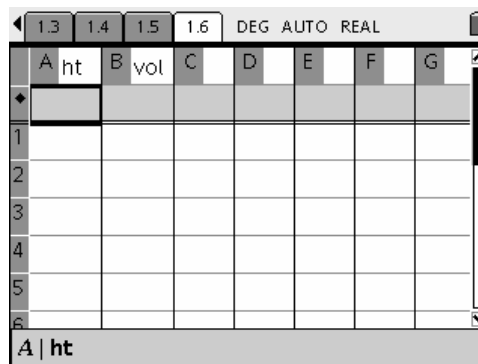



Figure 44

2. For column A (ht) we will set up a sequence to enter height values from 0 to 4.25 inches in steps of 0.25.
 - Move to the row where you see the  Symbol (between the column header and row 1. Here we will enter in our formula.
 - Type =seq(x, x, 0, 4.25, 0.25). Lower bound and upper bound and increment.

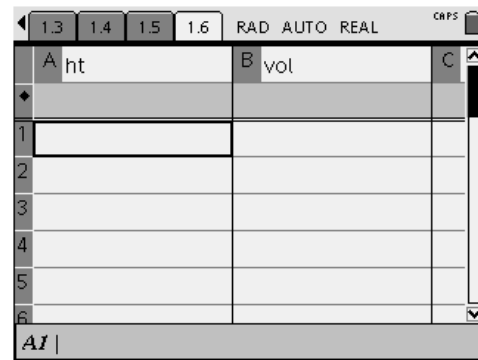



Figure 45

3. After you have pressed , Column A will fill with values for the height. See Figure 46.

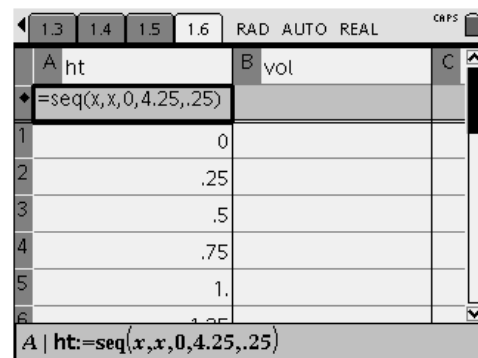



Figure 46

4. The volume can be calculated using the expression $\text{height} \cdot (11 - 2 \cdot \text{height}) \cdot (8.5 - 2 \cdot \text{height})$.
- Enter this expression, starting with an = sign, in the formula row for column B using “ht” to represent height (Figure 47).

A	ht	B	vol	C
◆	=seq(x,x,0,4.25,.25)	←	2*HT)*(8.5-2*HT)	
1	0			
2	.25			
3	.5			
4	.75			
5	1.			
6	1.25			

vol:=HT*(11-2*HT)*(8.5-2*HT)

Figure 47

5. Press  to complete the calculation. Column B should fill with values (Figure 48).

A	ht	B	vol	C
◆	=seq(x,x,0,4.25,.25)	=	ht*(11-2*ht)*(8.5-	
1	0		0.	
2	.25		21.	
3	.5		37.5	
4	.75		49.875	
5	1.		58.5	
6	1.25		63.75	

B1|=0.

Figure 48


6. Scroll down column B and look for the largest value for volume (Figure 49).

A	ht	B	vol	C
◆	=seq(x,x,0,4.25,.25)	=	ht*(11-2*ht)*(8.5-	
4	.75		49.875	
5	1.		58.5	
6	1.25		63.75	
7	1.5		66.	
8	1.75		65.625	
9	2.		63.	

B7|=66.

Figure 49

7. The maximum in this table occurs for a cut size of 1.5 inches. However, this is not necessarily the maximum value. The actual maximum could occur on either side of this value, so we could safely assume that the optimal value occurs for a cut size between 1.25 and 1.75 inches.

- Move to the column formula for height and press . Note that the formula is now ready for editing. See Figure 50.

A	ht	B	vol	C
◆	←	=	ht*(11-2*ht)*(8.5-	
1	0		0.	
2	.25		21.	
3	.5		37.5	
4	.75		49.875	
5	1.		58.5	
6	1.25		63.75	

ht:=seq(x,x,0,4.25,.25)

Figure 50

8. Scroll across the entry, change the values for the lower bound, upper bound and increment, as shown in Figure 51.

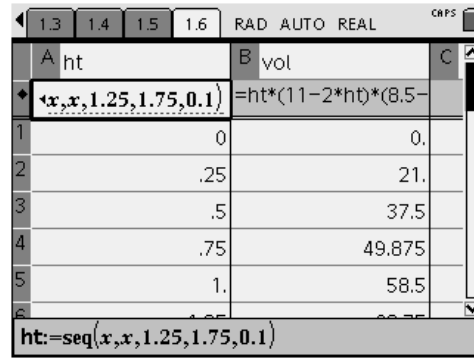


Figure 51

9. When you press Enter a dialog box will appear asking you to confirm that you want to replace the data. Select OK and the values in both columns will update as shown in Figure 52.

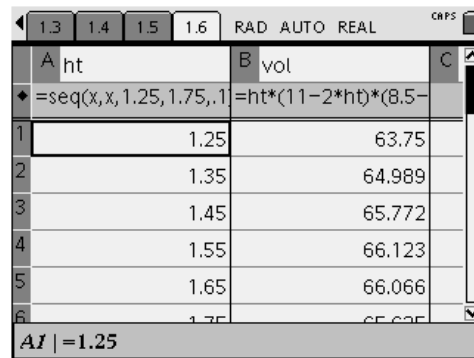


Figure 52

10. Scroll down column B looking for the maximum value of the volume (Figure 53).

11. As before, we could argue that the optimal volume occurs for a cut size is somewhere between 1.45 and 1.65 inches.

- Update the formula by reducing the increment to 0.01, the lower bound to 1.45 and the upper bound to 1.65.

12. Keep repeating this process to find the maximum value of the volume.

- This process could be repeated several times until the desired accuracy is obtained.

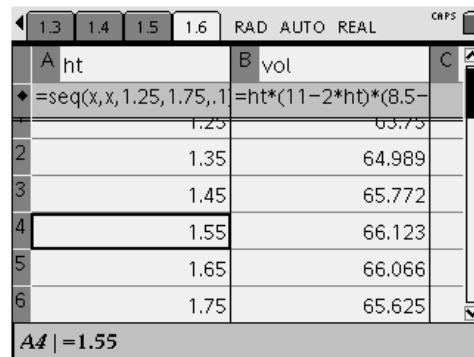


Figure 53

An Algebraic Solution Using CAS

- The final solution uses the CAS features to approach the problem with a Calculus solution. Open a Calculator page, Page 1.7.
 - You already have the function stored in $f1(x)$, so you can use that in a new calculator page (Figure 54).

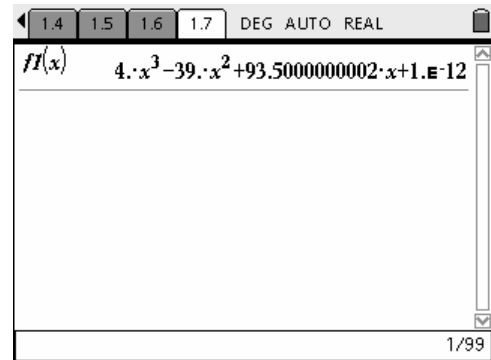


Figure 54

- Define a new function $first(x)$ as the derivative of $f1(x)$ with respect to x .
 - Press $\left[\text{menu} \right]$ followed by $\left[1 \right]$: Tools and $\left[1 \right]$: Define.
 - Type in the function name as $first(x)$.
 - Press $\left[= \right]$ then $\left[\text{menu} \right]$ followed by $\left[5 \right]$: Calculus and $\left[1 \right]$: Derivative.
 - Complete the expression as shown in Figure 55.

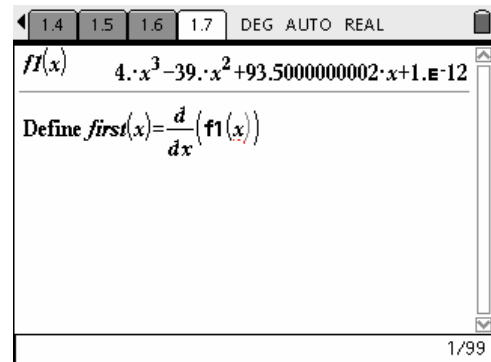


Figure 55

- Press $\left[\text{enter} \right]$ to complete the calculation.
 - Type the expression $first(x)$ to display the derivative function (Figure 56).

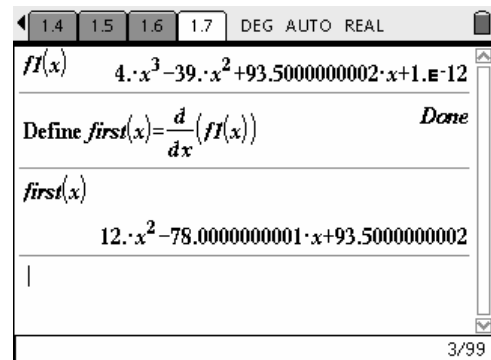


Figure 56

4. To find the optimal value, solve for the roots of the first derivative function.

- Press $\left[\text{menu} \right]$ followed by $\left[4 \right]$: Algebra and $\left[1 \right]$: Solve.
- Complete the expression as shown in Figure 57.

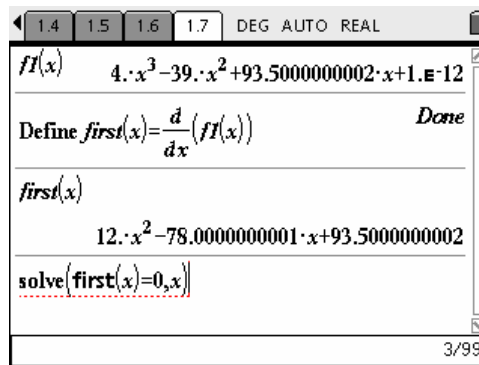


Figure 57

- Press $\left[\text{enter} \right]$ to complete the calculation (Figure 58).

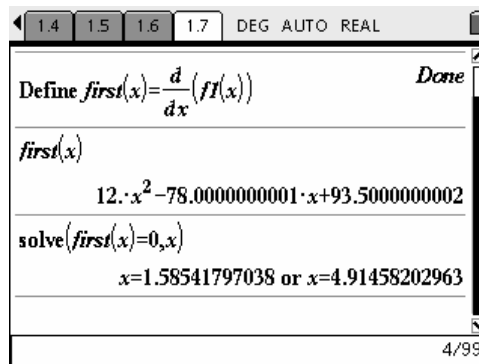


Figure 58

5. Two solutions arise since the function is a cubic and has two turning points. However, the solution $x = 4.91$ is outside the domain of x for this problem and should be discarded. Use the smaller value to find the value for the maximum volume.

- One way to do this is to type in the name of the volume function $f1()$, position the cursor between the parenthesis, move up to the previous line in the history area where the roots of $f1(x)$ occur and press $\left[\text{enter} \right]$.
- Then remove the extraneous solution, along with the phrases “ $x=$ ” and press $\left[\text{enter} \right]$ (Figure 59).

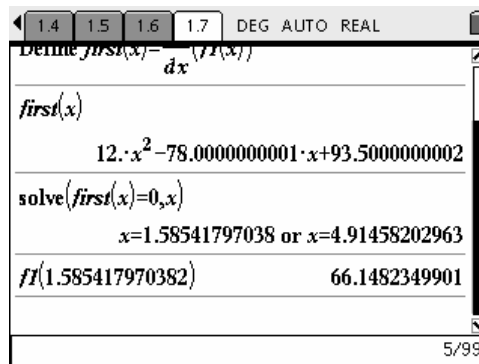


Figure 59

Lesson Plan Notes for CAS Activities for Advanced Algebra

Course(s): Algebra 2, Pre-Calculus and Calculus

Topic(s): Recursive sequences, division of polynomials, Factor Theorem, Remainder Theorem, function notation, graphing, solving equations, exact and approximate roots, complex roots, inequalities, scatter plots, regressions, slope, rates of change, limits, instantaneous rates of change, derivatives, maximums and minimums, intervals where a function is positive or negative and tangent lines.

Related NCTM Standards:

- Algebra Standards:
 - Students should generalize patterns using explicitly defined and recursively defined functions.
 - Students should use symbolic algebra to represent and explain mathematical relationships.
 - Students should identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.
 - Students should draw reasonable conclusions about a situation being modeled.
 - Students should approximate and interpret rates of change from graphical and numerical data.
 - Students should analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.

- Students should judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.
- Students should draw reasonable conclusions about a situation being modeled.

Suggested Total Time: Two hours.

Specific Pre-requisite knowledge:

Students should be familiar with the following:

- Some initial experimentation with the Graphs & Geometry tool where they were required to change the window, manipulate an equation, determine local maximum/minimum points, and calculate slopes.
- Students should be familiar with the algebra menu and the options that are available there.
- The activities in Parts 1 through 4 may have already been done by hand in class and students may be familiar with these topics. On the other hand, some teachers may prefer to use this activity to introduce these topics.

Materials Required/Classroom Set-up/Preparation:

The following equipment will be needed:

- Each student should have access to a TI-Nspire™ CAS + handheld device with the file “D2_4CASAdvancedAlgebra.tns” pre-loaded.
- Students should work in pairs or groups of three in order to help each other. It might be a good idea to seed each group with a strong student.

Lesson Notes:

- The most common error committed by students (and teachers) is not moving the cursor past the exponent. As soon as the exponent for any power is entered, you should press \blacktriangleright to get out of the exponent and enter the rest of the expression.
- The final activity on heart rates is a good way to introduce the Calculus menu.
- Some students may have an initial problem with the symbol for greater than or equal to. You need to enter two keys for this symbol. Once you execute the command, it appears on the screen as a single symbol.
- In the data for the three athletes, some students may be confused about what the “Number of heart beats” means, confusing it with the heart rate. One hopes that no athlete has a heart rate of 0 at any time and a heart rate of 660 would be of equal concern.
- The work in Activity 2 may need some more detail. You could consider providing students with a copy of the step-by-step instructions or doing demonstrations of some of the steps.

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CAS Activities for
Advanced Algebra
Student Worksheet

Name _____

Class _____

Part 1: Division by a polynomial

Questions: Divide the following:

- $(2x^3 - 2x^2 + 5x - 3)$ by $(x - 1)$
- $(15x^2 + 4x + 16 + 10x^3)$ by $(5x^2 + 2)$
- $(4x^3 + 2x^2 - 6x - 13)$ by $(2x + 1)$
- $(4x^3 + 2x^2 - 6x - 13)$ by $(2x + 1)$

Part 2: Remainder/Factor Theorem:

Questions: Find the value of the remainder for each division:

- $(x^2 + 2x + 4)$ by $(x - 2)$
- $(x^3 + 3x^2 - x + 8)$ by $(x + 1)$
- $(2x^3 + 3x^2 - 7x - 3)$ by $(2x + 5)$

Part 3: Finding Roots

Questions: Solve

- $2x^3 + 9x^2 + 10x + 3 = 0$
- $x^3 - 6x^2 + 6x + 8 = 0$
- $x^4 + 2x^3 - 3x^2 - 3x + 1 = 0$

Part 4: Inequalities

(for \leq use \leq and for \geq use \geq)

Questions: Find the intervals for the following:

- $x^5 - 6x^4 + 12x^3 - 8x^2 < 0$
- $x^3 - 3x^2 + 3x - 1 \geq 0$
- $x^3 - 6x^2 + 10x - 4 \leq 0$

Activity 1 with the TI-Nspire™ CAS Calculator Assignment

- Consider the polynomial function $f(x) = x^3 + 6x^2 + 9x + 4$.
 - Find the roots.
 - Find the maximum and minimum points.
 - Find the interval(s) when the function is positive.
 - Sketch the function.
- Answer question #1 for the function $f(x) = -3x^4 - 2x^3 + 15x^2 - 12x + 2$.

3. Determine the value(s) of k in each equation so that one root is double the other root.

a) $x^2 - kx + 50 = 0$

b) $2x^2 - 3x + k = 0$

4. If $f(x) = x^2$ and $g(x) = (x+2)(x-1)(x-3)$ find all x for which $f(x) < g(x)$.

Activity 2 Investigating Heart Rates with the TI-Nspire™ CAS Calculator

A coach wants to test the fitness of his athletes. He runs them through a six minute stress test while a monitor counts the number of heartbeats from time zero. The data from three teammates are shown below:

Time In minutes	Number of heart beats
0	0
1	55
2	120
3	195
4	280
5	375
6	480

Time in minutes	Number of heart beats
0	0
1	68
2	148
3	240
4	344
5	460
6	588

Time in minutes	Number of heart beats
0	0
1	85
2	180
3	285
4	400
5	525
6	660

- Enter the data into the Lists & Spreadsheet.
- Using three separate pages, draw a scatter plot of each table.
- Find an equation for each athlete's number of heart beats by either doing a regression on the calculator or by solving algebraically.
 - Graph the equations found.
- The coach has established a guideline that the athlete's heart rate must not exceed 100 beats per minute at exactly 3 minutes.
 - Do all the athletes seem to meet the criteria?
- When does each athlete's heart rate reach 100 beats per minute?
 - Drag the tangent line to the graph of the regression equation to graphically estimate when this happens for each athlete.
- Find the instantaneous rate of change of the number of heart beats for each athlete's data by finding the value of the derivative at time 3 minutes.
 - Are these answers close to your estimates in question 4?
- Using your equations, when does each athlete's heart rate reach 100 beats per minute?
 - Are these answers close to your estimates in question 5?
- As an alternative approach, add a point on the curve and construct the tangent line at this point.
 - Measure the slope of the tangent line and drag the point until the slope is close to $m = 100$.

CAS Activities for Advanced Algebra

Materials

- TI-Nspire™ Math and Science Learning Handheld
- D2_4CASAdvancedAlgebra.tns file

Overview

This activity introduced various CAS commands suitable for Algebra and Pre-Calculus and Calculus.

Activity 1—Operations with Polynomials

Part 1: Division by a polynomial

Press **(menu)** followed by **(4)**:Algebra, **(6)**:Fraction Tools and **(1)**:Proper Fraction. Enter the expression in the parentheses using separate sets of parentheses for the numerator and denominator. Press Enter. See Figure 1.

Part 2: Remainder/Factor Theorem

Enter the polynomial followed by the **(|)** key (the small gray key below and to the right of the **(ctrl)** key). Enter the value of x and press **(enter)**. See Figure 2.

Part 3: Finding Roots

Press **(menu)** followed by **(4)**:Algebra and **(1)**:Solve. Type in the equation within the parentheses and press **(enter)**. See Figure 3.

1.1 RAD AUTO REAL

$$\text{propFrac}\left(\frac{2 \cdot x^3 + 3 \cdot x^2 - 5 \cdot x + 2}{x + 3}\right)$$

$$\frac{-10}{x + 3} + 2 \cdot x^2 - 3 \cdot x + 4$$

1/99

1.1 RAD AUTO REAL

$$x^3 + 2 \cdot x^2 + 3 \cdot x + 2 | x = 1$$

8

1/99

Figure 2

1.1 RAD AUTO REAL

$$\text{solve}(x^3 - 27 \cdot x^2 + 180 \cdot x - 324 = 0, x)$$

$$x = 3 \text{ or } x = 6 \text{ or } x = 18$$

$$\text{solve}(x^3 - 4 \cdot x^2 + 2 \cdot x + 3 = 0, x)$$

$$x = \frac{-(\sqrt{5}-1)}{2} \text{ or } x = \frac{\sqrt{5}+1}{2} \text{ or } x = 3$$

2/99

Figure 3

Approximate Roots:

Enter the Solve command as before but press **ctrl** followed by **enter** instead of just **enter**. See Figure 4.

Complex Roots:

Press **menu** followed by **4**:Algebra, **8**:Complex and **1**:Solve. Enter the equation as before and press **enter** to complete the operation. See Figure 4.

Part 4: Inequalities

Press **menu** followed by **4**:Algebra and **1**:Solve. Enter the inequality as before using **<** and **=** for \leq . When needed, use **>** and **=** for \geq . See Figure 5.

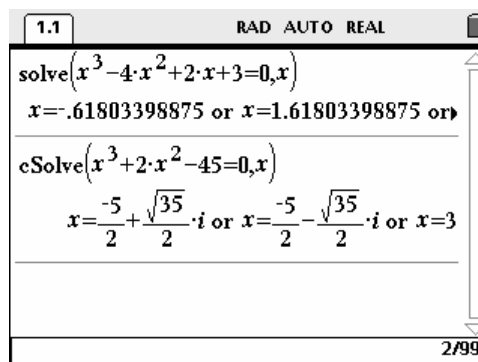


Figure 4

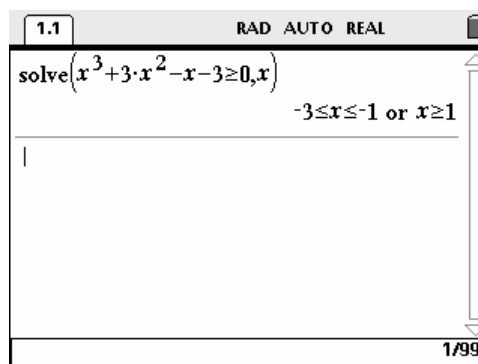


Figure 5

Activity 2 Investigating Heart Rates

A coach wants to test the fitness of his athletes. He runs them through a six minute stress test while a monitor counts the number of heartbeats from time zero. The data from three teammates are shown below:

Time in minutes	Number of heart beats
0	0
1	55
2	120
3	195
4	280
5	375
6	480

Time in minutes	Number of heart beats
0	0
1	68
2	148
3	240
4	344
5	460
6	588

Time in minutes	Number of heart beats
0	0
1	85
2	180
3	285
4	400
5	525
6	660

1. Open a new Lists & Spreadsheet page and enter the data into columns A, B, C and D. Note the column headings (Figure 6).

	A time	B ath1	C ath2	D ath3	E
1	0	0	0	0	
2	1	55	68	85	
3	2	120	148	180	
4	3	195	240	285	
5	4	280	344	400	
6					

Figure 6

2. Graph a scatter plot of each relationship.
 - Open three new Graphs & Geometry pages for the scatter plots of the relationships.
 - Press menu followed by $\{3\}$:Graph Type and choose $\{3\}$:Scatter Plot.
 - Press enter to open the x field and select time.
 - Press tab to move to the y field.
 - Press enter and select ath1.
 - Press tab to move to the OK button and press enter .
 - Repeat for athlete 2 and athlete 3.
 - See Figures 7, 8 and 9.

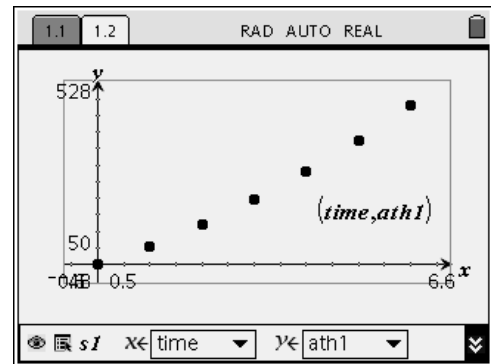


Figure 7

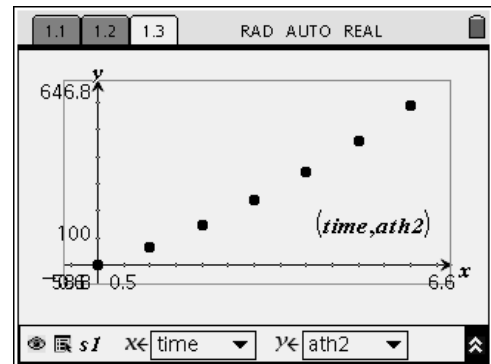


Figure 8

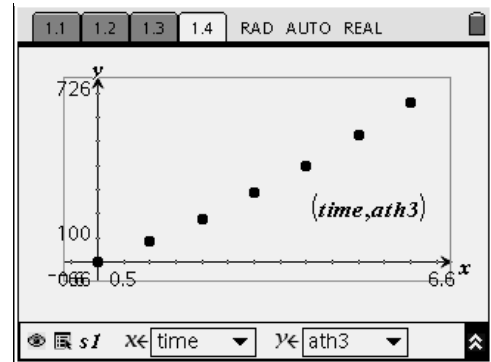


Figure 9

3. Find an equation for each athlete's number of heart beats versus time by either doing a regression on the calculator or by solving algebraically. Graph the equations found.

- Open a new Calculator page and press **(menu)** followed by **7**:Statistics, **1**: Stat Calculations and **6**:Quadratic Regression.
- Press **▼** to open the x List field. Choose time for this field for each regression.
- Press **(tab)** to move to the y List field.
- Choose the appropriate athlete for each regression.
- See Figures 10, 11 and 12.

4. The coach has established a guideline that the athlete's heart rate must not exceed 100 beats per minute at exactly 3 minutes. Do all the athletes seem to meet the criteria? Draw in a line and manipulate the line until it is tangent to the function at time 3 minutes. Find the slope of the tangent line.

- Move back to the first scatter plot.
- Press **(menu)** followed by **3**:Graph Type and choose **1**:Function. Move to f1(x) and press **(enter)** to graph the curve with the scatter plot. See Figure 13.
- Press **(tab)** to move to the entry line and scroll to f4(x).
- Type in the equation $f4(x) = 80x$ as an initial guess for the line tangent to the curve. A line will appear on the screen.
- Press **(esc)** to move the cursor into the graphing window.
- Move the cursor to the end of the line to change the slope of the line and to the middle of the line to change the y-intercept.

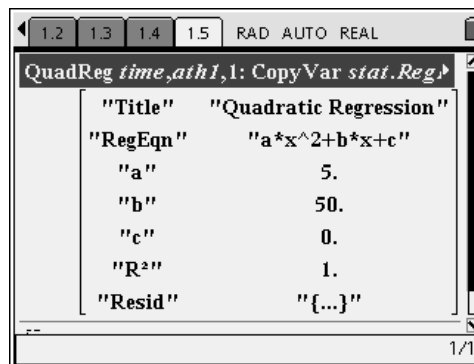


Figure 10

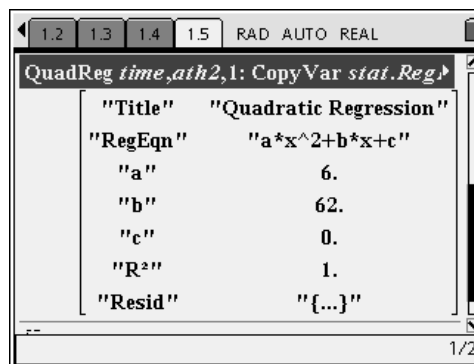


Figure 11

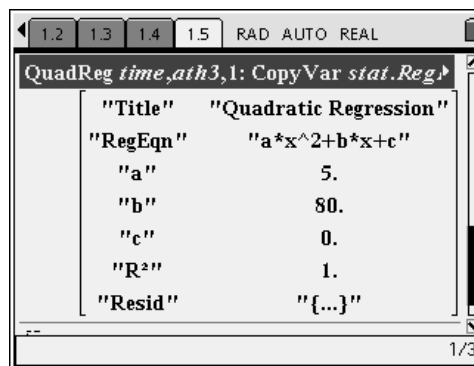


Figure 12

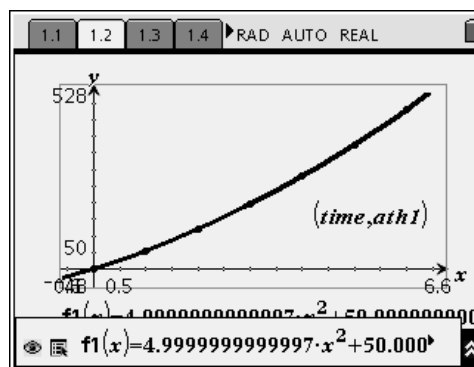


Figure 13

- Continue to make adjustments using these two tools until you have a line that appears to be tangent to the curve at $x = 3$. See Figure 14.
- Repeat this for the other two scatter plots. See Figures 15, 16, 17 and 18.

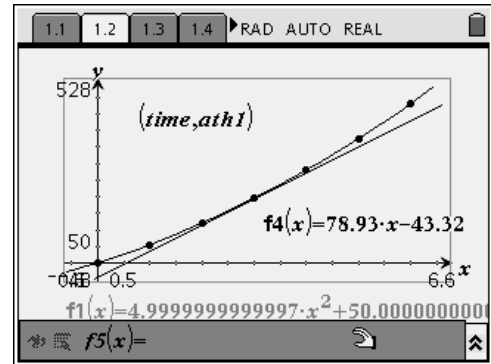


Figure 14

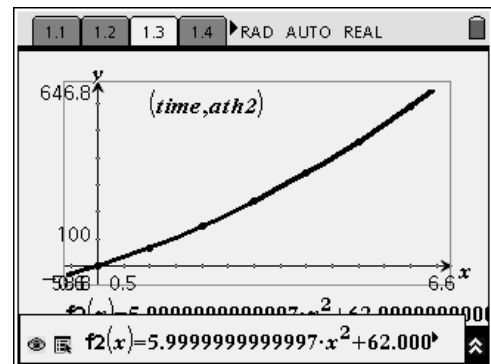


Figure 15

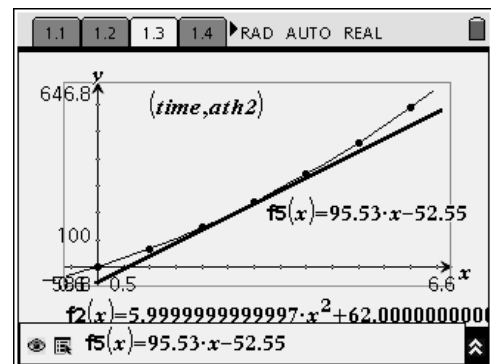


Figure 16

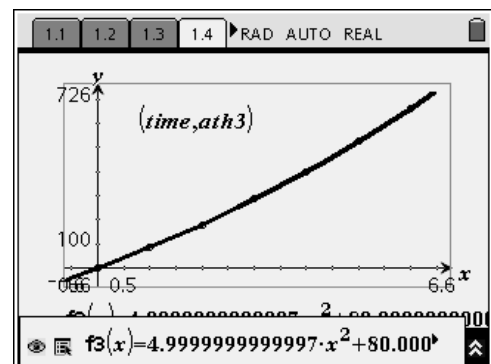


Figure 17

5. When does each athlete's heart rate reach 100 beats per minute? Drag the tangent line to graphically estimate when this happens for each athlete.

- Drag the end of the line until the slope is close to $m = 100$.
- Then, drag the line until it appears to be tangent to the curve.
- Using the tic marks on the x-axis, record the approximate value of x . See Figure 19.
- Repeat for the other two curves.
- See Figures 20 and 21.

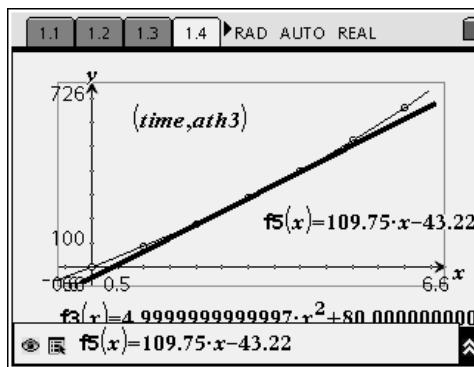


Figure 18

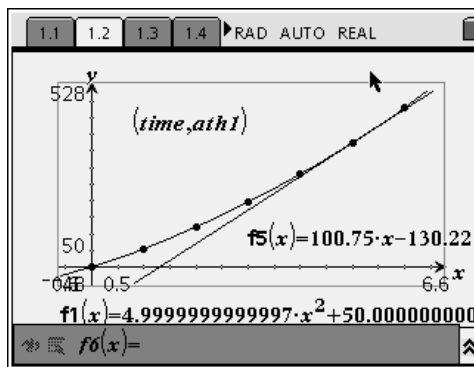


Figure 19

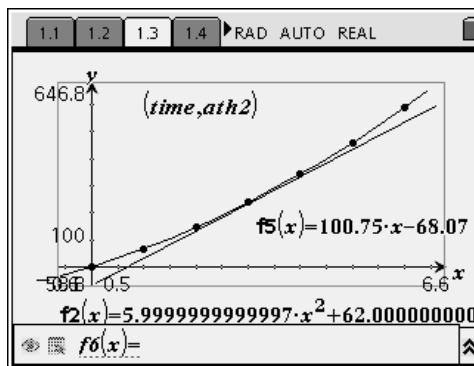


Figure 20

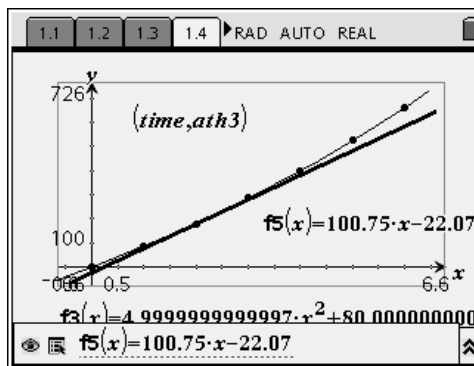


Figure 21

6. Find the instantaneous rate of change of the number of heart beats for each athlete's data by finding the value of the derivative at time 3 minutes. Are these answers close to your estimates in question 4?
- Open a new Calculator page.
 - Press **(menu)** followed by **1**:Tools and **1**:Define.
 - Enter the function name $fp1(x)$ and press the **=** key.
 - Press **(menu)** followed by **5**:Calculus and **1**:Derivative.
 - Complete the derivation expression as shown in Figure 22.
 - On the next line, evaluate the new function at $x = 3$. See Figure 22.
 - Repeat for the other two relationships. See Figures 23 and 24.
7. Using your equations, when does each athlete's heart rate reach 100 beats per minute? Are these answers close to your estimates in question 5?
- Open a new Calculator page.
 - Press **(menu)** followed by **4**:Algebra and **1**:Solve.
 - Set each derivative function equal to 100. See Figure 25.

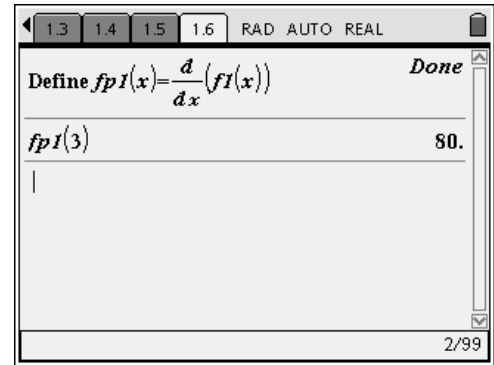


Figure 22

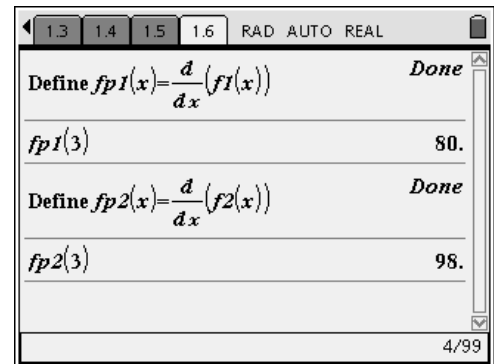


Figure 23

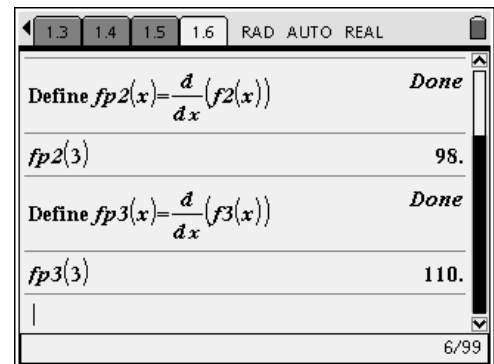


Figure 24

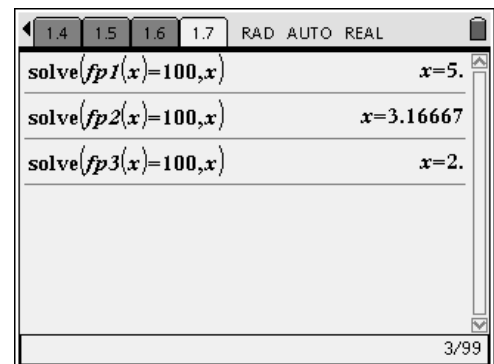


Figure 25

8. As an alternative approach, add a point on the curve and construct the tangent line at this point. Measure the slope of the tangent line and drag the point until the slope is close to $m = 100$.

- Open a new Graphs & Geometry page and plot $f_1(x)$.
- Press **(menu)** followed by **6**: Points and Lines and choose **2**: Point On.
- Move to the curve and click. A point and its coordinates will appear.
- Press **(menu)** followed by **6**: Points and Lines and choose option **7**: Tangent.
- Move to the point on the curve and click on the point.
- A line tangent to the curve at the point will appear. You may need to drag the end points out to see the tangent line better.
- Press **(menu)** followed by **7**: Measurement and choose option **3**: Slope.
- Click on the tangent line and move the slope value to an open area.
- Label the slope as shown in Figure 26.
- Drag the point until the value of the slope (m) is close to 100. See Figure 27.

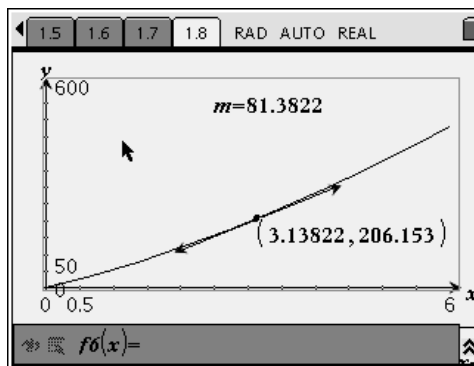


Figure 26

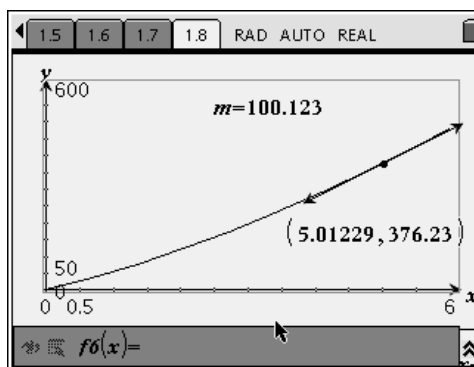


Figure 27

Lesson Plan Notes for the Tale of the Tape

Course(s): Algebra 1 and Calculus

Topic(s): Scatter plots, functions, graphing, equations, recursive sequences, derivatives, and local maximums

Related NCTM Standards:

Algebra Standards:

1. Students should generalize patterns using explicitly defined and recursively defined functions.
2. Students should use symbolic algebra to represent and explain mathematical relationships.
3. Students should identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.
4. Students should draw reasonable conclusions about a situation being modeled.
5. Students should approximate and interpret rates of change from graphical and numerical data.
6. Students should analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.
7. Students should judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.

8. Students should draw reasonable conclusions about a situation being modeled.

Suggested Total Time: One hour

Specific Pre-requisite knowledge:

Students should be familiar with the following:

- Using spreadsheet formulae
- Recursive sequences
- Constructing a scatter plot
- Finding the function of best fit
- Derivatives of functions

Materials Required/Classroom Set-up/Preparation:



The following equipment will be needed:

- Each student should have access to a TI-Nspire™ math and science learning handheld with the file “D3_1TaleofTape.tns” pre-loaded.
- If the CAS solution is to be explored, then students should have access to a TI-Nspire™ CAS math and science learning handheld.

Lesson Notes:

The step-by-step solutions present three approaches for solving this problem. The first two could be done by an Algebra 1 student while the CAS solution is for Calculus students only.

1. If CAS is not an option, an alternative could be to find the point of intersection with the graphs and the x-axis.

2. As students are working through the solutions, you may want to keep the following issues in mind:
 - It is likely that every student or group will have a different solution for the moveable parabola. This should not concern them. Stress that they are not trying to get any particular solution (which you might demonstrate) and that almost every solution has validity. The only solutions that should be excluded are ones where the maximum point occurs after the point (440, 60).
 - Anticipate some problems as students move through the regression and scatterplots. There is likely to be some confusion as to when they should press  to open a field and when to press  on the NavPad cursor control.
 - Encourage students to try different values for the coordinates of the third point in the second approach.
 - The CAS model is the most straightforward, although younger students would have no appreciation for the mathematics involved in this solution.
3. Students will need to be kept on task. Each model might take some time, and they might become discouraged.

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The Tale of the Tape

Student Worksheet

Name _____

Class _____

In 1997, Mark McGwire hit a monstrous home run off pitcher Randy Johnson. The ball was hit from a height of about 3 feet and it landed in the stands 440 feet from home plate, 60 feet above the playing surface. At the time, it was claimed that the ball would have traveled 538 feet if the path were unobstructed.

Your problem is to investigate the path of the ball three ways and to determine what distance you think the ball might have traveled if not stopped by the stands.

Before you begin the investigation, you might like to try websites such as www.mlb.com which has several videos of home runs archived. The site www.HitTrackerOnline.com has archived video from long home run hits from the past few years.

Approach 1 – Using a Parabola

In the first approach to this problem, you will use a moveable parabola to try to physically fit a curve to the two points. At one point, you will be asked to determine how close your curve fits the two points.

1. Enter the function evaluations below.

$f(0) =$ _____ and $f(440) =$ _____

2. What is the meaning of the x -intercept in the context of this question?

The Tale of the Tape

Materials

- TI-Nspire™ Math and Science Learning Handheld
- D3_1TaleofTape.tns

Overview

In this activity, students will model the path of a baseball and solve problems related to the distance the ball travels.

Introduction

In 1997, Mark McGwire hit a home run off of Randy Johnson. The claim was made that the ball would have traveled 538 feet if it hadn't landed in the stands.

The ball was hit from a height of about 3 feet, and it landed in the stands 440 feet from home plate, 60 feet above the playing surface.

What we have to do is to create a model for this situation and determine how far this ball might have traveled.

There are three different approaches to the problem:

1. Using the movable parabola feature.
2. Using a regression equation.
3. Using the CAS. In this approach, we add in another piece of information and make use of some physics formulae.

Approach 1: The Movable Parabola

1. Open a new document.
2. Add a Notes application page.
3. Type the information related to this problem as shown on page 1.1 in the .tns file.
4. Add a second and third Notes application page to this document to include the additional information.
5. Add a Lists & Spreadsheet application page.
6. Enter the distances into column A, and label this column *dis*.
7. Enter the heights into column B, and label this column *ht*. See Figure 1.
8. Add a Graphs & Geometry application page, and press **(menu)** **3** **3** for Menu 3: Graph Type, 3: Scatter Plot.
9. Choose *dis* for the x-entry and *ht* for the y-entry.
10. You will need to adjust the window by pressing **(menu)** **4** **1** for Menu 4: Window, 1: Window Settings.
11. Complete the dialog box using appropriate values. Press **(enter)** (Figure 2).
12. Insert a Notes application page, and include the information given in the .tns file on page 1.6.
13. Press **(ctrl)** **▲** to display the page sorter.
14. Highlight page 1.5 (Figure 3).
15. Press **(ctrl)** **(C)** to copy the page.

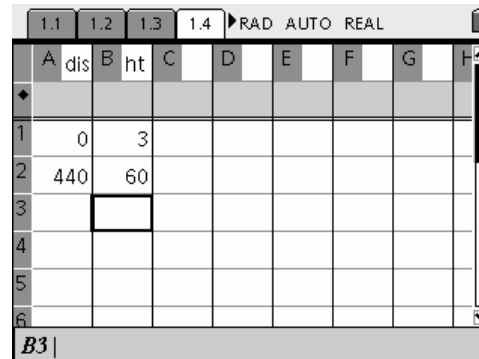


Figure 1

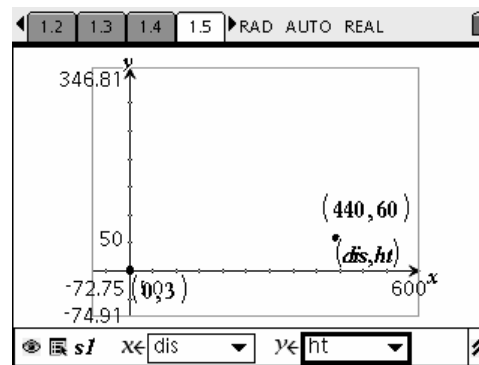


Figure 2

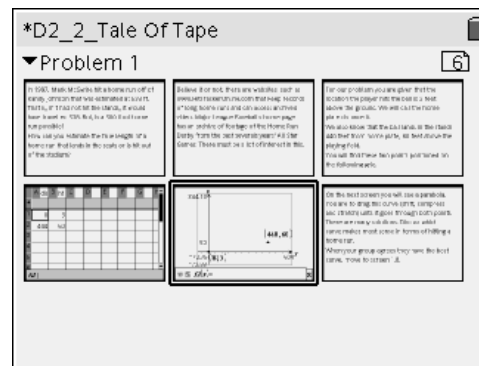


Figure 3

16. Move to the right to highlight page 1.6.
17. Press ctrl V to insert a copy of page 1.5 after page 1.6.
18. On page 1.7, press menu 3 1 for Menu 3:Graph Type, 1: Function.
19. For the equation, enter a function that opens down.
20. Discuss the rationale for this beginning equation with the students.
 - The function in Figure 4 started as $f_1(x) = -0.001(x-250)^2 + 300$.
21. Move your cursor to one of the ends of the parabola, and note the symbol that appears. See Figure 5.
22. Press ctrl ☒ to grab the parabola.
 - You will be able to bend the graph in both directions.
23. When you are satisfied with the shape of the parabola, move the cursor close to the vertex, and note that a different symbol appears. See Figure 6.
24. Press ctrl ☑ to grab the point, and you will be able to drag the parabola to a new location.
25. Move the parabola to a position where it appears to pass through the two points.
 - There are an infinite number of graphs that are possible solutions. One graph is shown in Figure 7.

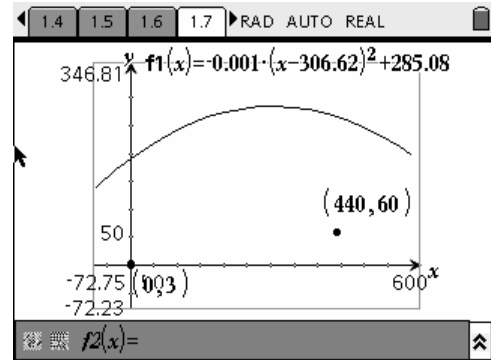


Figure 4

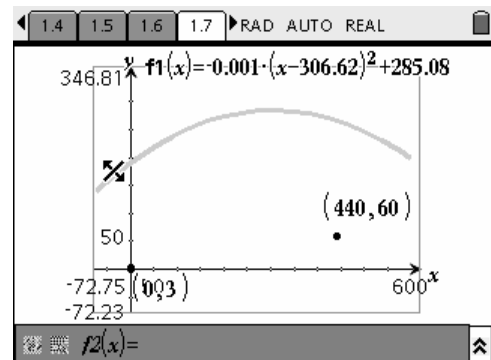


Figure 5

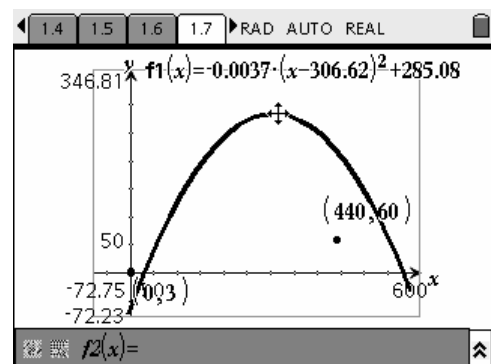


Figure 6

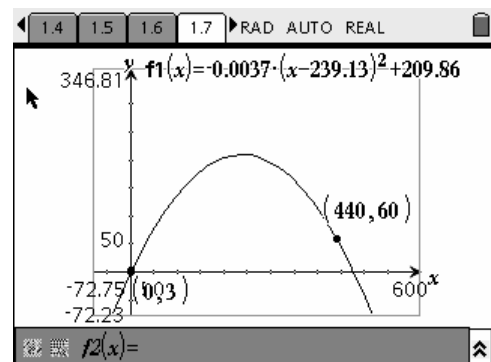





Figure 7

26. Add a Notes application page. Type the information related to this problem as shown on page 1.8 in the .tns file.
27. To test whether the function graphed on page 1.7 passes through the two points, students will be instructed to insert a Calculator application page, and find the actual values for $f(0)$ and $f(440)$ (Figure 8).
- Comments may be added to a Calculator page by pressing    for Menu 1:Tools, 6:Insert Comment.
 - If these function values are close to 3 and 60, respectively, you should continue.
 - If not, go back to the Graphs & Geometry page, and change the function again.
28. Add a Notes application page. Type the information related to this problem as shown on page 1.10 in the .tns file.
29. If using a CAS device, add another Calculator application page, and solve for the x -intercept of $f(x)$. See Figure 9.

Approach 2: Using a Regression

Note: Pages 2.1 and 2.2 in the .tns document are Notes pages. Page 2.3 is a copy of the spreadsheet on page 1.4 with the screen split vertically and a Graphs & Geometry application added.

The first two rows of the spreadsheet are the same as before, using the two points provided in the problem. The third point, (220, 100), is completely arbitrary. See Figure 10.

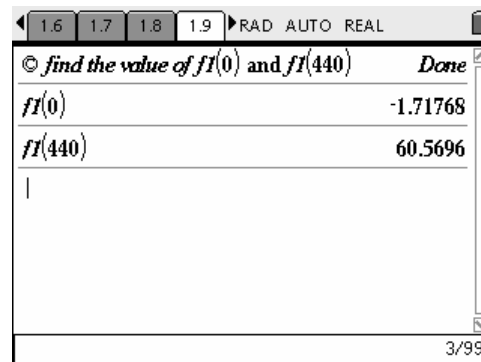


Figure 8

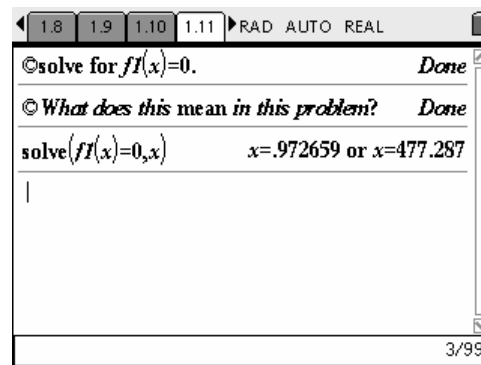


Figure 9

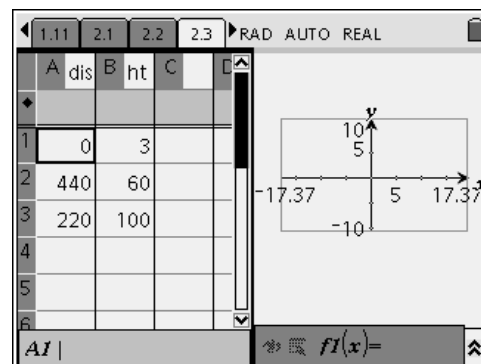


Figure 10

1. While in the Lists & Spreadsheet work area, press **(menu)** **(4)** **(1)** **(6)** for Menu 4:Statistics, 1:Stat Calculations, 6:Quadratic Regression.
2. Press **(tab)** to move between fields.
3. Complete the top part of the dialog box as shown in Figure 11.
4. As you move down the Quadratic Regression dialog, fill the 1st Result Column entry field with d [] .

Note: The [] comes from typing [and]—it is not a special symbol.

5. Press **(tab)** to move to the bottom of the dialog box, and press **(enter)** on the OK button.
 - You should see the results in columns D and E. See Figure 12.

6. Press **(ctrl)** **(tab)** to move to the Graphs & Geometry work area.
7. Press **(menu)** **(3)** **(3)** for Menu 3:Graph Type, 3:Scatter Plot.
8. At the bottom of the work area, for the x entry, choose d i s, and for the y entry, choose h t. See Figure 13.

9. Press **(menu)** **(4)** **(9)** for Menu 4:Window, 9: Zoom Stat. See Figure 14.
10. Press **(menu)** **(3)** **(1)** for Menu 3:Graph Type, 1:Function.

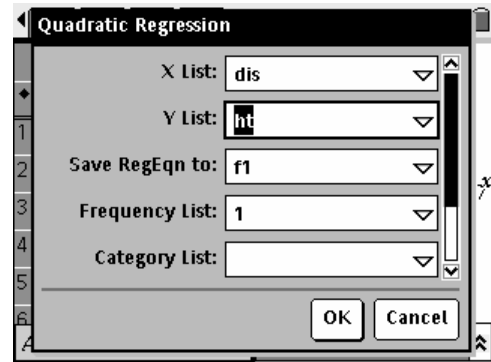


Figure 11

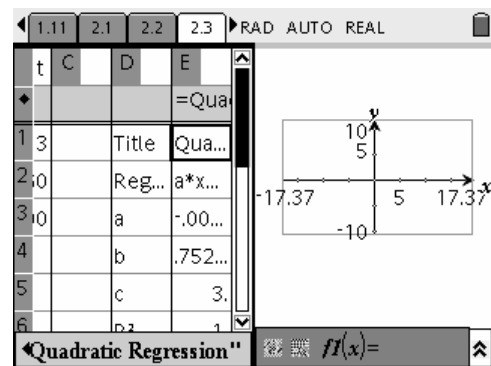


Figure 12

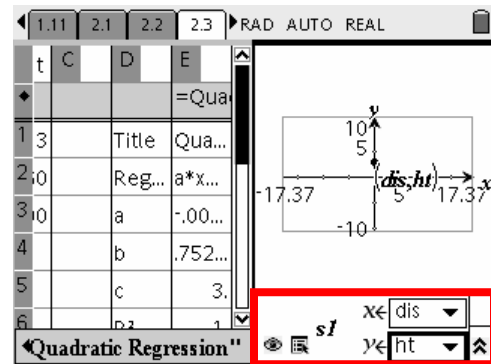


Figure 13

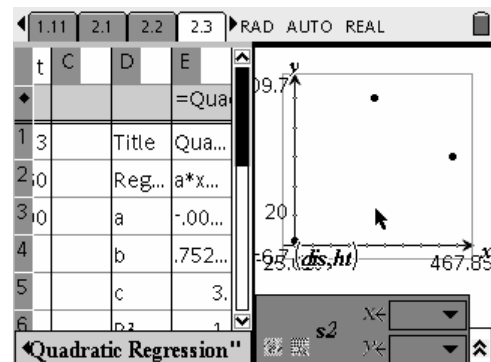




Figure 14

11. Use the NavPad cursor controls to move up to $f_1(x)$ in the entry line, and then press . See Figure 15.
12. Add a Notes application page. Type the information related to this problem as shown on page 2.4 in the .tns file.
13. If using a CAS device, insert a Calculator application page, and solve for the x -intercept(s) of $f_1(x)$. See Figure 16.

Note: An interesting aside is shown in this approach when you go back to the Lists & Spreadsheet page (2.3) and change the height for the third point.

As soon as you press , you will see the point change in the Graphs & Geometry page, as well as in the regression equation. See Figure 17. However, the last solution for the x -intercept will need to be re-calculated.

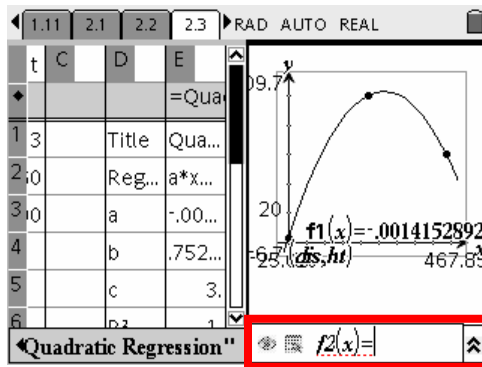


Figure 15

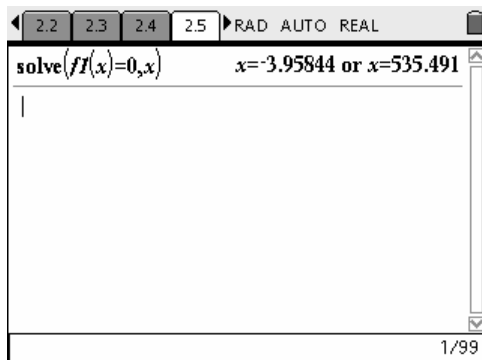


Figure 16

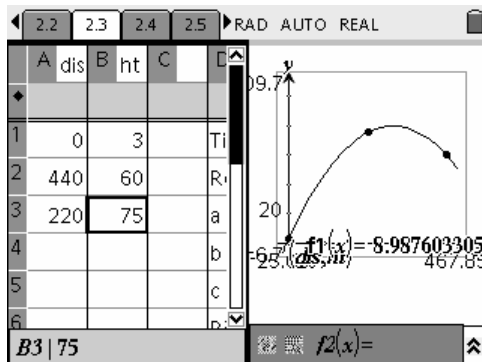


Figure 17

Approach 3: Using the CAS

To create the TI-Nspire document in Problem 3 insert six Notes application pages. Type the information related to this problem as shown on page 3.1 to 3.6 of the .tns file.

The following illustrates potential solutions as students insert pages and work through the investigation. The page numbering will change as students insert pages.

Note: Additional information is that the ball took 5.1 seconds to travel from home plate to the stands.

- On page 3.2, students are instructed to insert a Calculator application page, and enter the calculation for the forward velocity (page 3.3 - Figure 18).
- Press ctrl followed by enter for an approximate answer.
- Store this value to variable v .
 - To get the right arrow, press ctrl followed by stor var . See Figure 18.
- On page 3.5 students will be instructed to insert another Calculator application page.
- Press menu 1 1 for Menu 1:Tools, 1:Define.
 - Define the function $h(t)$ as shown in Figure 19.
 - Use v as the coefficient of the second term, and be sure to enter a multiplication symbol in between v and t .

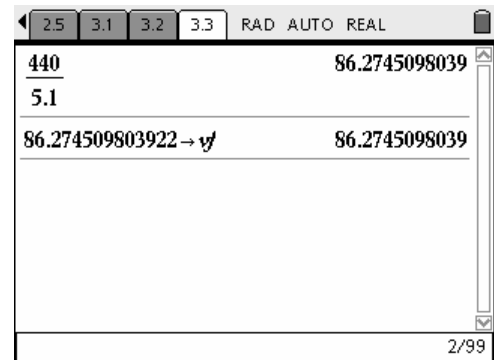


Figure 18

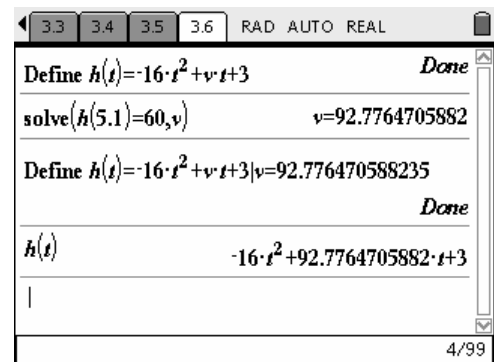


Figure 19

6. In the second line, press MENU 4 1 for Menu 4:Algebra, 1:Solve.
7. On the third line, re-define the function by copying the command from the first line, pressing the I key, and copying the result from the last line.
8. Enter $h(t)$ and press ENTER , the updated function will be displayed. See Figure 19.
9. On page 3.7 students will be instructed to insert another Calculator application page.
10. On the first line, solve for the x -intercept of the function (Figure 20).
 - Since the first answer is negative, that solution should be discarded.
11. On the next line enter V F $\text{STO} \rightarrow \text{VAR}$.
12. To get the constant, move up to the previous line, and highlight the results. Press ENTER to paste the results, and use the CLEAR key to edit out all of the variables and the negative decimal answer.
13. Press ENTER . See Figure 20.
14. On page 3.9 students will be instructed to insert a new Calculator application page.
15. Define $dh(t)$ to be the derivative of $h(t)$ with respect to t (Figure 21).
 - The second line is not necessary, but it does show the derivative function.
16. In the third line, solve for the zero of the derivative function.
17. In the fourth line, note the use of the I key between the name of the function and the result. See Figure 21.

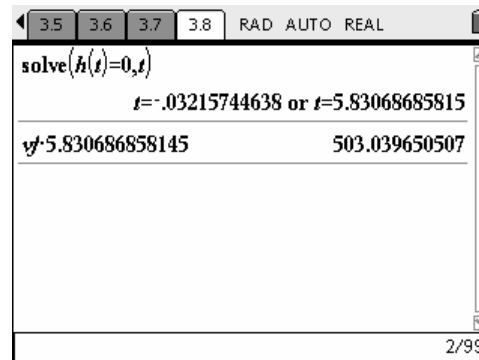


Figure 20

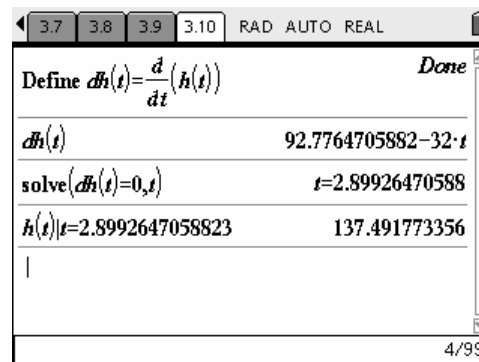


Figure 21

Lesson Plan Notes for Derivatives for Algebra1

Subject: Calculus

NCTM Standards

All students should...

Grades 9-12 Algebra Standards:

1. generalize patterns using explicitly defined and recursively defined functions
2. analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior
3. understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on more-complicated symbolic expressions
4. understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions
5. interpret representations of functions of two variables
6. use symbolic algebra to represent and explain mathematical relationships
7. judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology
8. identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships
9. draw reasonable conclusions about a situation being modeled

Activity Overview

As students drag point X along the x -axis, the tangent line “slides” along the graph of the parabola and the slope of the tangent line changes. The slope of the tangent line depends on the shape of the graph and the point of tangency. Students should be able to verbalize these relationships from their observations.

- They should then construct a locus that shows the path of point P .
- They should be able to describe how this locus summarizes information about the slope of the parabola.
- Students will then change the parabola to other functions such as a cubic or a sine wave and repeat the process.

The technique of constructing perpendicular lines through point X and the transferred point on the y -axis is called “cross-hairing” (like the cross-hairs in a telescopic sight or surveyor’s transit). In a mathematical sense, these two perpendicular lines project their individual values across the plane allowing the change of the two variables to be viewed as the pathway of this point (Figure 1).

Materials Needed

“D3_2Derivative.tns” document

Derivative as a Function student worksheet

Prerequisite Knowledge

Students should be familiar with the concept of the derivative as the slope of a tangent line. However, they should not know about the derivative as a function since this activity will help them discover this new way of thinking about derivatives.

Pedagogy

Students are sometimes confused in Calculus when the term *derivative* is used in one context to represent a numeric value and used in another context to represent a function. This activity should help them understand these two uses of the derivative.

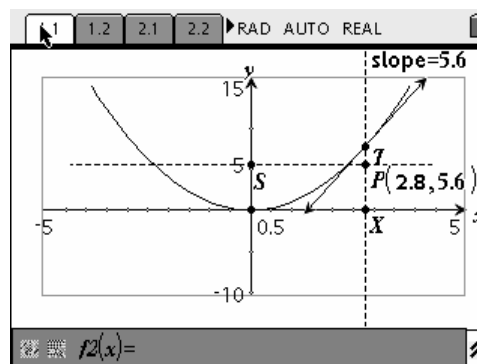




Figure 1

Properties that may need some prompting from the teacher are related to special positions of the tangent line, e.g., when its slope is zero, when the direction of rotation changes, why the graph of the derivative looks the way it does, or what happens off the edges of the screen.

The idea of mapping the value of the slope of the tangent line to the y -axis is the key element in the visual technique of this activity. Once this value is mapped to the y -axis, the stage is set to produce the graph of a function “derived” from a variable dependent on the actual shape of the original function’s graph.

Instructions to Create the Document

1. Open a new document.
2. Add a Graphs & Geometry application.
 - Enter the function $f1(x) = x^2$ in the function edit line (Figure 2).
 - Use the **Window Settings** option in the **Window** menu to set a $[-5,5] \times [-10,15]$ viewing window.
 - Use the **Hide/Show** tool in the **Tools** menu to hide the equation $f1$ in the viewing window.
3. Use the **Point On** tool in the **Points & Lines** menu to place a point on the x -axis.
 - Take care to place the point between hash marks.
 - Immediately after placing the point on the axis, press   to attach a label to the point.
4. Place a line perpendicular to the x -axis at point X using the **Perpendicular** tool in the **Construction** menu.
5. Use the **Intersection Point(s)** tool in the **Points & Lines** menu to mark the intersection of the perpendicular line and the graph of $f1(x) = x^2$.
 - Label the point “ T ” (Figure 3).
6. Create a tangent line to the curve at T with the **Tangent** tool in the **Points & Lines** menu.

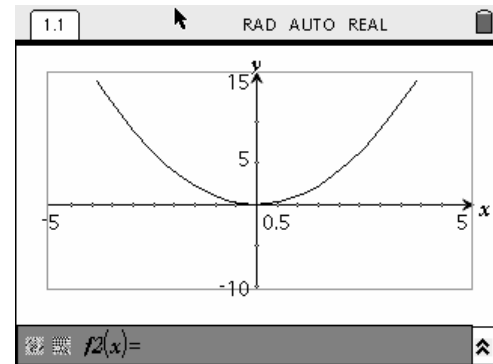


Figure 2

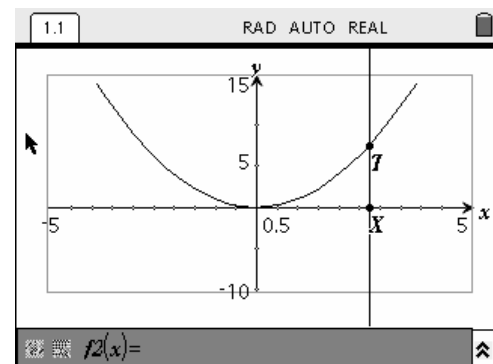




Figure 3

- Be certain the point T is blinking (selected) when you place the tangent line.
- Find the slope of the tangent line with the **Slope tool** in the **Measurement menu**.
 - After clicking the tangent line, the value of the slope should appear.
 - Move this slope to the upper right-hand corner of the screen and click again (Figure 4).
 - Use the **Text tool** in the **Tool menu** to insert the label “*slope =*” in front of the slope value.
 - You may need to drag the label to adjust the placement.
 - Adjust the precision of the slope to one decimal place by hovering the pointer tool over the value of the slope and pressing  several times (Figure 5).
 - Now, use the **Measurement transfer tool** in the **Construction menu** to transfer the value of the slope on to the y -axis. A point should appear on the y -axis.
 - Label this point S .
 - Use the **Perpendicular tool** in the **Construction menu** to construct a perpendicular line to the y -axis through point S .
 - Use the **Attributes feature** in the **Tools Menu** to change the style of the two perpendicular lines to *dashed* (Figure 6).
 - Find the intersection of the perpendicular lines with the **Intersection Point(s) tool** in the **Points & Lines menu**.
 - Label this point P .
 - Use the **Coordinates and Equations tool** in the **Tools menu** to find the coordinates of point P .
 - Adjust the precision of the coordinates to one decimal place by hovering the pointer tool over each coordinate and pressing .
 - You may need to move the coordinates slightly but keep them close to point P .
 - Drag point X along the x -axis.
 - The tangent line, the slope, and point P should adjust accordingly (Figure 7).

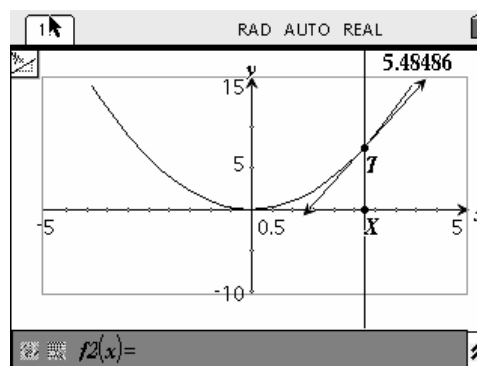


Figure 4

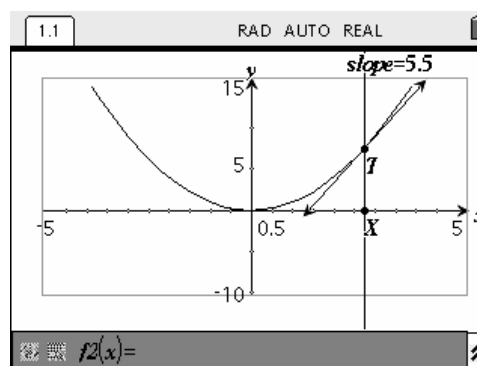


Figure 5

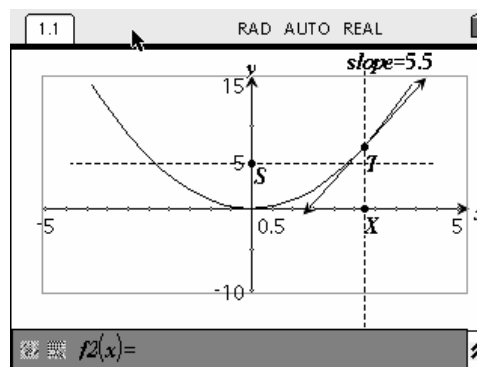


Figure 6

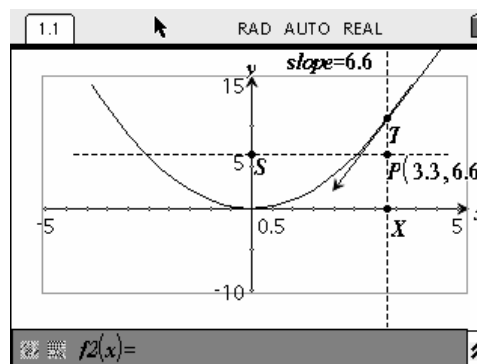


















Figure 7

12. Store the x -coordinate of point P to a variable named px .
- To accomplish this task, select the x -coordinate with the **Pointer tool**, press , select **1:Store Var**, and then press    .
 - The value of the x -coordinate should now appear in bold print. Press  to return to the **Pointer tool** mode.
13. Store the value of the slope in a variable named $slope$.
- To do this, select the slope value with the **Pointer tool**, press , select **1:Store Var**.
 - Since the name “slope” already appears in the text box, just press . Press  to return to the **Pointer tool** mode.
14. Press  and insert a **Lists & Spreadsheet** application.
15. Name the first column “xc” and the second column “slp” (Figure 8).
16. Link column A to the variable px for manual data capture with the following procedure:
- Move the cursor to the formula entry cell for column A and then press  and select **3:Data**.
 - Now select **2:Data Capture** and then **2:Manual Data Capture**. A formula should open in the formula entry cell.
 - Press  and select **3:Link To:**
 - Highlight variable px , press  to paste this variable back to the formula and then press  again to save the formula (Figure 9).
17. Use a similar process to link column B to the variable $slope$ (Figure 10).
18. Press   to return to the **Graph & Geometry** application.

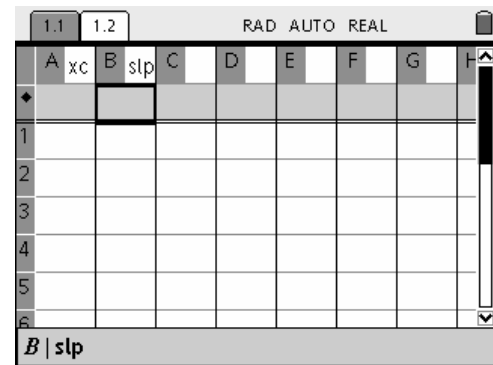


Figure 8

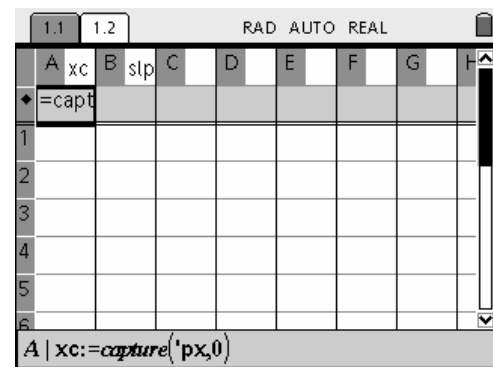


Figure 9

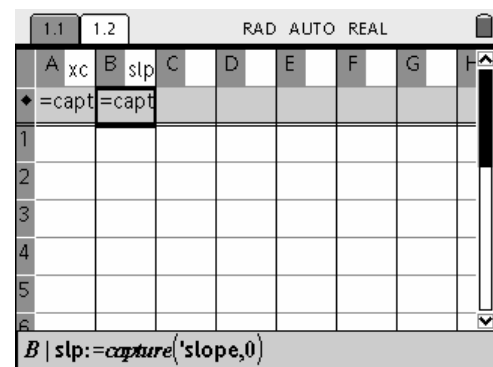


Figure 10

19. Press **menu** and select **3:Graph Type** and then **3:Scatter Plot**.
 - Select *xc* for the x-values and *slp* for the y-values (Figure 11).
20. Select **1:Function** from the **Graph Type menu** to return to function graphing mode.
21. Even though you are not finished, save the file as “tanslope.”
 - To save: Press **ctrl** **home**, select **1:File** and then **4:Save As**.
 - Type in tanslope for the file name and press **enter**.
22. Next, you are going to add another problem and repeat steps 2 through 19. However, in this problem, you will graph the cubic function $y = x^3 - 5x + 2$ in a $[-5,5] \times [-10,10]$ viewing window.
 - Press **ctrl** **home** then select **2:Edit** and **8:Insert Problem** (Figure 12).
23. Now repeat steps 2-19 with the cubic function $y = x^3 - 5x + 2$ in place of the parabola $y = x^2$.
 - Use a $[-5,5] \times [-10,10]$ window instead of a $[-5,5] \times [-10,15]$ window (Figure 13).
24. Save the updated activity by pressing **ctrl** **home** then select **1:File** and **3:Save**.
25. Now you will add one more problem and repeat steps 2 through 19. However in this problem you will add the sine function $y = 5\sin(x)$ in a $[-7,7] \times [-10,10]$ viewing window.
 - Press **ctrl** **home** and then select **2:Edit** and **8:Insert Problem**.
 - Repeat steps 2 through 19 for the sine function (Figure 14).
26. Select the **Save** feature from the **File menu** to save the finished activity.

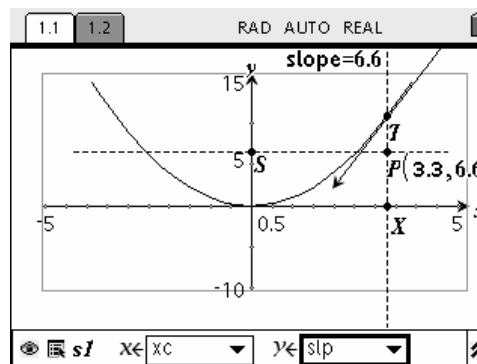


Figure 11

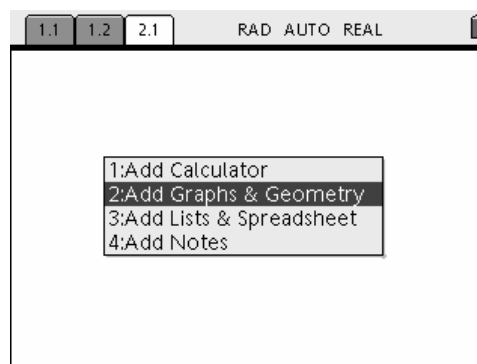


Figure 12

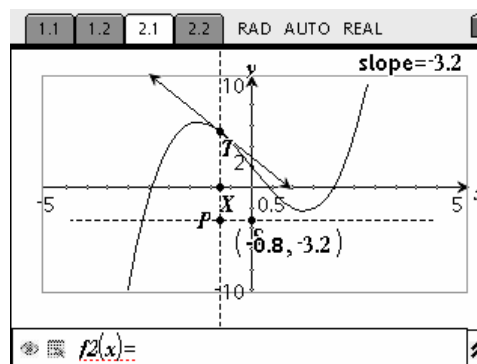


Figure 13

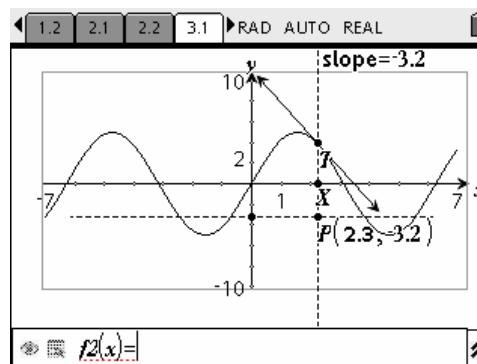


Figure 14

Derivatives for Algebra1

Student Worksheet

Name _____

Class _____

In this activity you will explore the changes to the slope of a tangent line as the point of tangency changes. This will be done numerically and summarized visually. The visual summary can be thought of as a new slope function called a derivative. Next you will investigate further what happens to the slope function as the original function changes.

1. Open the file D3_2Derivative.tns. This is the graph of $y = x^2$ (Figure 1).

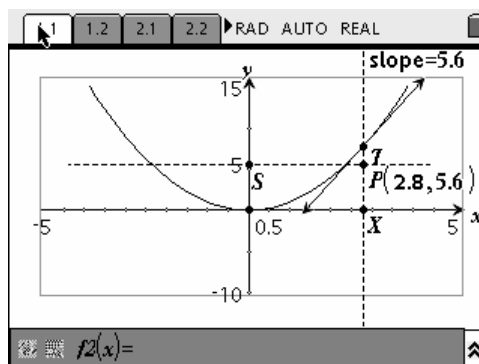


Figure 1

Drag point X along the x -axis to move the tangent line. Answer the following questions.

- When is the slope of the tangent line positive?
 - When is the slope of the tangent line negative?
 - When is the slope of the tangent line zero?
2. What do the x and y coordinates of the point P have to do with the tangent line?
 3. Drag point X and notice the path point P follows.
 - You can plot a point at the current location of point P by pressing **ctrl** **.**.
 - Use the Hide/Show tool to hide the label (xc,slp) that appears when you first press **ctrl** **.**.
 - When you move point P away, you should see a point marking the old location of P .
 - Marking some of the locations of point P should help you see the path point P follows.
 - You can press **ctrl** **▶** to move to the next page and see the coordinates of the points you have marked.
 - Press **ctrl** **◀** to move back to the graph. Describe the path of point P .

4. Make a locus of point P as point X changes (Figure 2).

- Press **menu**. From the Construction menu, choose Locus.
- Click on point P and then point X .
- Sketch your results in the window provided.

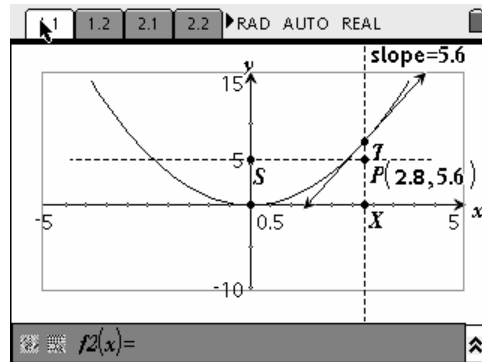


Figure 2

5. This locus is called a derivative. What does the derivative tell us about the graph of the parabola?

6. See if you can find an equation for the derivative.

- Enter the equation in $f2(x)$ to check.
- Does it match the locus?
- Write your equation for the derivative:

7. Predict what will happen to the derivative as you make the following changes to the parabola. Don't actually make the changes to the parabola on your calculator until you have written down your predictions.

- Vertical shift
Prediction:
Actual:
- Vertical stretch
Prediction:
Actual:
- Vertical shrink
Prediction:
Actual:

8. Press **ctrl** \blacktriangleright twice to move to Problem 2.1 (Figure 3).

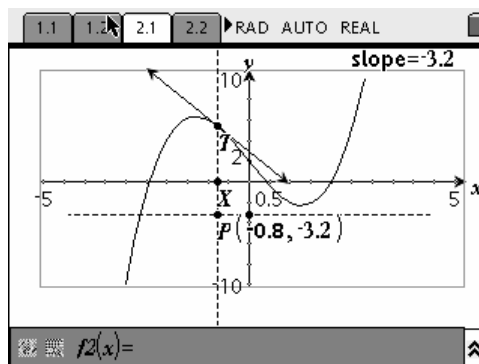


Figure 3

- This is the graph of $y = x^3 - 5x + 2$. Consider what will happen to the slope point P as you drag the tangent line.
- When will point P be below the x -axis?
 - When will point P be above the x -axis?
 - When will point P be on the x -axis?
9. Predict the graph of the derivative of $y = x^3 - 5x + 2$.
- Sketch your prediction in the window above.
 - Then check your answer by making a locus of points.
10. See if you can find an equation for the derivative.
- Enter the equation in $f2(x)$ to check.
 - Does it match the locus?
 - Write your equation for the derivative:

11. Press **ctrl** \blacktriangleright twice to move to Problem 3.1 (Figure 4).

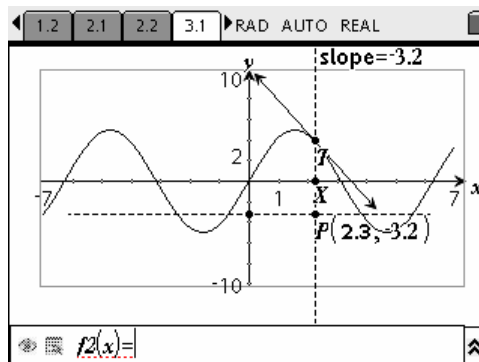


Figure 4

This is the graph of $y = 5\sin(x)$. Consider what will happen to the slope point P as you drag the tangent line.

- When will the slope be positive?
- When will the slope be negative?
- When will the slope be zero?

12. Predict the graph of the derivative of $y = 5\sin(x)$.

- Sketch your prediction in the window above.
- Then check your answer by making a locus of points.

13. See if you can find an equation for the derivative.

- Enter the equation in $f2(x)$ to check.
- Does it match the locus?
- Write your equation for the derivative:

14. Transform the sine wave (i.e., vertical shift, vertical stretch, vertical shrink).

- What happens to the derivative?

15. Explain why sometimes when you are asked to find a derivative in a calculus book the answer is a single number and other times the answer is a function.

Extension

Add a calculator page and use the CAS derivative command to find the derivative of $y = x^2$.

- Does your result match your equation in step 7?
- Use CAS to find the derivatives of $y = x^3 - 5x + 2$ and $y = 5\sin(x)$.
- Do your answers match the results in steps 11 and 14?

Chords in Circles

Concepts

- Chords
- Loci
- Ellipses

Materials

- TI-Nspire™ Math and Science Learning Handheld

Overview

In this activity, students will explore:

- The relationship between the length of a chord and its distance from the center of the circle; and
- The relationship between the lengths of chords and their distances from the center of the circle.

NCTM Standards

All students should...

Grades 9-12 Algebra Standards:

1. interpret representations of functions of two variables
2. use symbolic algebra to represent and explain mathematical relationships
3. use a variety of symbolic representations, including recursive and parametric equations, for functions and relations
4. identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships
5. draw reasonable conclusions about a situation being modeled

Grades 9-12 Geometry Standards:

1. analyze properties and determine attributes of two- and three-dimensional objects
2. explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them
3. establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others
4. use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations
5. investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates

6. draw and construct representations of two- and three-dimensional geometric objects using a variety of tools
7. use geometric models to gain insights into, and answer questions in, other areas of mathematics

Grades 6-8 Geometry Standards:

1. use geometric models to represent and explain numerical and algebraic relationships

Introduction

Students can investigate these properties using the pre-made TI-Nspire document file, “D3_3Chords.tns” which includes some constructions done for them. Alternatively teachers may desire students to do all the constructions themselves. The instructions that follow here illustrate all the constructions.

Part 1

1. Open a new document, and add a Graphs and Geometry application page (Figure 1).
2. Press **menu** **2** **1** for Menu 2:View, 1:Hide Axes and then **menu** **2** **3** for 3:Hide Entry Line (Figure 2).
3. Construct a circle by pressing **menu** **8** **1** for Menu 8:Shapes, 1:Circle.
4. After clicking to choose the location of the center, immediately label the center by pressing **caps** **A** (Figure 3).
5. Using the NavPad cursor controls, move away from the center to increase the radius,

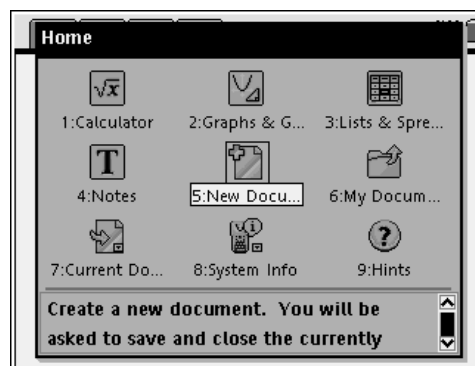


Figure 1

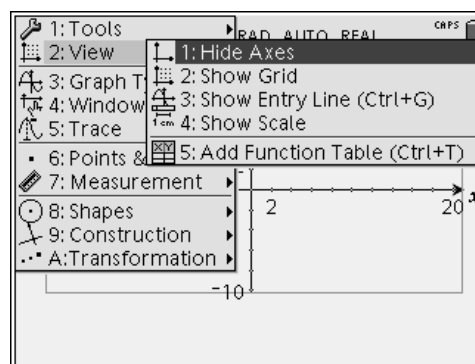


Figure 2

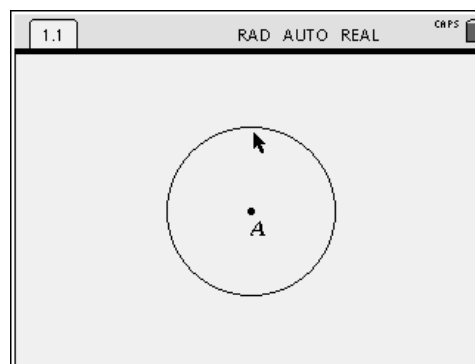















Figure 3

6. Press  a second time to set the radius length.
7. From the Menu, press   for 6:Points and Lines, 5:Segment (Figure 4).
 - Construct a chord on the circle.
 - Label the endpoints of the chord immediately after pressing  to place each endpoint of the segment.
8. Find the midpoint of chord BC by pressing    for Menu 9:Construction, 5:Midpoint.
9. Label the midpoint D (Figure 5).
10. Construct a perpendicular line. Press    for Menu 9:Construction, 3:Perpendicular Bisector.
11. Construct a perpendicular line to chord BC through point A (Figure 6).
12. Instruct students to drag point C.
13. Choose 2:Hide/Show from the 1:Tools menu, and hide the line AD.
14. Construct segment AD by choosing 5:Segment from the 6:Points and Lines menu (Figure 7).
15. Measure the lengths of chord BC and segment AD by pressing    for Menu 7:Measurement, 1:Length.

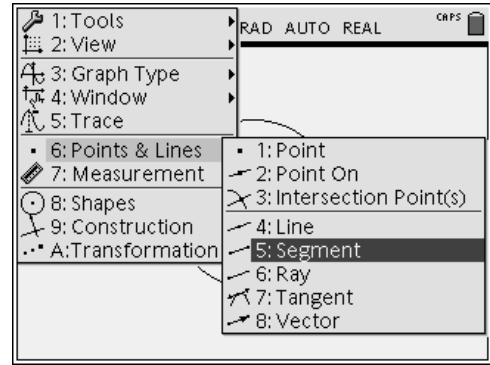


Figure 4

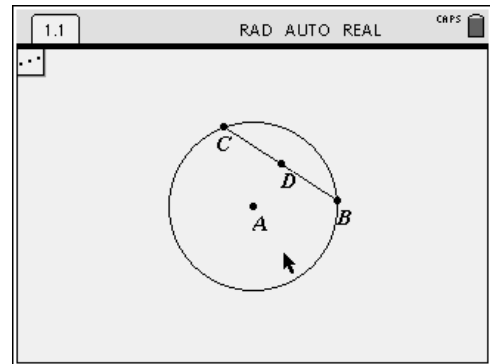


Figure 5

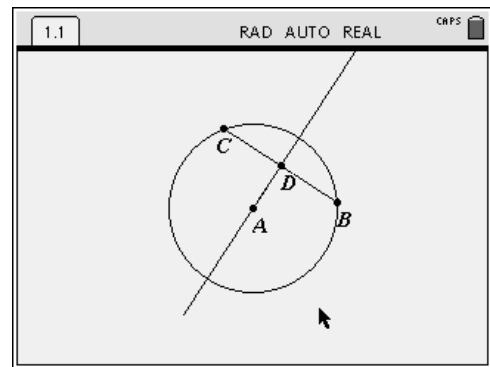


Figure 6

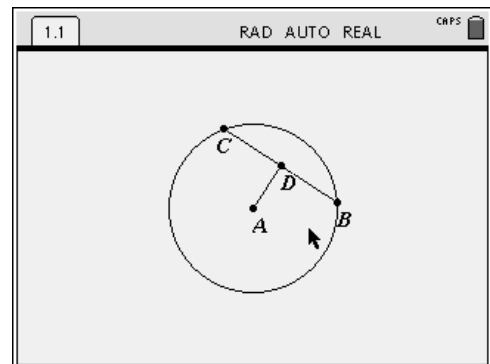











Figure 7

16. Drag the measurements to the upper right-hand corner of the screen (Figure 8).
17. To label each measurement, press  twice to select the measurement, move the cursor to the left (in front of the measurement), and type the appropriate label.
18. Instruct students to drag point C.
19. Select 1:Show Axes from the 2:View menu (Figure 9).
 - It might be necessary to drag the origin to the lower left corner and/or to drag the circle with labels and other measurements.
20. Transfer length BC to the x-axis and length AD to the y-axis. Press    for Menu 8:Constuction, 9:Measurement Transfer (Figure 10).
21. Using the Measurement transfer tool, select measurement BC by pressing  and then select the x-axis by pressing .
22. Repeat by transferring measurement AD to the y- axis.
 - A point should appear on each axis.
23. Open a text box, and select the point on the x-axis.
24. Label this point BC.
25. Repeat for the point on the y-axis, and label it AD (Figure 11).
26. Press    for Menu 9:Construction, 3:Perpendicular.

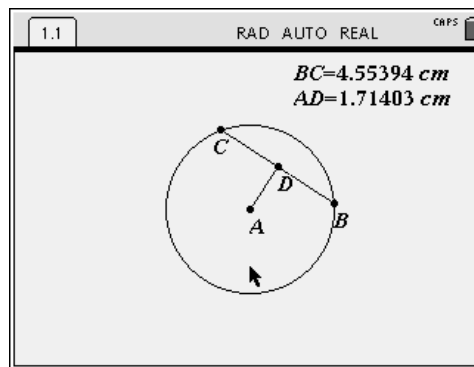


Figure 8

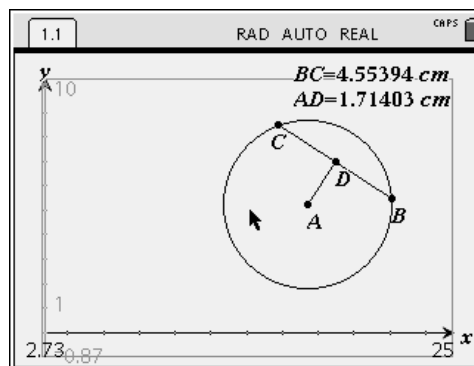


Figure 9

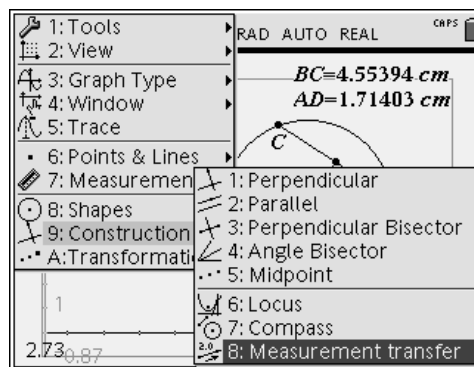


Figure 10

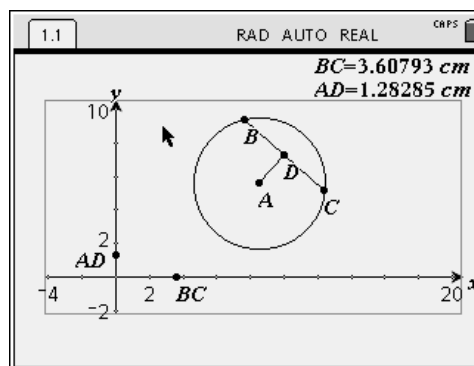


Figure 11

27. Construct a line through the point labeled AD perpendicular to the y-axis and another line through the point labeled BC perpendicular to the x-axis (Figure 12).
28. Find the intersection of the two lines by pressing **(menu)** **6** **3** for Menu 6:Points and Lines, 3:Intersection Point(s).
29. Select each line by using the **(enter)** key or the **(*)** key.
 - A point should appear.
30. Label the point immediately by typing letter G (Figure 13).
31. Drag or animate point B or C, and watch point G.
32. Hide the perpendicular lines by selecting 2:Hide/Show from the 1:Tools menu and clicking on each line with the cursor (Figure 14).
33. To construct the locus of point G as point B moves on the circle, press **(menu)** **9** **6** for Menu 9:Construction, 6:Locus.
34. First, select point G and then point B (Figure 15).
35. Drag point B or C, and then drag the circle to observe the behavior of point G.

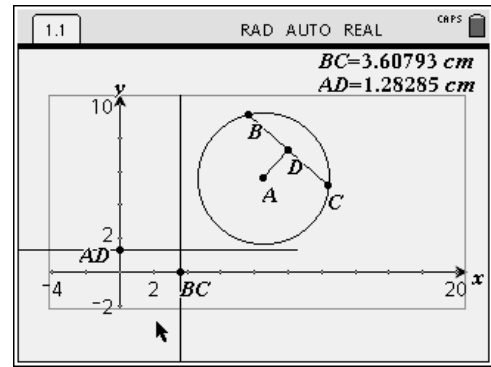


Figure 12

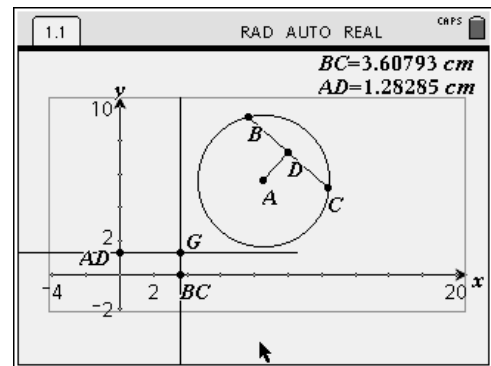


Figure 13

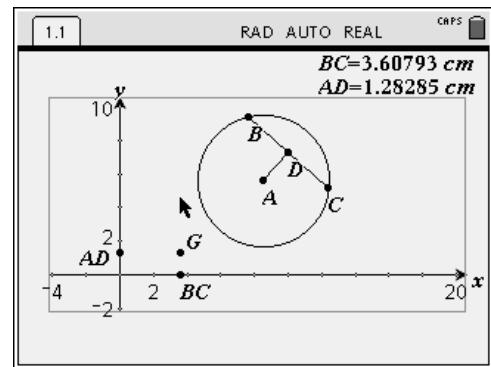


Figure 14

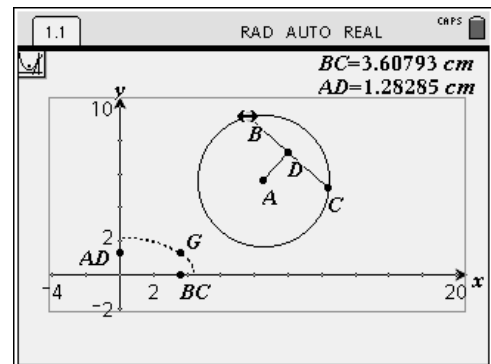


Figure 15

36. Find the coordinates of point G by pressing

for Menu 1:Tools,

6:Coordinates and Equations and selecting point G (Figure 16).

- See the Extension before beginning Part 2.

Part 2

1. Construct chord HJ in circle A.
2. Find its midpoint, and label it K.
3. Construct segment AK.
4. Measure the length of chord HJ and the length of segment AK (Figure 17).
5. Drag point H or J around the circle, and make the length of chord HJ close to the length of chord BC (Figure 17).
6. **Optional:** Instruct students to plot length HJ and length AK on the x- and y-axes, respectively.
 - Repeat the locus construction with these measurements. Compare the two loci.

Extension:

1. After constructing the locus, challenge students to find the equation of the ellipse that fits the locus in terms of diameter and radius (Figures 18 and 19).
 - The solution is given here, but not in the body of the activity. *As shown, the ellipse is centered at the origin and has a major axis of twice the diameter and a minor axis of twice the radius.*

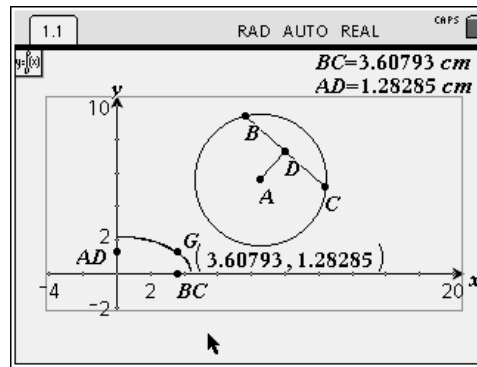


Figure 16

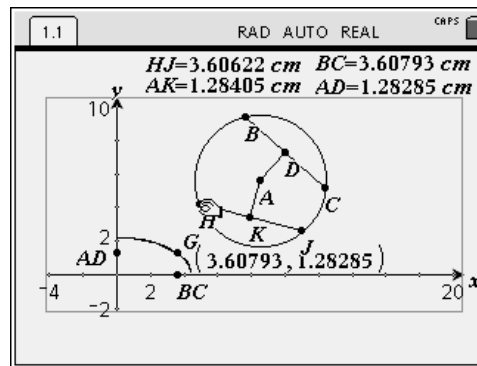


Figure 17

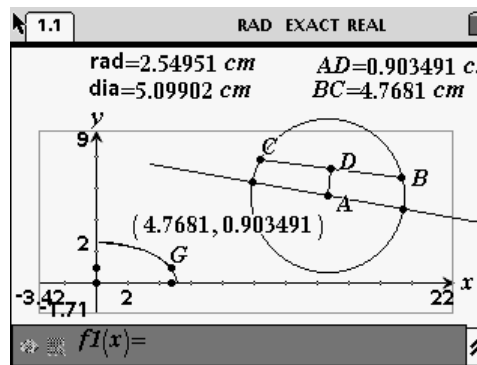


Figure 18

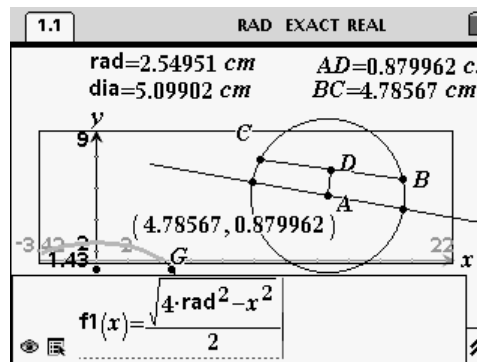


Figure 19

Chords in Circles

Student Worksheet

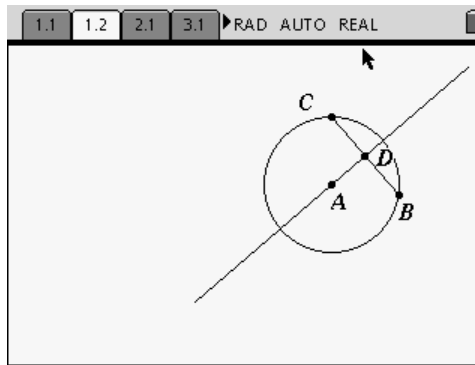
Name _____

Class _____

You will explore the relationships between the length(s) of a chord(s) and distance(s) from the center of the circle. Open the file **D3_3Chords.tns** on the TI-Nspire™ handheld.

Part 1

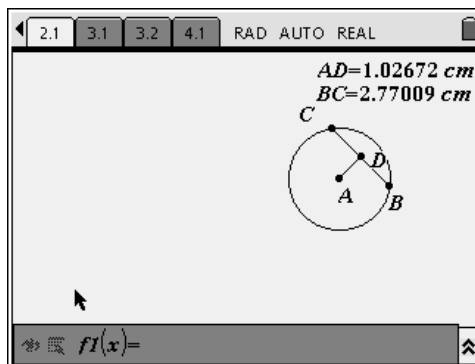
On page 2 of Problem 1, you are given circle A with chord BC and midpoint D of chord BC both drawn. The perpendicular bisector of chord BC is also drawn (Figure 1).

**Figure 1**

1. Drag points B or C around the circle, and make a conjecture about the perpendicular bisector with respect to the center of the circle.
2. Answer Question 1 on the worksheet at the end of this handout.

In Problem 2 (page 2.1), the perpendicular bisector has been hidden and segment AD drawn.

3. First, measure the lengths of chord BC and segment AD by pressing menu 7 1 for Menu 7:Measurement, 1:Length. Drag the measurements to a corner of the screen.
4. To label each measurement, press enter twice to select the measurement, move the cursor to the left (in front of the measurement), and type the appropriate label (Figure 2).

**Figure 2**

5. Drag point C, and answer Question 2 on the worksheet at the end of this handout.
6. Next, proceed to problem 3.

In Problem 3 (page 3.1), the length of chord BC has been transferred to the x-axis, and the length of segment AD has been transferred to the y-axis (Figure 3).

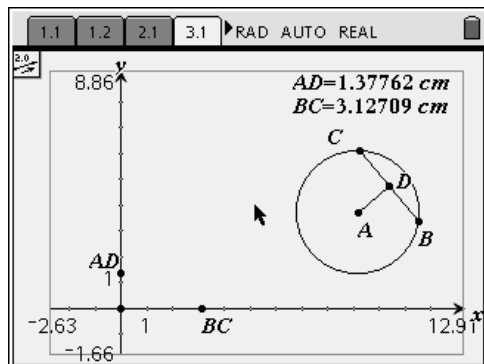


Figure 3

7. Drag point C or B to see the values change.
8. On page 3.1, press **(menu)** **9** **3** for Menu 9:Construction, 3:Perpendicular Bisector.
9. Select the point labeled BC on the x-axis and then select the x-axis.
10. Repeat, to construct the line perpendicular to the y-axis through point AD (Figure 4).

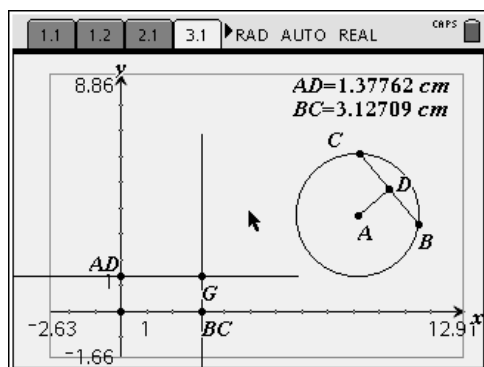


Figure 4

11. Find the intersection of the 2 lines by pressing **(menu)** **6** **3** for Menu 6:Points and Lines, 3:Intersection Point(s).
12. Select each line by using the **(enter)** key or the **(2nd)** key. A point should appear.
13. Label the point immediately by typing letter G.
14. Drag point C, and watch the path of point G. Look for a pattern.

On page 3.2, the perpendicular lines are hidden, and the path of point G (a locus of points) is shown (Figure 5).

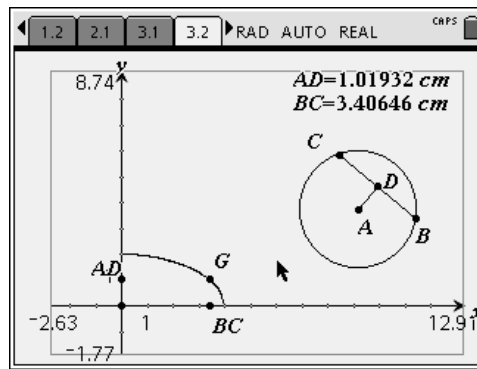


Figure 5

15. Drag point C or B, and then drag the circle.
16. Observe the behavior of point G.
17. Find the coordinates of point G by pressing menu $\{1\}$ $\{6\}$ for Menu 1:Tools, 6:Coordinates and Equations and selecting point G.
18. Drag point C, and answer the Question 3 on the worksheet at the end of this handout.

Part 2: Investigating Congruent Chords

Problem 4 (page 4.1) has chord HJ added to circle A. The length of chord HJ and the distance from A to K (the midpoint of chord HJ) have been measured and segment AK has been constructed (Figures 7 and 8).

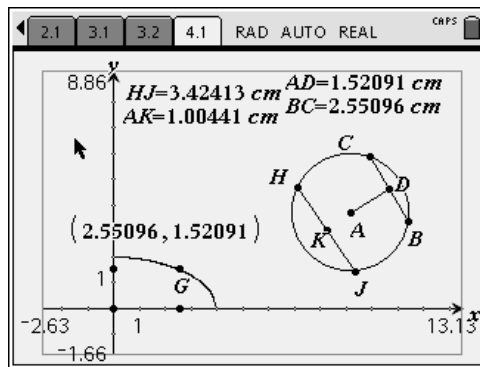


Figure 7

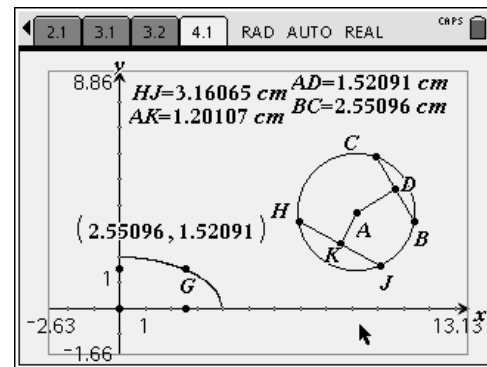


Figure 8

19. Drag H or J around the circle to make the length of chord HJ close to the length of chord BC.
20. Answer Question 4 on the worksheet at the end of this handout.

Chords in Circles

Student Worksheet

Name _____

Class _____

1. As point C moves around the circle, what is true about the perpendicular bisector of chord BC?

2. As point C moves around the circle, how does the length of chord BC relate to the distance of the chord from the center of the circle?

What is true about chord BC if the distance is 0?

3. As point C moves around the circle, describe the path of G by answering each of the following:
 - a. When point G is at the y-intercept of the locus, what is true about BC and AD?

 - b. When point G is at the x-intercept of the locus, what is true about BC and AD?

 - c. As point G moves from left to right, what happens to the x-coordinate of the locus? What does this have to do with the chord BC?

4. Make a conjecture about the relationship of congruent chords and their distances from the center of the circle?

The Minimum Wage

Student Worksheet

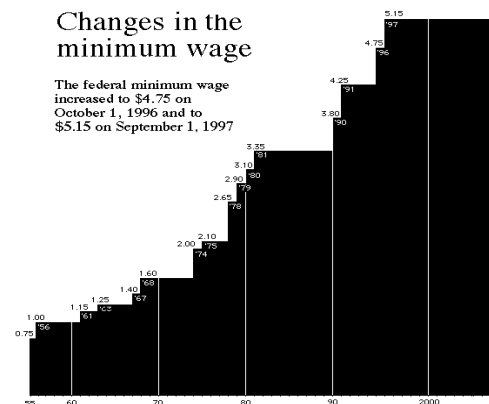
Name _____

Class _____

Using Technology to Advise the Congress

Minimum wages for certain industries have been a part of the American labor movement since the 1930's. The first nationally mandated Federal Minimum Wage was established in 1955 at \$0.75/ hour. Over the ensuing years, the minimum wage has been increased at irregular intervals reflecting the political and economic climate of the country. (See the graph.) In August, 1996 Congress passed and President Bill Clinton signed a new law that changed the minimum wage in the United States from \$4.25 per hour to \$4.75 effective October 1, 1996 and to \$5.15 effective September 1, 1997. This was the last change to the Federal law up until 2006 when it became a campaign issue.

During each election cycle since 1997 this hotly debated issue has come to the forefront for new consideration. After the 2006 midterm elections, the new Democratic Speaker of the House, Nancy Pelosi, promised that raising the minimum wage in the U.S. would be one of the top priorities for the newly elected Congress. She further stated that the current minimum wage in 2006, \$5.15/hour, represents the lowest level of buying power for workers in the entire history of the federal minimum wage in the U. S.



As is the case with all actions of the government, the minimum wage law always has many supporters and detractors. Small business owners usually oppose increases in the minimum wage claiming it is inflationary, will cost jobs, and ultimately run them out of business. Others contended that most increases are still not enough to balance the effects of higher costs of living. This argument seems particularly valid since the minimum wage has not changed in 10 years, since 1997. Who is correct? How much should the minimum wage be increased, if at all? Does the history of this program shed any light on what should happen in the future? In this problem, we will examine ways that mathematics and technology can be used to help analyze important decisions that affect our everyday lives.

Investigate:

1. Open the TI-Nspire™ math and science learning handheld document titled **D3_4MinWage.tns**. Page 1.1 contains some background information for the problem.
 - Read the assignment on page 1.2 (Figure 1).

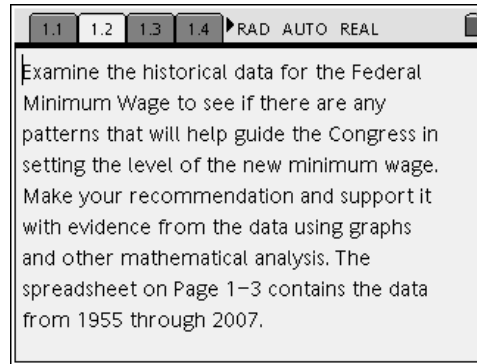


Figure 1

- The data set for the history of the Minimum Wage in the U.S. is on page 1.3 in a spreadsheet format (Figure 2).

	A year	B wage	C	D
1	1955	.75		
2	1956	1		
3	1957	1		
4	1958	1		
5	1959	1		

Figure 2

2. Use the variety of mathematical tools contained in the TI-Nspire™ handheld or computer software to complete the assignment.
 - Add pages and/or problems to this document to do a complete analysis.
 - Make your recommendation for the Congress to act upon when considering what to do with the minimum wage for the United States.

Consumer Price Index (Minimum Wage Problem Continued)

1. There is a second problem in the TI-Nspire™ document which adds new information to the minimum wage argument based on the Consumer Price Index (Figure 3).

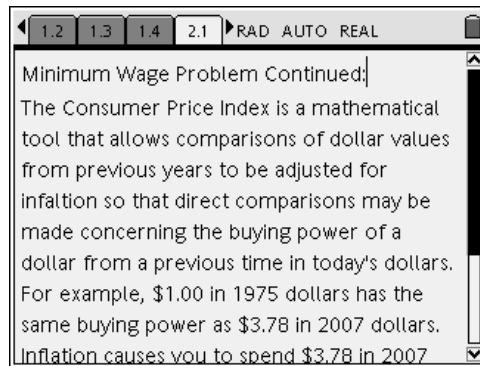


Figure 3

- Analyze the minimum wage data in light of this new information to see if the assertion about the low value of the 2007 minimum wage is true or not.
2. Reanalyze the minimum wage data in light of the information provided by using the Consumer Price Index to adjust each year's minimum wage for the corresponding values in 2007 dollars (Figure 4).

The screenshot shows a TI-Nspire document window with a title bar containing page numbers 2.1, 2.2, 2.3, and 2.4, and a mode selector set to 'RAD AUTO REAL'. The main area displays a table with the following data:

	A year	B wage	C cpi...	D
1	1955	.75	5.69	
2	1956	1	7.48	
3	1957	1	7.24	
4	1958	1	7.04	
5	1959	1	6.99	
6	1960	1	6.87	

The cursor is positioned at the bottom left of the table, near the text 'DI |'.

Figure 4

- Apply this analysis to your recommendation to the Congress about what to do with the minimum wage in the near future.

Update:

A bill is proposed has within its provisions a section providing an increase in the Federal Minimum Wage for the U.S. According to USA Today, the current \$5.15 minimum wage will increase in \$0.70 increments to \$7.25 in three steps. Two months after the legislation is signed the minimum wage will increase to \$5.85. One year later, it will take another \$0.70 jump to \$6.55. And finally, a \$0.70 jump another year later to \$7.25.

Compare your predictive models to what will now happen to see which model was the most accurate.

NCTM Standards

Grades 9-12 Numbers and Operations Standards:

All students should...

1. develop a deeper understanding of very large and very small numbers and of various representations of them

Grades 6-8 Algebra Standards:

All students should...

1. represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules
2. relate and compare different forms of representation for a relationship
3. use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships
4. model and solve contextualized problems using various representations, such as graphs, tables, and equations
5. use graphs to analyze the nature of changes in quantities in linear relationships

Grades 9-12 Algebra Standards:

All students should...

1. generalize patterns using explicitly defined and recursively defined functions
2. interpret representations of functions of two variables
3. use symbolic algebra to represent and explain mathematical relationships
4. judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology
5. identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships
6. draw reasonable conclusions about a situation being modeled.

Grades 6-8 Data Analysis and Probability Standards:

All students should...

1. select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatterplots
2. discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatterplots
3. make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots of the data and approximate lines of fit

The Swimmer Problem

Student Worksheet

Name _____

Class _____

Problem Statement

The lifeguard sits near the shore on a straight stretch of Strait Lake beach. Approximately 150 m down the beach, a swimmer, approximately 60 m off shore, yells for help.

If the lifeguard can run 8 m/sec on land and swim 2 m/sec in the water, what is the best path for the lifeguard to take to reach the swimmer in the shortest amount of time (Figure 1)?

You will be asked to solve this problem geometrically, numerically, graphically, and algebraically (Figure 2).

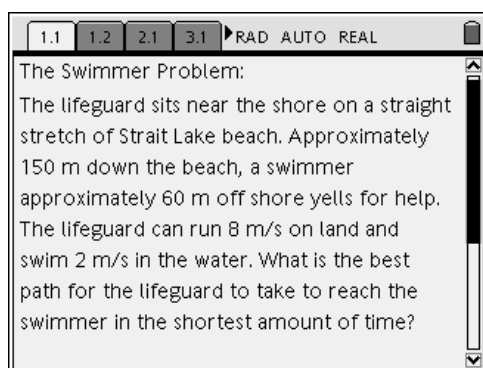


Figure 1

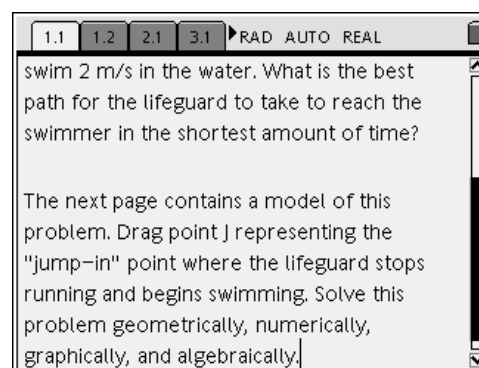


Figure 2

Investigate

1. Move to page 1.2 of this problem in the TI-Nspire™ .tns document

D3_5SwimmerProblem.tns.

- This page contains an interactive model of the problem situation.
- Point **J** represents the “jump- in” point where the lifeguard stops running along the “shoreline” and begins swimming.

2. Drag point J to simulate the various points where the lifeguard could choose to stop running and start swimming toward the swimmer in trouble.

Figures 3 through 5 represent various possible solutions.

- Figure 3 could be called the “swim only” solution where the lifeguard simply jumps directly into the water and swims all the way to the swimmer.
- What are the advantages and disadvantages of this solution?

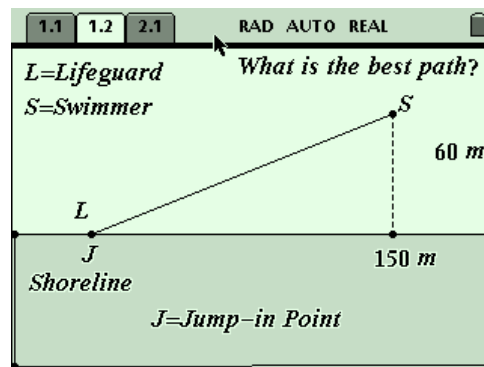


Figure 3

- Figure 4 represents what might be called the “equal method” solution where the lifeguard runs and swims equal amounts.

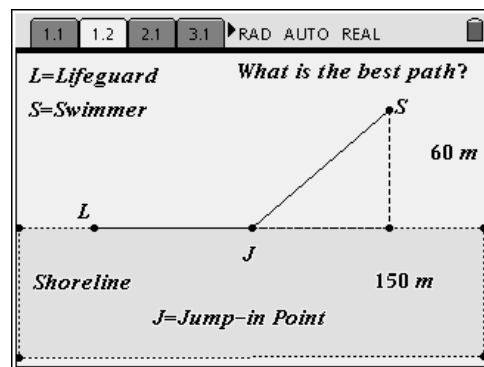


Figure 4

- Figure 5 represents the “perpendicular” solution where the lifeguard runs down the beach 150 meters, turns at a right angle to jump in the water and then swims the 60 m to the swimmer in trouble.
- You cannot move the point J to the left of L or more than 150 m down the beach.
- Why could you immediately dismiss those “solutions?”

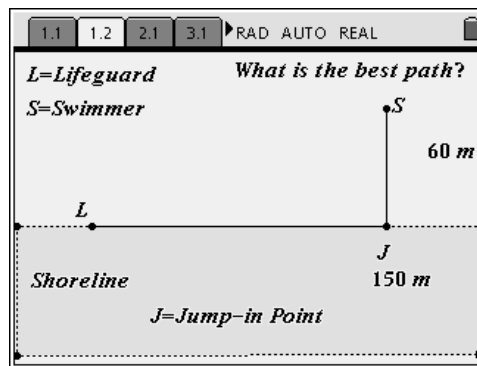


Figure 5

3. What are the characteristics of the “best” path in this problem? Explain.
4. Make a conjecture about the best path for the lifeguard to follow to save the swimmer. Explain your reasoning.

5. In the TI-Nspire™ document, Problems 2, 3, 4, and 5 each contain a copy of the Swimmer Problem's interactive model. Use these 4 work spaces to solve the original problem:
 - 1) Geometrically (Problem 2)
 - 2) Numerically (Problem 3)
 - 3) Graphically (Problem 4)
 - 4) Algebraically (Problem 5 – CAS option)
 - Add as many pages to each problem space as you need to solve the problem by the selected method.
6. After you have completed all of the solution techniques, add a new Problem, Problem 6, and complete Notes which compare the various results for accuracy and understandability.
7. Save the entire document with all of your work as **Swimmer Problem [your last name]**, and submit this work to your teacher for evaluation.

The Swimmer Problem

Materials

- TI-Nspire™ Math and Science Learning Handheld
- D3_5SwimmerProblemTch.tns

Overview

This activity describes the steps for creating the document and a potential solution for the various approaches to solving the problem.

NCTM Standards

All students should...

Grades 9-12 Algebra Standards:

1. generalize patterns using explicitly defined and recursively defined functions
2. interpret representations of functions of two variables
3. use symbolic algebra to represent and explain mathematical relationships
4. judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology
5. identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships
6. draw reasonable conclusions about a situation being modeled

Grades 9-12 Geometry Standards:

1. use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations
2. investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates
3. use geometric models to gain insights into, and answer questions in, other areas of mathematics

Introduction

One of the most important parts of the problem solving process is Polya's first step—understand the problem. Students must experiment with the model to come to a good understanding of what the problem is asking.

What is the “best” path for the swimmer? Why is time a critical factor in this problem? What difference is there between running along the beach and swimming in the water?

Simply dragging point **J**, the transition point from running on the beach to swimming, is the first step in this process of understanding the problem.

Several different strategies for the lifeguard should be discussed for advantages and disadvantages. For example, the “swim only,” solution where the lifeguard jumps into the water immediately and swims directly to the swimmer, is the shortest path, a straight line. Some students will ignore the fact that the lifeguard can travel much faster on the beach than swimming in the water. Is this the “best” path? Likewise, the “perpendicular” solution where the lifeguard runs down the beach to a position directly inshore from the swimmer, makes a right angle turn, and swims out to rescue the swimmer, is an “obvious” solution. This plan minimizes the swimming leg, taking advantage of the lifeguard's speed on the beach. Other solutions could be investigated as well.

Proceeding in this manner, eventually, students will see the need to display measurements that reveal how long it actually takes for the various solution techniques so an accurate judgment can be made about the “best” solution.

A prepared TI-Nspire™ document file is included to show a potential solution. It is named **D3_5SwimmerProblemTch.tns**.

You may need to offer assistance, guidance, or hints as students work on the various solutions. A rubric can be used for assessment purposes.

1) Geometric Solution

1. Measure the two legs of the lifeguard's path to the swimmer—the running and the swimming portions (Figure 1).
2. Compute the total time it takes for the lifeguard to get to the swimmer for any position of point **J**.
 - From the problem statement, we know that the lifeguard can run at 8 m/sec and swim at 2 m/sec.
 - Using the basic distance-rate-time formula,

$D = r \cdot t$, we know that each time factor is

$$t_{\text{running}} = \frac{D_{\text{running}}}{8}$$

$$\text{and } t_{\text{swim min g}} = \frac{D_{\text{swim min g}}}{2},$$

or the total time is the sum of these individual times, or $Tt = t_{\text{running}} + t_{\text{swim min g}}$

3. Drag point **J** along the line until the Total time is at its smallest value (Figure 2).
 - Notes may be added to the screen using the Text tool to identify findings.

Geometrically, the solution is that it will take the lifeguard 47.7976 seconds to swim to the drowning swimmer if she runs 134.25 m down the beach before jumping into the water and swimming the rest of the way.

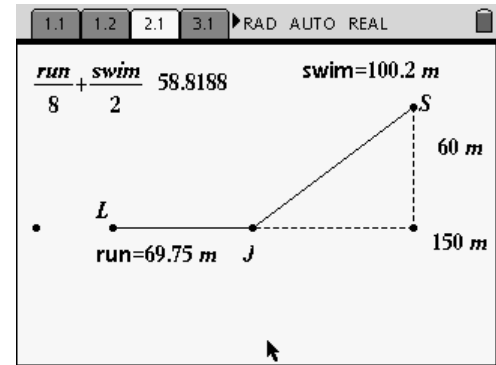


Figure 1

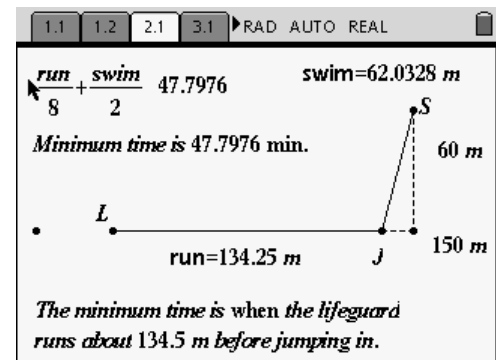



Figure 2

2) Numerical Solution

1. Compute the total time (Ttime) for the lifeguard to reach the swimmer for any distance running down the beach based on the run and swim distances and speeds.
2. Store these three variables into the memory using the  key (Figure 3).
 - The variable names can be entered during the storing operation (Figure 4).
 - Dragging point J generates various outputs for the run, swim, and Ttime in addition to updating the stored values automatically (Figure 5).
3. Open a new page, and add a Lists & Spreadsheet application page (Figure 6).
4. Label three columns `running`, `swimming`, and `totaltime` to serve as collection lists for the various outputs.

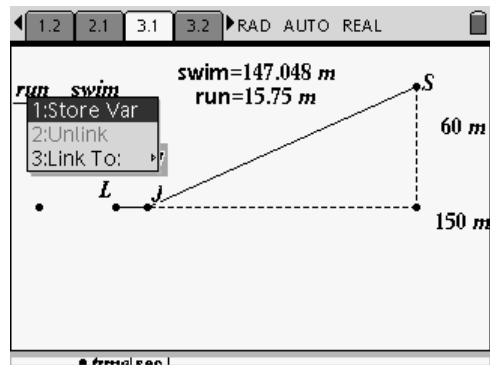


Figure 3

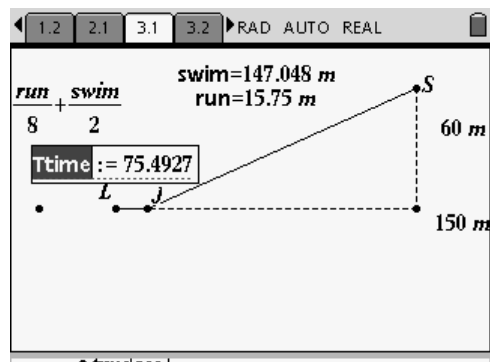


Figure 4

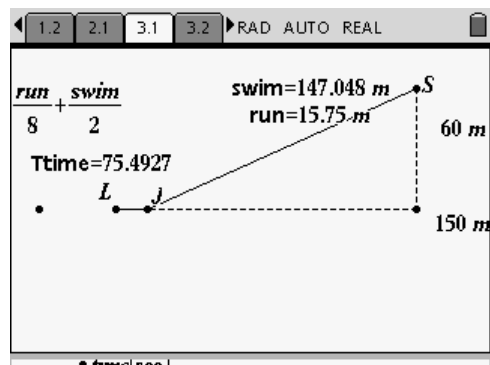


Figure 5



Figure 6



Note: You cannot name the lists in the spreadsheet with the same names used to store the variables from the geometric construction. The list names `running`, `swimming`, and `totaltime` are chosen to reflect the variables `run`, `swim`, and `Ttime` that will be stored in them, respectively.

The spreadsheet will be used for several purposes:

- 1) It will collect data from the geometric construction as point J is dragged.
- 2) It will also send collected values directly to the graph screen to be plotted in a scatter plot as the data are collected.
- 3) And, it will be the place where regression analysis will be done on the collected data.

5. Collect the data using the Manual Data Collection tool on the Data menu (Figure 7).

- This command allows the user to specify when and exactly how much data is actually collected.
- Figure 8 shows this command entered into the “list rule” row of the spreadsheet (the gray cell below the column name) in the three named columns that are ready to collect data from shared variables `run`, `swim`, and `Ttime`, which will be generated in the geometric construction on page 3.1.

6. To activate the Manual Data Collection command, press  .

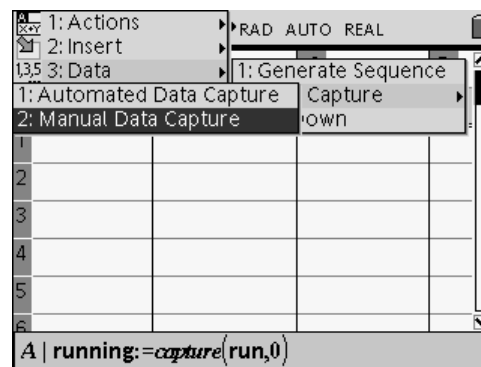


Figure 7

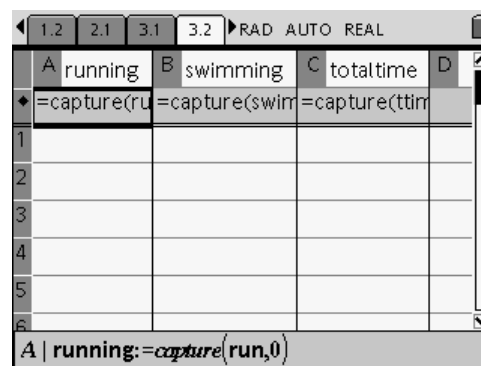


Figure 8

- Figure 9 shows the first set of data collected in the spreadsheet, and Figure 10 shows the state of the geometric construction when those data were collected.

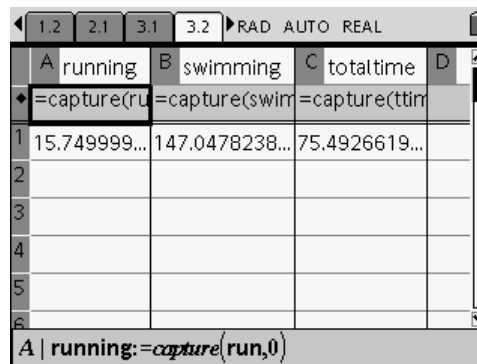


Figure 9

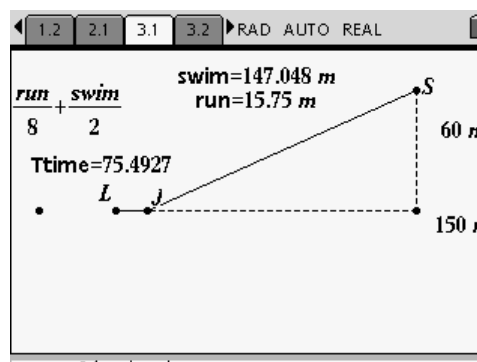


Figure 10

- Before collecting more data points, return to page 3.1, the Graphs & Geometry screen, press ctrl home 6 1 for Menu 6:Page Layout, 1:Custom Split (Figure 11).

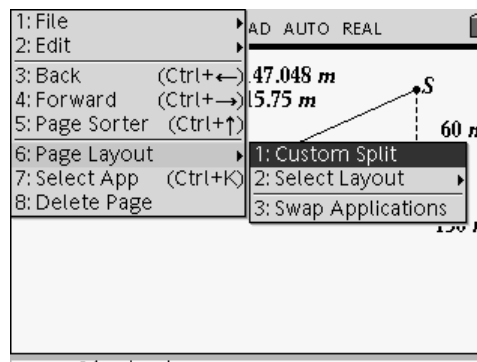


Figure 11

- Use the + and - to select a horizontally split screen as shown in Figure 12.
 - Now you have a split screen view of the geometry construction modeling the problem at the same time you see the scatter plot of the collected data as they are being collected in the spreadsheet.

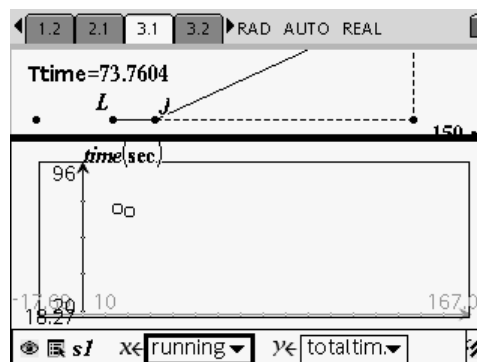


Figure 12

- Figure 12 (previous page) shows the split screen with a scatter plot (with the axes scaled appropriately) of the lists **running** and **totaltime**, the independent and dependent variables for this problem, respectively.
 - Only two data sets have been collected from the geometric construction at this point in the exploration.
9. Switch to the geometric construction screen using the **ctrl** **tab** key sequence.
 10. Drag point J to another position along the line representing the edge of the beach, and press **ctrl** **.** to collect more data, showing the total time as a function of the distance the lifeguard runs along the “beach” before jumping into the water. Repeat to collect more data.
- Figures 13 and 14 show the scatter plot as the data are being collected. Clearly, there is a minimum value for the total time variable.
 - The question now becomes what type of function might model this data pattern?

One way to fit a function to the data is to use one of the “transformable” function types like $f1(x) = x^2$ (Figure 15).

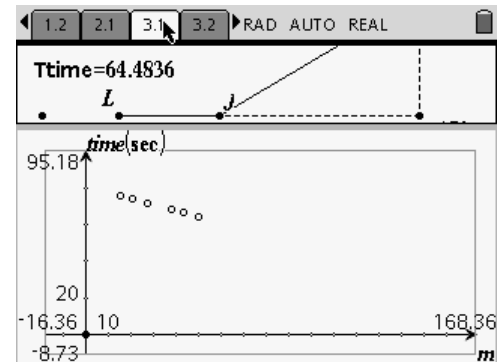


Figure 13

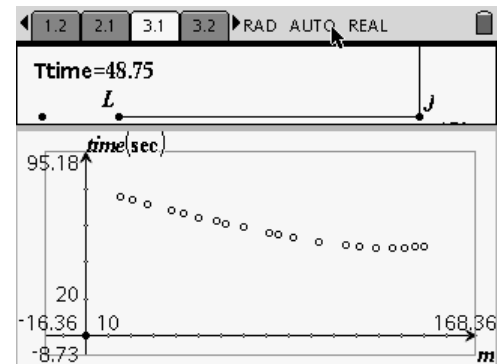


Figure 14

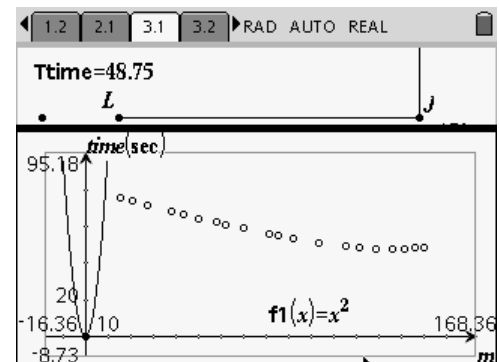


Figure 15

11. Drag and stretch the function until it “fits” the data in the scatter plot (Figure 16).

- The values in the function definition can also be changed manually by directly editing the equation.

12. Press **menu** **6** **2** for Menu 6:Points & Lines, 2: Point-On to put a point on the graph of the function (Figure 17).

13. Drag the point along the graph until it reaches the minimum value of the function.

- At that point, a lower case “m” appears indicating a local minimum and the coordinates of the point “jump” to a calculated value for the local minimum, not a simple coordinate pair on the function graph (Figure 18).
- The minimum value from the numerical analysis of the transformed quadratic function seen in Figure 18 is for the lifeguard to run 134.5 meters down the beach before jumping into the water, giving a total time to reach the swimmer of 47.91 seconds.
- The top part of Figure 18 shows point P dragged to the geometric solution of running 134.25 meters down the beach for a total time to reach the swimmer of 47.7976 seconds.
- These solutions are relatively close together.

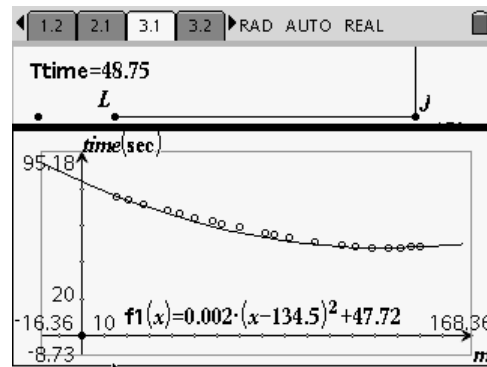


Figure 16

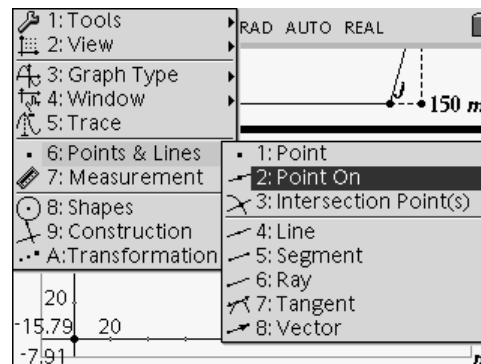


Figure 17

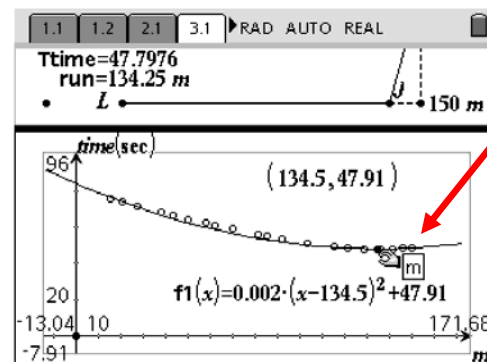


Figure 18

Another way to model the data set is with regression tools.

14. Switch to the spreadsheet, and press

for Menu 4:Statistics,

1:Stat Calculations. . . , 6:Quadratic Regression (Figure 19).

- Once the model is chosen, a dialogue box will open where you can choose the lists to regress and the storage location for the resulting function definition, or you can keep the defaults (Figure 20).

- Once the lists are specified, the regression results appear in the spreadsheet (Figure 21).

15. Return to the scatter plot, and graph the regression model stored in function f2(x).

16. Using the 2:Point-On tool from the 6:Points & Lines menu again, drag the point to the local minimum value (Figure 22).

- If the local minimum is not on the screen, readjust the scale of the x -axis by dragging the hash marks on this axis only.

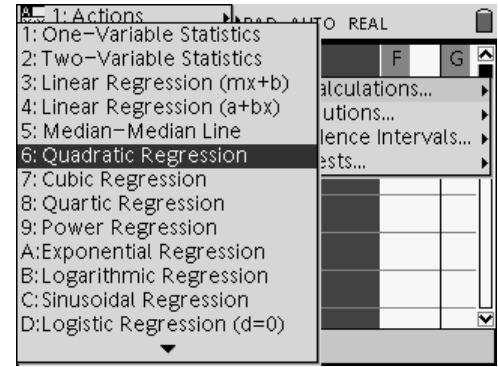


Figure 19

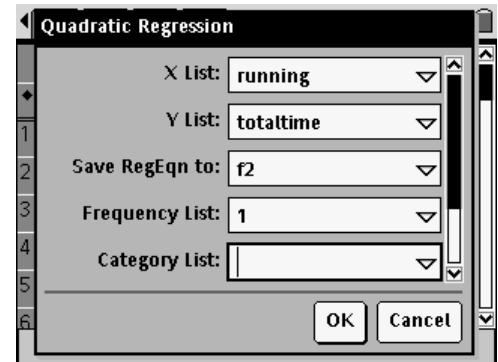


Figure 20

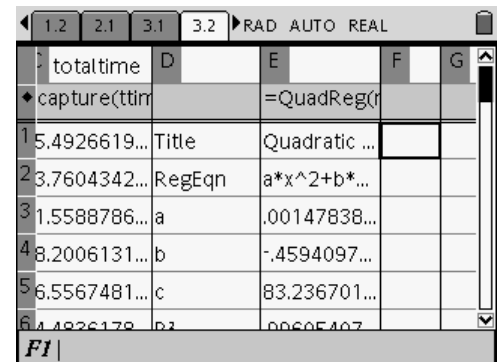


Figure 21

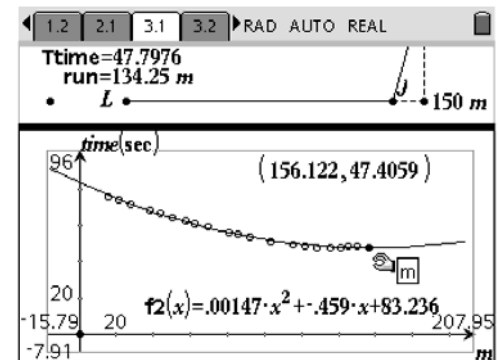


Figure 22

As seen in Figure 22 (previous page), the local minimum is for the lifeguard to run 156.122 meters down the beach for a total time to the swimmer of 47.4059 seconds.

This solution, while mathematically correct for this model, means that the lifeguard actually ran past the “perpendicular” point at 150 meters by over 6 meters and then turned and swam back to the swimmer.

Compare this solution to the geometric solution still showing at the top of Figure 22 (previous page).

- Which is more accurate?

As another model, Figures 23 and 24 show the cubic regression model and its corresponding local minimum value of running 135.551 meters down the beach for a time to the swimmer of 48.1111 seconds.

This may be a viable model until you notice that the coefficient of the x^3 term in the regression equation is nearly equal to zero, making the equation nearly quadratic.

Other models from the list of possible regression equations can be used to “fit” the data set. But remember, a rule of thumb when using regression modeling is that the “best” model is usually the “simplest” mathematical model.

An example of this would be to use the $n-1$ degree polynomial model to fit n data points exactly.

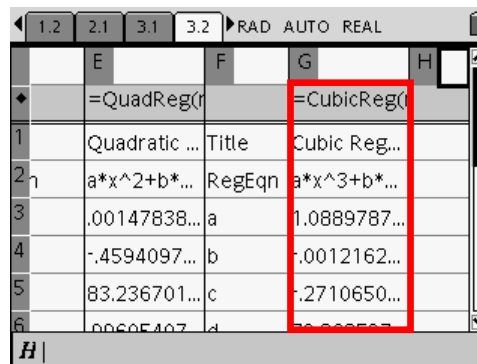


Figure 23

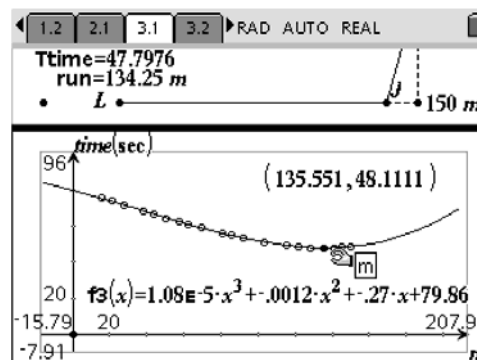


Figure 24

If you had 50 points in the data set, as long as none of the points are on top of each other or vertically aligned, as in this data set, then there is a 49th degree polynomial that exactly fits the 50 points. A 49th degree polynomial is not a “simple” mathematical model.

If you are working with a TI-Nspire™ CAS learning handheld, there is another way to find local minimum values of functions.

Figure 25 shows the command `fMin()` used to find the x -value of the local minimum value of the first quadratic function we transformed to fit the data set.

17. Once the x -value of local minimum value is established, evaluate the function at this value to produce the function value (Figure 25 above).

Repeat this procedure for other named functions such as the two regression models, quadratic and cubic, that are stored currently in `f2(x)` and `f3(x)`.

- The results are identical as those found using the 2:Point-On tool on the graph of the functions.

Note: When a function can have more than one minimum value such as the cubic model `f3(x)` in this example, the `fMin()` tool requires 4 parameters including the function, the variable, a lower bound for the search area, and an upper bound for the search area (Figure 25 above).

<code>fMin(f1(x),x)</code>	$x=134.500$
<code>f1(134.5)</code>	47.910
<code>fMin(f2(x),x)</code>	$x=156.122$
<code>f2(156.12244897959)</code>	47.406
<code>fMin(f3(x),x,120,150)</code>	$x=135.551$
<code>f3(135.55137835756)</code>	48.111

Figure 25

18. Whether you are using a TI-Nspire™ CAS handheld or not, add a Page to the document, and add a Notes application.
19. Write a summary of the results of the numerical solution for future reference (Figure 26).

3) The Graphical Solution

This solution begins with an analysis of the problem so that an algebraic equation that represents the problem situation can be written.

Figure 27 shows a geometric construction of the problem (cut and pasted onto a new page (4.1) of this document).

This model is still dynamic by dragging point J. However, the measurements have been replaced by variable representations. The distance the lifeguard runs before jumping into the water is represented by x and is the independent variable in the problem.

The swimming distance for the lifeguard is the hypotenuse of a right triangle with legs of 60 m and $(150-x)$ m. By the Pythagorean Theorem, the hypotenuse is equal to $\sqrt{60^2 + (150-x)^2}$. Given a running speed of 8 m/sec and a swimming speed of 2 m/sec, the total swimming time is given by the function

$$Tt(x) = \frac{x}{8} + \frac{\sqrt{60^2 + (150-x)^2}}{2} \text{ (Figure 5b).}$$

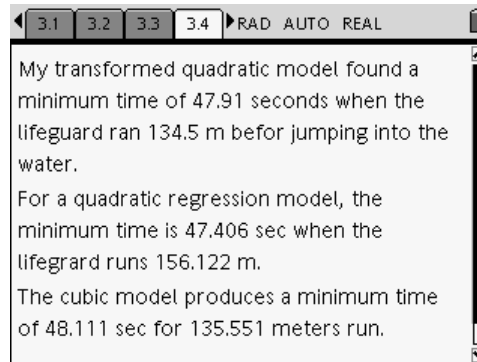


Figure 26

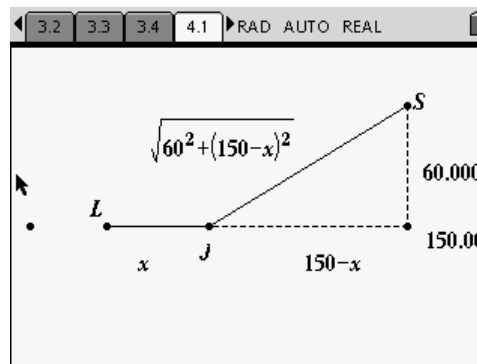


Figure 27

1. Highlight the right side of the function in Figure 28 using the Text tool in combination with the Caps Shift key and the arrow keys.

2. Press **ctrl** **↵** **2** **4** for Menu 2:Edit, 4:Copy (or press **ctrl** **C** on the handheld keyboard) to copy the highlighted function definition (Figure 29).

3. Add a new page to the document, and add a Graphs & Geometry application (Figure 30).

4. Paste the saved function definition into the Function Entry Box using the 2:Edit menu or **ctrl** **V** from the keyboard (Figure 31).

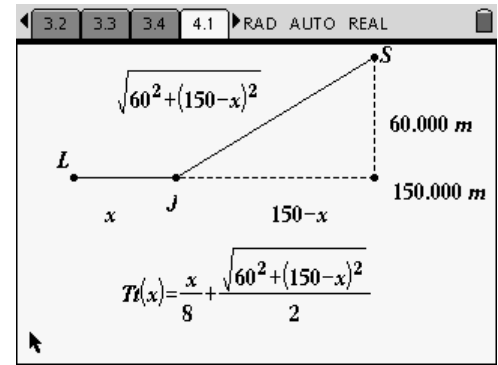


Figure 28

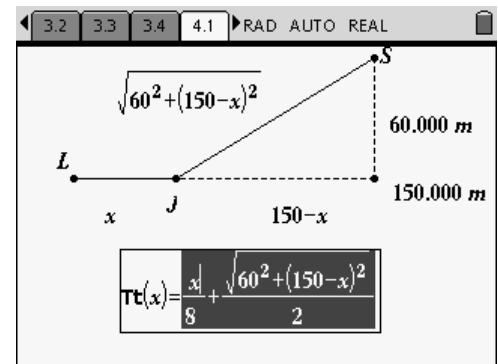


Figure 29

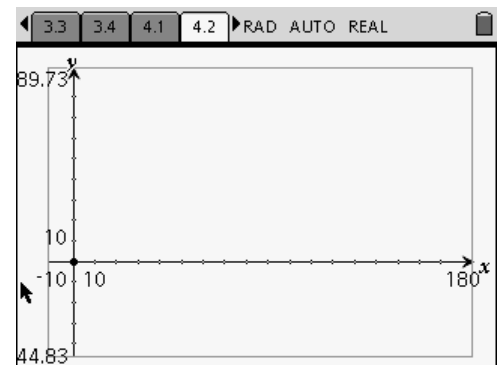


Figure 30

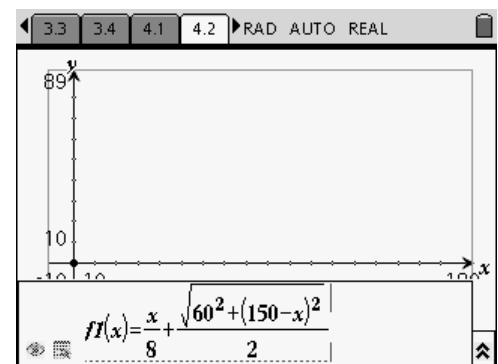


Figure 31

- Draw the graph of the function, adjusting the window as needed (Figure 32).

A graphical solution is shown in this example using the Trace tool to get a point on the function that can be dragged to the local minimum instead of the Point-On tool. The end result is the same for each tool.

- Press [menu] [5] for Menu 5:Trace (Figure 33).

- A point with its coordinates appears on the graph of the function. This point is under control of the left and right arrow keys.
- As the Trace point moves along the function, the coordinate pairs change to indicate the approximate position of the point (Figure 34).

- Press the [enter] key at any time to place a point at the current location of the Trace point.

- Move the Trace point away to see the new point on the graph (Figure 35).
- You can repeat this operation as many times as you want, leaving a point where the Trace point was located.
 - These new points are exactly the same as points created with the Point-On tool.

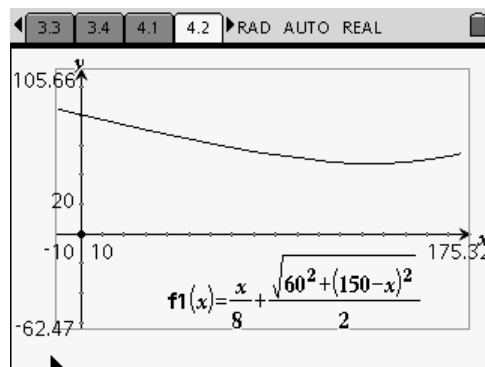


Figure 32

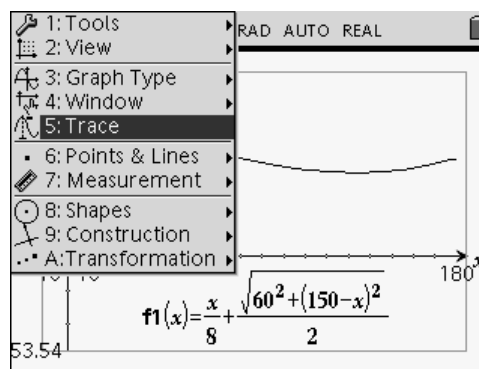


Figure 33

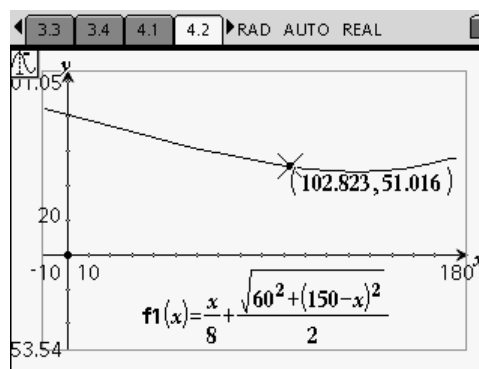


Figure 34

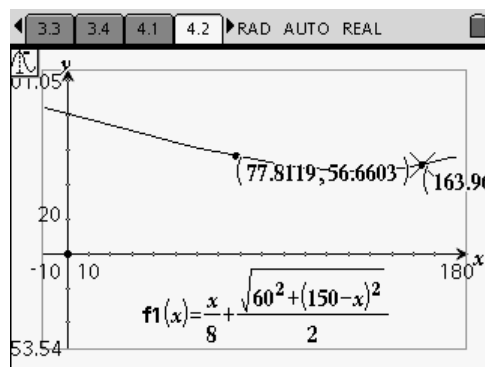


Figure 35

10. Press the **(esc)** key to end the Trace, leaving any copies of points still on the screen (Figure 36).

11. Drag the coordinate pair away from the remaining point on the graph (Figure 37).

12. Drag the point toward the function's local minimum value until a lower case "m" appears on the screen and the point "jumps" to the computed minimum value (Figure 38).

- Text comments can be added to the screen to annotate what the graph shows.

13. Compare this solution—the lifeguard runs 134.5081 meters down the beach before jumping into the water for a total time of 47.7974 seconds to get to the swimmer—to other solutions found by the geometric and numerical methods.

- Notes can be made on a new page with the Notes application (Figure 39).

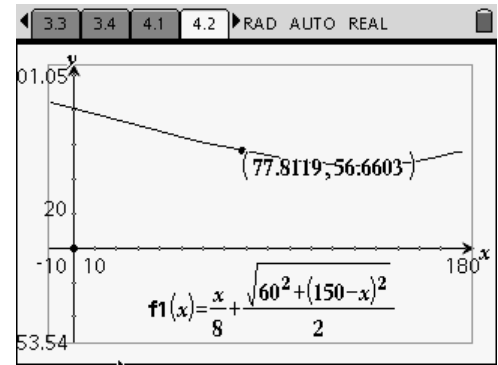


Figure 36

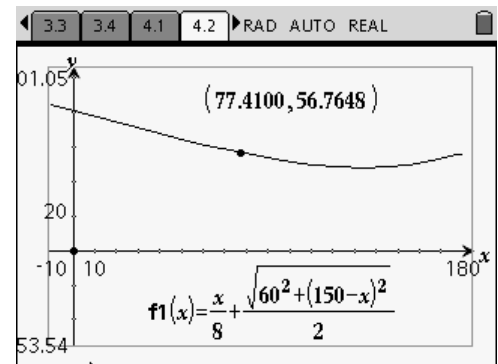


Figure 37

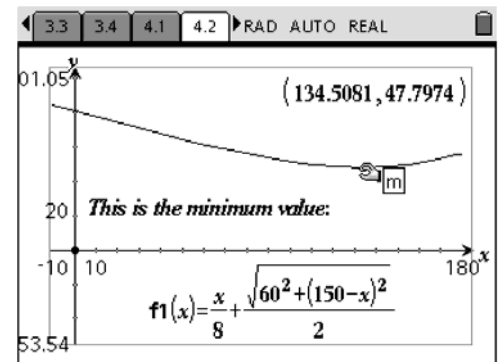


Figure 38

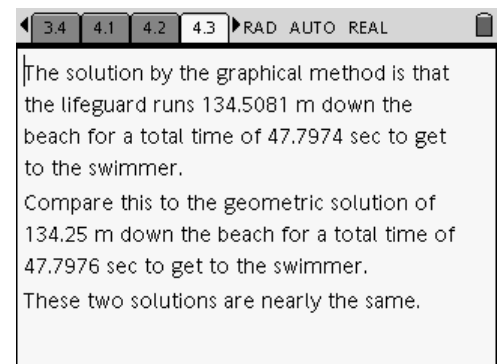


Figure 39

4) The Algebraic Solution (using CAS)

The algebraic solution to the problem begins in a similar way as the graphical solution by representing the problem situation as a function definition (Figure 40).

1. This can be page 4.1 copied and pasted onto a new page 5.1.
2. Open another new page (5.2 in this example), and add the Calculator application (Figure 41).

3. Enter the Define command by either selecting it from the Tools menu or typing the word “define” on the screen using the keyboard (Figure 42).

4. Copy and paste the function definition from page 5.1, or rewrite it on the Calculator screen with the keyboard (Figure 43).

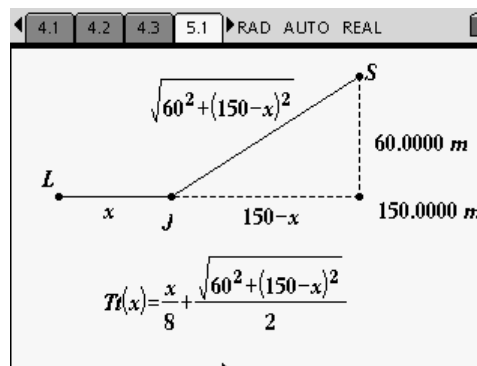


Figure 40

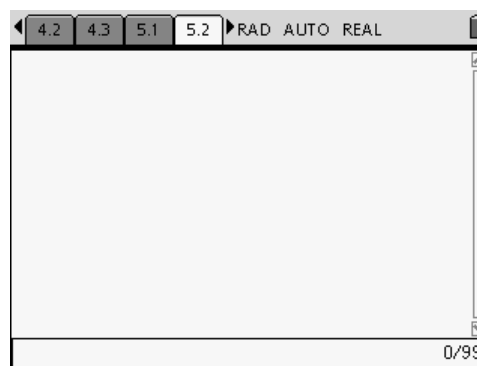


Figure 41

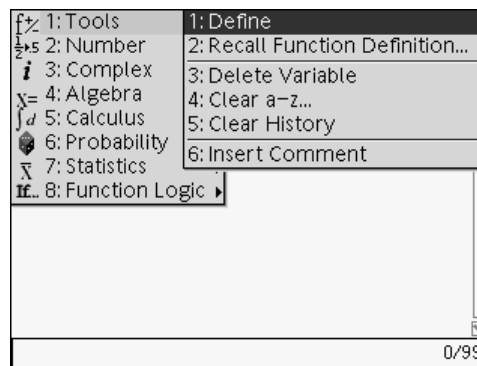


Figure 42

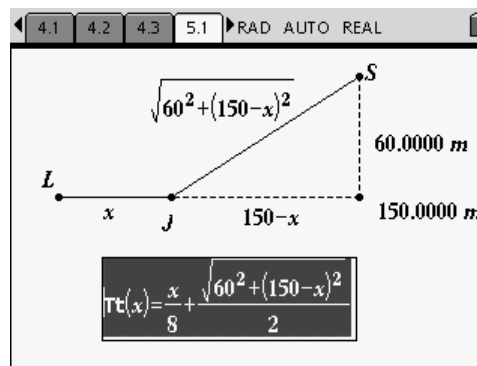




Figure 43

5. Execute the Define command to define the function $Tt(x)$ (Figure 44).

- Notice that the capital letter “T” is changed to lower case. Function definitions are not case sensitive, so the program changes everything to lower case (Figure 45)

6. Open the Operations Template menu by pressing   (Figure 46).

7. Choose the first derivative tool, and enter the function name $Tt(x)$ with respect to the variable x (Figure 47).

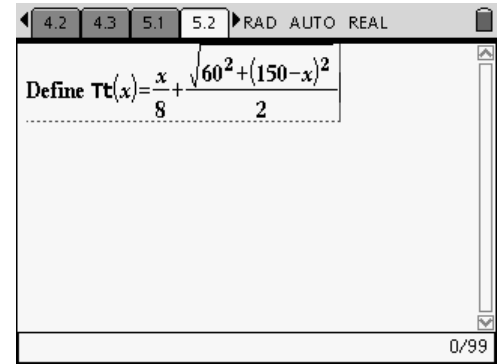


Figure 44

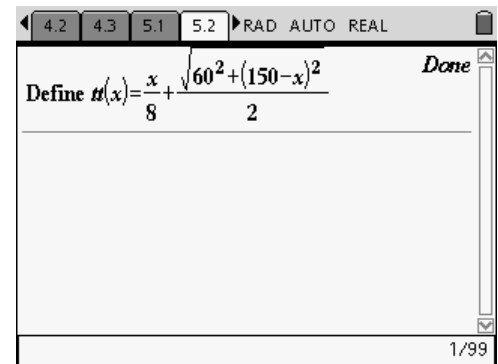


Figure 45

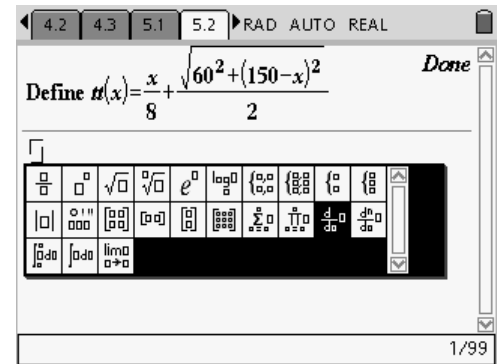


Figure 46

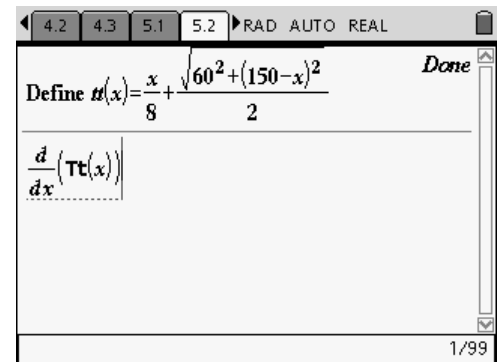


Figure 47

8. Execute this command to find the first derivative of the function $Tt(x)$ (Figure 48).

9. Enter the Solve command from the Algebra menu (or by typing it on the screen) (Figure 49).

10. Press the up arrow to highlight the previous answer, the first derivative of the function $Tt(x)$ (Figure 50).

11. Press Enter to copy this definition into the Solve command.

12. Set this expression equal to zero (0) and solve with respect to the variable x (Figure 51).

13. Execute the Solve command to find the value of x when the derivative is equal to zero.

- This is where, according to the definition of the first derivative of a function, the tangent line to the function has a slope of zero. This is a local maximum or minimum value.

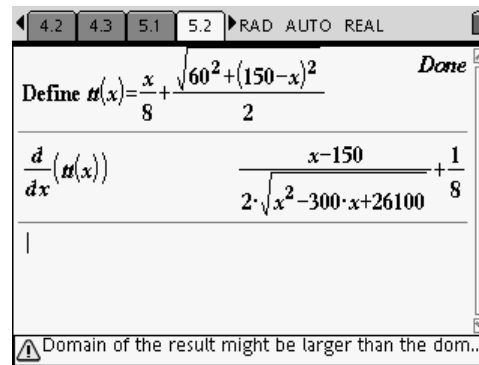


Figure 48

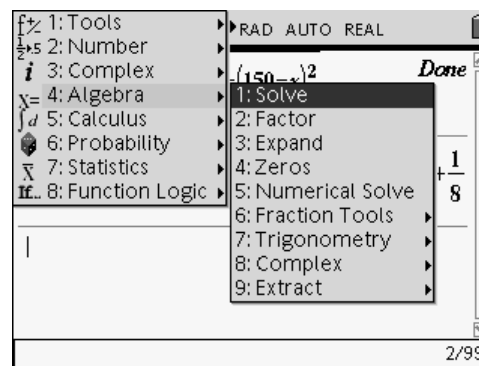


Figure 49

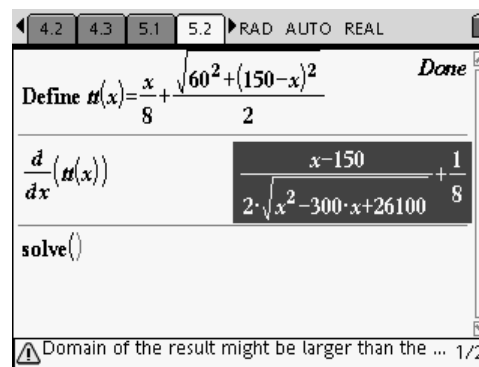


Figure 50

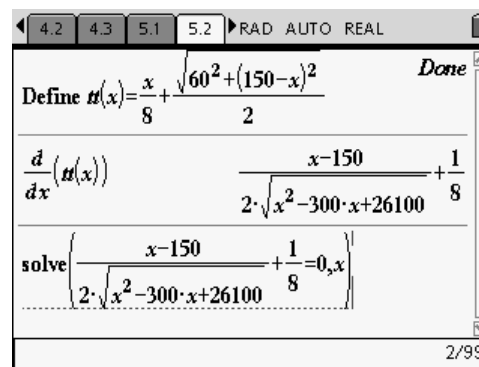


Figure 51

- Figure 52 shows the output when the Enter key is pressed.
 - If your handheld is set in Auto mode, the exact answer will appear as in Figure 52.
14. Press ctrl enter to solve the equation again but with an answer that is an approximate decimal value (Figure 52).
- If the value of x we just found is the point in the domain where a local minimum exists, then the function value at this point will be the solution to our problem.
15. Evaluate the function $Tt(x)$ at the value found for x (Figure 53).
- The value for x can be copied down by the same technique used for the derivative in the previous step.
16. Scroll up, and press enter .
17. Finally, as done in the numerical solution, we can use the 6:Function Minimum [fMin()] command from the 5:Calculus menu (Figure 54).

After the minimum value is calculated, the function value is computed in the same way as the previous step (Figure 55).

Notice that the numerical values obtained by using the fMin() tool are identical to those found by solving the first derivative set equal to zero for x . This is the expected result.

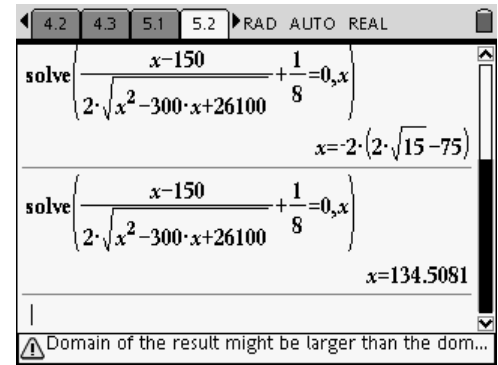


Figure 52

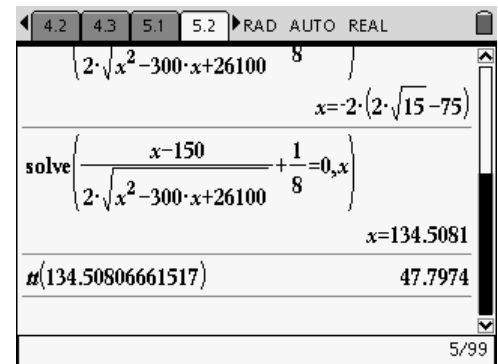


Figure 53

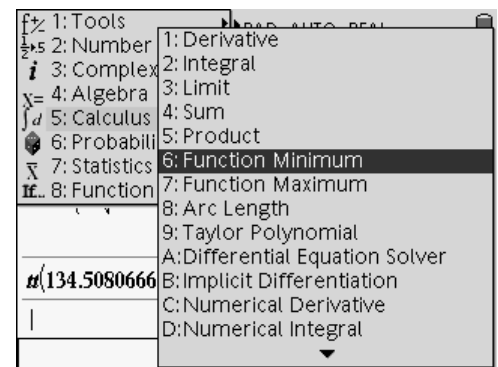


Figure 54

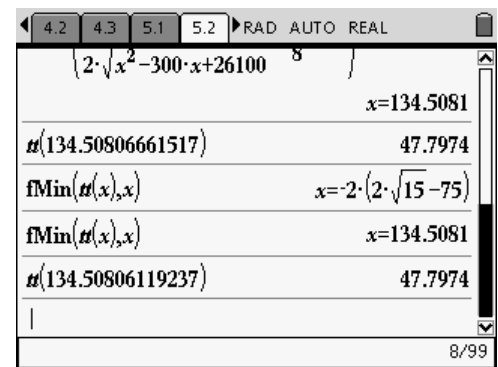


Figure 55

Summary of Solutions

After 4 different solution techniques, how do the various results compare?

1. Add a new problem to the document, and open a Notes application (page 6.1 in this example).
 - This summary will take on the personal characteristics of each individual student.
 - Figures 56 through 58 represent one possible example.
 - Others could include a spreadsheet used to summarize all the numerical information (Figure 59).

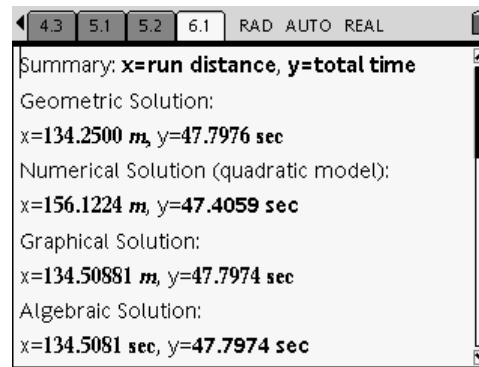


Figure 56

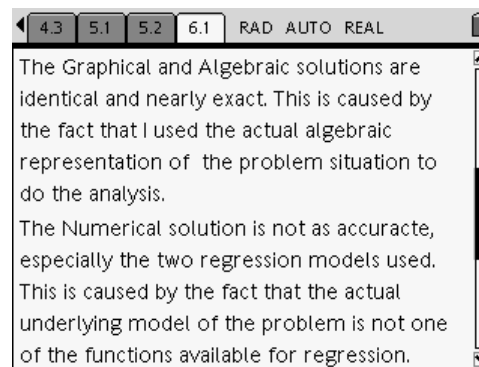


Figure 57

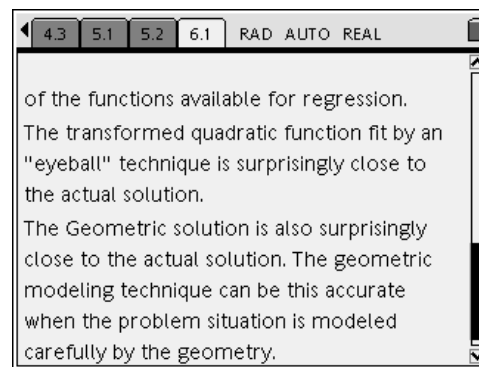


Figure 58

	A solution	B run	C totaltime	D
1	geometric	134.2500	47.7976	
2	num-trans	134.5000	47.9010	
3	num-quad	156.1224	47.4059	
4	num-cubic	135.5514	48.1111	
5	graphical	134.5081	47.7974	
6	algebraic	134.5081	47.7974	

Figure 59

Car Depreciation Problem

Student Worksheet

Name _____

Class _____

As you complete the “D3_6CarDepreciation.tns” TI-Nspire™ math and science learning handheld file, answer the following questions.

For your sixteenth birthday, your parents decide to buy you a car. Your parents pay \$9000 for your new car. A year after buying the car, the car is worth \$7650. When you've had the car for two years, the car is worth \$6502.50. Use this information to complete the questions on the following pages.

1. What is the independent variable?
2. What is the dependent variable?
3. What kind of relationship could exist between the variables? Explain.
4. What kind of function could model the value of the car as it ages?
5. Why did you choose this type of function over another?

Sketch a copy of the graph of what your data looked like when you plotted it (Figure 1).

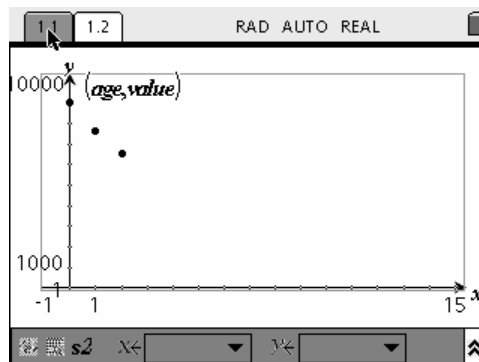


Figure 1

6. What do you notice about the value of the car as it gets older? Explain.
 7. At what rate does the value of the car depreciate each year?
 - Explain how you found your answer.
 8. What is an appropriate domain for your function? Explain.
 9. What is an appropriate range for your function? Explain.
 10. Find an exponential function in the form $y = ab^x$ to model the car's value as it ages.
 11. What is the meaning of 'a'?
 12. What is the meaning of 'b'?
- Sketch a copy of graph of what your function looked like when you graphed it (Figure 2).

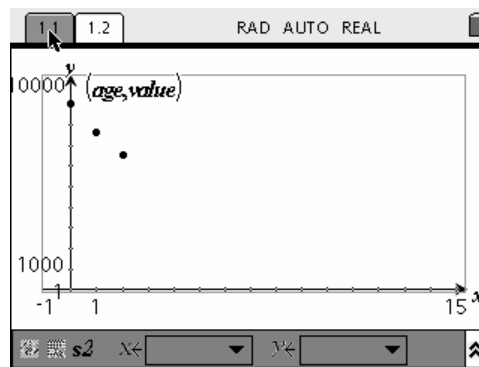


Figure 2

21. Sketch a copy of the graph the function $f_2(x)=1000$ (Figure 3).

- Where does the model for the car's value as it ages and the function $f_2(x)=1,000$ intersect?

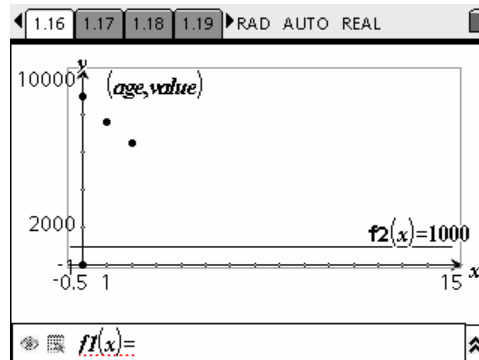


Figure 3

22. What is the meaning of this intersection point? Explain.

23. What would happen to the values in the spreadsheet and the model for the data change if the original price of the car was \$7500 and the rate of depreciation remained the same?

24. What would happen to the values in the spreadsheet and the model for the data change if the original \$9000 value of the car decreased by 7% each year?

Dream Car Depreciation

Student Worksheet

Name _____

Class _____

Introduction

To investigate the value of your dream car over time find the price of the car from ages 0 to 8 on the Internet. (from the present to 8 years ago.) The price should decrease over time.

1. Add a Lists & Spreadsheet application page to a new document on TI-Nspire™ handheld.
2. Move the cursor until it is in the name box next to A or B, depending on which column you are naming (Figure 1).

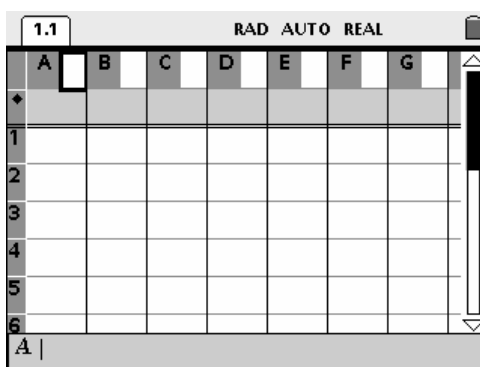


Figure 1

3. Name the first two columns “age” and “value”, respectively (Figure 2).

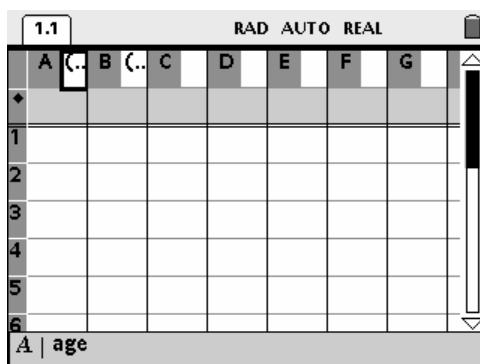


Figure 2

4. Change the width of the columns by following these steps:

- Press **menu** and then press 1:Actions (Figure 3).
- Press 2:Resize (Figure 4).

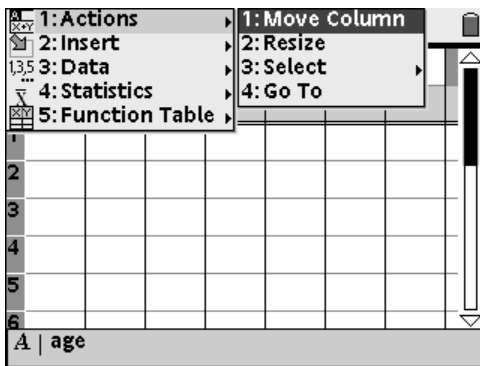


Figure 3

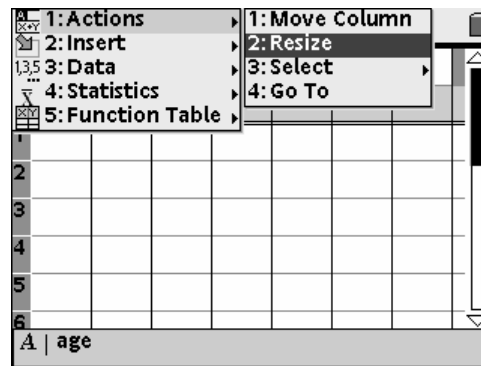


Figure 4

- Press **enter** to set the column width (Figures 5 and 6).

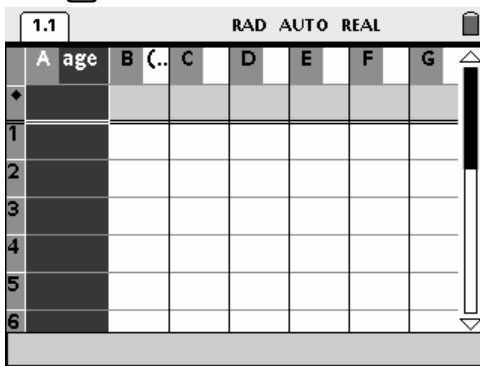


Figure 5

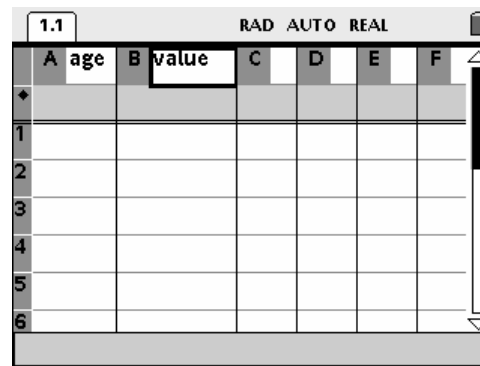


Figure 6

5. Enter the data you found for ages 0 through 8 into the table (Figure 7).

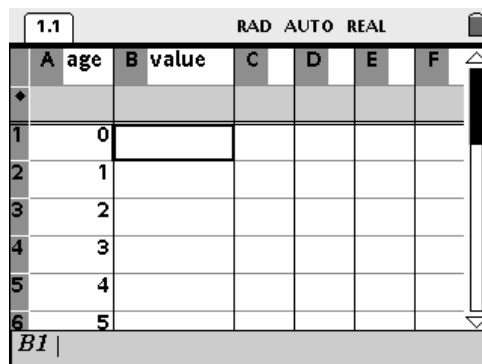


Figure 7

6. Add a Graphs & Geometry page to your document.
7. Change the function type to “scatter plot” by pressing **(menu)** (Figure 8).

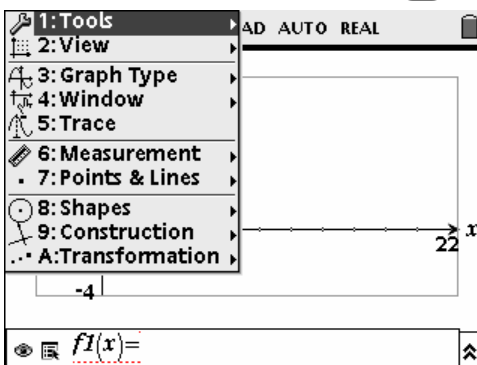


Figure 8

8. Press 3:Graph Type, and then select 3:Scatter Plot (Figure 9).

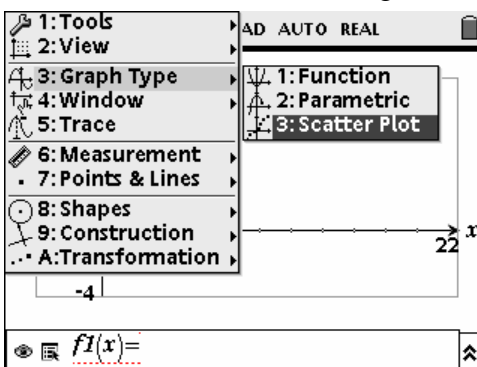


Figure 9

9. Press the **(enter)** key to bring down the menu.
 - Select the list name for the variable x (Figure 10).
 - Press **(tab)** to go over to the y variable and follow the same steps (Figure 11).

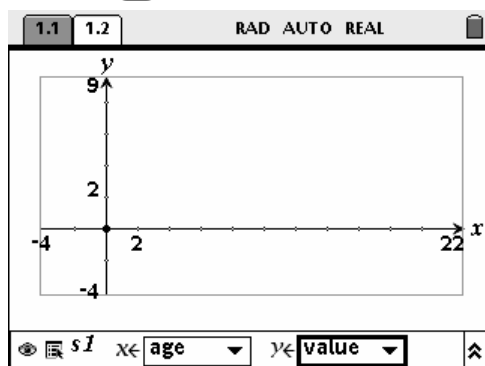


Figure 10

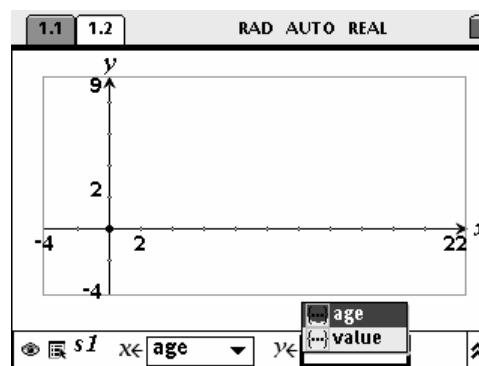


Figure 11

10. Adjust your window to fit your data.

11. Press **(menu)** and then press 3:Graph Type to change your graph back to function mode (Figure 12).

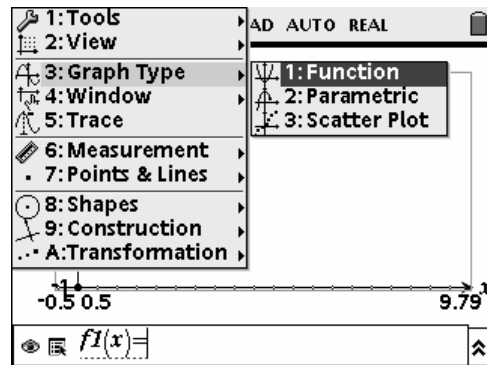


Figure 12

12. Find a function in the form $y=a*b^x$ for the data that you found online.

Go back to the first page of your document and in the third column find the rate of decrease in the values for each year to the next (Figure 13).

1.1		1.2		RAD AUTO REAL		
A	age	B	value	C	decrease	D
1	0					
2	1					
3	2					
4	3					
5	4					
6	5					

Figure 13

13. Calculate the average of these rates of decrease.
14. How do the values of 'a' and 'b' in your exponential decay function compare to the values in your table? Explain.
15. When will the car be worth \$5000?

How much will the car be worth in three and one-half years?

16. Why might there be a difference between the value of the car at a certain year and the value predicted by your exponential function for that year?

Lesson Plan Notes for Car Depreciation

Courses: Algebra 1 and Algebra 2

Topics: Exponential Decay

Suggested Total Time: 45 minutes

Overview

The student activity Car Depreciation Problem could serve as a formative assessment opportunity on exponential functions. The second student activity, Dream Car Depreciation could then be a summative assessment piece. These notes which follow include how to create the student document file.

Related NCTM Standards

Grades 9-12 Algebra Standards:

1. Students should generalize patterns using explicitly defined and recursively defined functions.
2. Students should understand relations and functions and select, convert flexibly among, and use various representations for them.
3. Students should understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions.
4. Students should draw reasonable conclusions about a situation being modeled.

Specific Pre-requisite knowledge:

- Characteristics of exponential functions and how they differ from other functions.
- How to solve exponential functions using logarithms.

Materials Required/Classroom Set-up/Preparation:

All TI-Nspire™ handhelds will need the “D3_6CarDepreciation.tns” file loaded on them.

It is recommended that students work in pairs on this file so they can discuss their findings.

Lesson Notes

- Quickly review the basics of exponential decay and growth functions, how the parameters of these functions affect their graphs, and how exponential functions differ from other functions. You might also want to review how to solve exponential functions.
- Split students into partner groups and give each group a TI-Nspire™ handheld.
- Students should open the TI-Nspire™ file, “D3_2CarDepreciation.tns”.
- While students are working, keep an eye on their progress and help them make connections between their model, spreadsheet, and the graphs.
- As an alternative to the given values of the car, students could look up the value of any car of their choice, examine the change in the value of the car over time, and find the rate of decay for their own car. They can then use their own data to answer the questions within the file.
- When all students have completed the file, bring everyone together for a whole class discussion about what they learned. Make sure that each student has reached the following conclusions:
 - An exponential decay function can be used to relate a car’s value over time.
 - Within the exponential decay function, $y = ab^x$, the value of ‘a’ is determined by the car’s original price. The value of ‘b’ is determined by the rate of depreciation. The value of ‘b’ is the percentage of the value that a car maintains year after year.
 - The age of a car can be calculated using logarithms when you know its value.
 - The value of a car can be calculated when you know its age.

Creating the TI-Nspire™ Handheld Calculator File

Note:


This activity has instructions on creating the TI-Nspire document file on the handheld. However, due to the large amount of text entry you may desire to use the computer software.

1. Add a notes page and type the following text.

- Car Depreciation Problem

For your sixteenth birthday, your parents decide to buy you a used car. Your parents pay \$9000 for your car. A year after buying the car, it is worth \$7650. When you have had the car for two years, the car is worth \$6502.50. Use this information to complete the questions on the following pages.

2. Add a notes page and change the template to be “Q&A”.

- Do this by pressing , 1: Templates, 1: Q&A (Figures 1-3).

3. Type the following text into the question box.

1. What is the independent variable?
2. What is the dependent variable?
3. What kind of relationship could exist between the variables? Explain.

4. Add a notes page, change the template to be “Q&A”, and type the following text.

4. What kind of function could model the value of the car as it ages?
5. Why did you choose this type of function over another?

5. Insert a notes page and type the following text.

- On the next page, complete the table by filling in the value of the car as it ages.

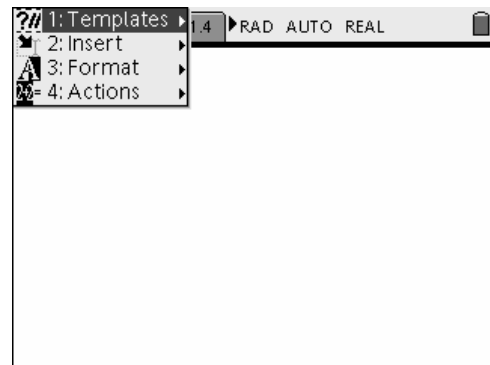


Figure 1

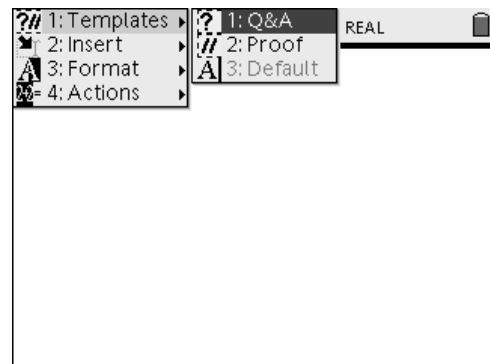


Figure 2

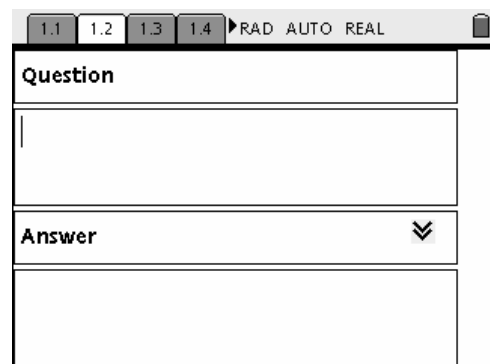


Figure 3

6. Add a lists and spreadsheet page.
 - Label columns A and B as “age” and “value” respectively.
 - Resize the columns so you can see the complete labels (Figure 4).

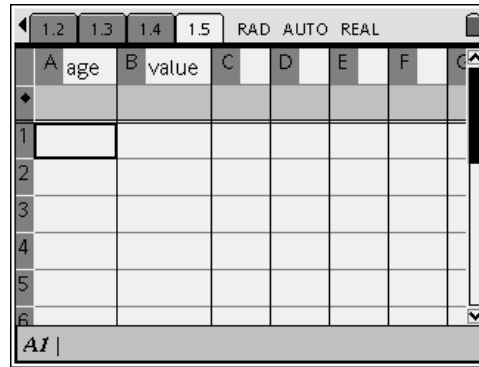


Figure 4

7. Under the age column, type in ages 0 through 10 and under the value column type in the value of the car for the first three ages (Figure 5).

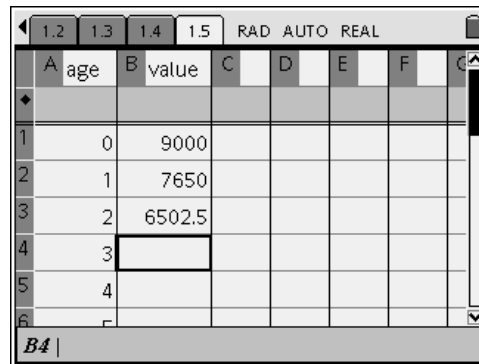


Figure 5

8. Split the screen using the “Custom Split” feature. Do this by following these steps:

- Press **ctrl** .
- Press 6: Page Layout (Figure 6).

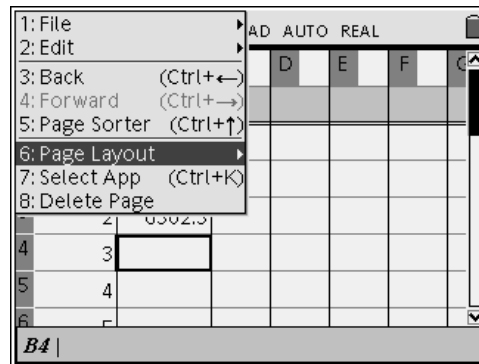


Figure 6

- Press 1: Custom Split (Figure 7).

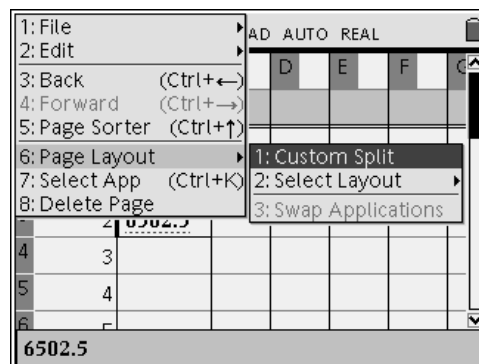



Figure 7

- Use the navigation keys to move the split line to the left so that only the first two columns in the spreadsheet are being shown.
- Press  when you have the screen split where you want it (Figure 8).

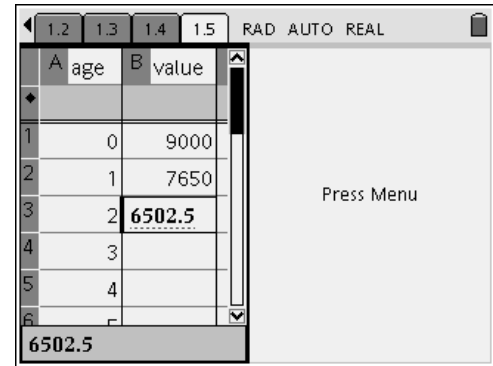


Figure 8

9. Switch to the right side of the screen and add a Graphs & Geometry page.
 - Change the graph type to a scatter plot.
 - Let the x variable be 'age' and let the y variable be 'value' (Figure 9).

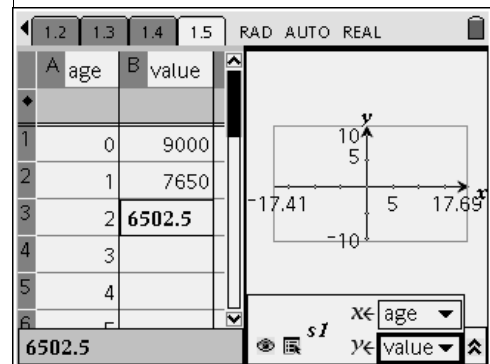


Figure 9

10. Change the window parameters to be Xmin: -1, X max: 15, Ymin: -0.5, Ymax: 10000 (Figure 10).
 - You may wish to move the scatter plot label (age, value).

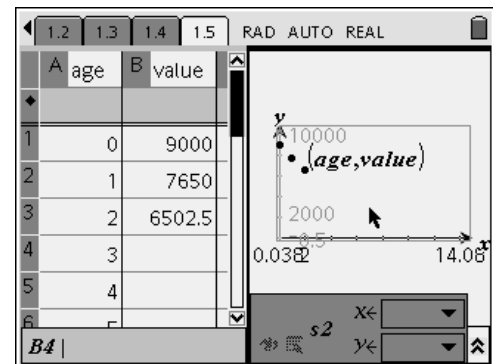


Figure 10

11. Hide the function entry line (Figure 11).
12. Add a notes page, change the template to be "Q&A" and type the following text.
 6. What do you notice about the value of the car as it gets older? Explain.
 7. At what rate does the value of the car depreciate each year? Explain how you found your answer.

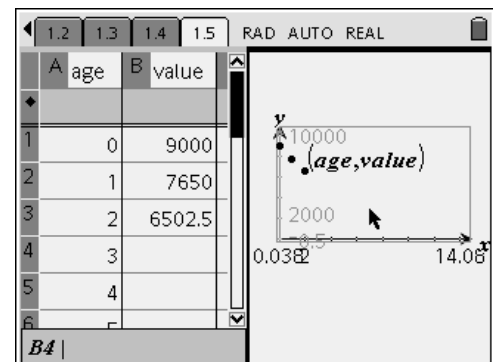


Figure 11

13. Add a notes page, change the template to be “Q&A” and type the following text.
 8. What is an appropriate domain for your function? Explain.
 9. What is an appropriate range for your function? Explain.
14. Add a notes page, change the template to be “Q&A” and type the following text.
 10. Find an exponential function in the form $y=a*b^x$ to model the car’s value as it ages.
 11. What is the meaning of ‘a’?
 12. What is the meaning of ‘b’?
15. Add a notes page and type the following text.
 - On the next page, type the function into the function line to see if your function was correct.
16. Insert a page, but do not add an application.
17. Go back to the scatter plot that appears earlier in the problem.
 - Press Ctrl K to select the application and then press Ctrl C to copy the application.
 - Return to your blank page and press Ctrl V to paste the scatter plot into the page.
18. Change the graph type to “function” (Figure 12).
19. Add a notes page, change the template to be “Q&A”, and type the following text.
 13. Was your function correct? Why or why not?
 14. If your function was incorrect, what can you do to fix your model? Explain.
20. Insert a notes page and type the following text.
 15. Once you have a model that you think will work, go back to the previous graph page and change your function to check your answer. What is your final function?
21. Add a notes page, change the template to be “Q&A”, and type the following text.
 16. What will the value of the car be when you’ve had it for 10 years?
 17. How long will you have owned the car when it is worth \$1000?

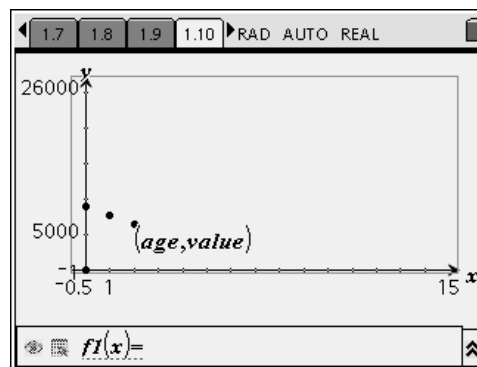


Figure 12

22. Add a notes page, change the template to be “Q&A” and type the following text.
18. Will the car ever be worth \$0? Explain.
 19. If the car had been brought to the used car lot three years before your parents bought it, how much was it worth when it was first brought to the used car lot?
23. Add a notes page, change the template to be “Q&A”, and type the following text.
20. It appears as though the graph never crosses the x-axis. Why does this happen?
24. Add a notes page, change the template to be “Q&A”, and type the following text.
- On the next page, the function $f_2(x) = 1000$ has been graphed. Press “Enter” when your function $f_1(x)$ is active to see its graph.
21. Where does the model for the car’s value as it ages and the function $f_2(x) = 1000$ intersect?
 22. What is the meaning of this intersection point? Explain.
25. Insert a page. Copy the scatter plot and function, $f_1(x)$, from earlier in the problem onto the page. Graph the function $f_2(x)=1000$.
26. Add a notes page, change the template to be “Q&A”, and type the following text.
23. What would happen to the values in the spreadsheet and the model for the data if the original price of the car was \$7500 and the rate of depreciation remained the same?
 24. What would happen to the values in the spreadsheet and the model for the data if the original \$9000 value of the car decreased by 7% each year?
27. Add a notes page and type the following text.
- Create a spreadsheet for the car’s value if the original price had been \$7500 and the car’s value depreciated by 7% each year.
 - Then, analyze how the new data and the scatter plot compare to the data and scatter plot from the original problem.
 - It might help to change the graphing style of one of the scatter plots by changing the attributes.

28. Add a new page and copy the spreadsheet from the beginning of the document.
 - Label column C “car2.”
 - Enter the data for the second car in column car2.
29. Add a new page and copy the scatterplot of value versus age from the beginning of the document.
 - Graph the data for car2 versus age.
 - Change the graphing style of one of the scatter plots.
30. Add a notes page, change the template to be “Q&A”, and type the following text.
 - How are the two scatter plots similar? Explain.
 - How are the two scatter plots different? Explain.
31. Add a notes page, change the template to be “Q&A”, and type the following text.
 - Find a function that would model the data for the second car.
 - Explain how you found the values of ‘a’ and ‘b.’
32. Add a notes page, change the template to be “Q&A”, and type the following text.
 - Is it possible for these two cars to cost the same amount after they have been purchased? Why or why not?
 - How would the graphs look if the two cars cost the same amount?

Maximize the Area of a Garden

Concepts

- Perimeter
- Area
- Optimization
- Quadratic Functions
- Derivatives






Overview

This activity describes the steps necessary to create the student .tns file. Students will be investigating what is the maximizing area possible with a fixed perimeter.

Materials

- TI-Nspire™ Math and Science Learning Handheld

Getting Started

1. Open a new TI-Nspire™ document, and add a Notes page.
2. Enter the title for the activity on the Notes page (Figure 1).
3. Add another new Notes page, and complete a description of the problem (Figure 2).
4. Add a third new Notes page.
5. Press      for 6:Page Layout, 2:Select Layout and 3:Layout 3 (Figure 3).
 - This will create a horizontal split of your page.
 - The upper pane will still be active.

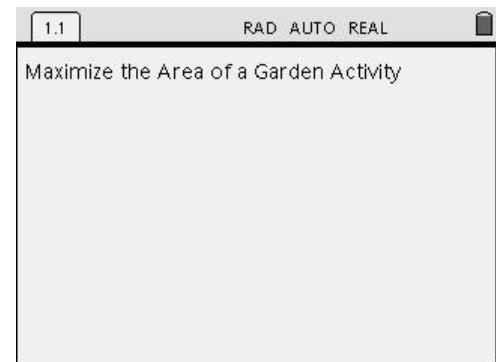


Figure 1

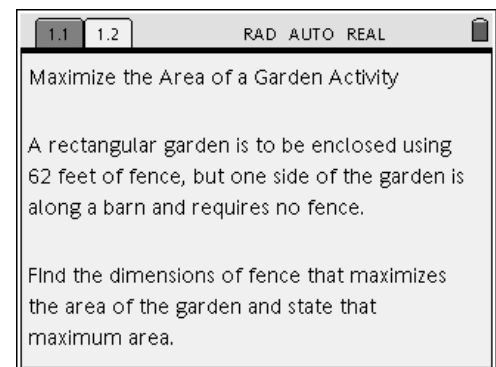


Figure 2

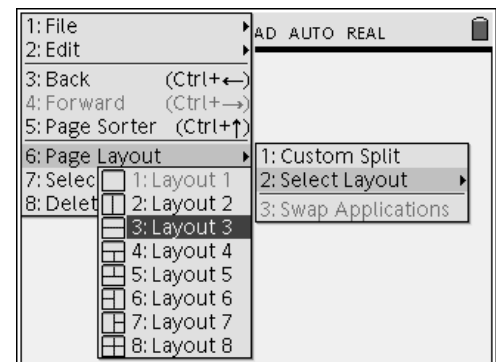

























Figure 3

6. Complete the final set of instructions in the upper pane as shown in Figure 4.
7. Press   to switch from the upper pane to the lower pane.

Graphs & Geometry: Create a Rectangle

1. Add a Graphs & Geometry page.
2. Press    for Menu 2:View, 1:Hide Axes.
3. Press   to hide the entry line.
4. Press    for Menu 6:Points & Lines, 4:Line.
5. Add a line across the screen.
6. Press    for Menu 1:Tools, 3:Attributes.
7. Click on the line you drew on the screen, and the Attributes Menu will appear.
8. Press  on the first option to make the line appear thick (Figure 5). Press .
9. Press    for Menu 8:Shapes, 3:Rectangle.
 - The rectangle tool requires you to identify three corners of your rectangle.
10. To begin, click on the thick line from the last step, move the cursor down, and press  to anchor the second point.
11. Finally, move the cursor right to the third point, and press  a final time (Figure 6).
12. Press    for Menu 1:Tools, 5:Text.

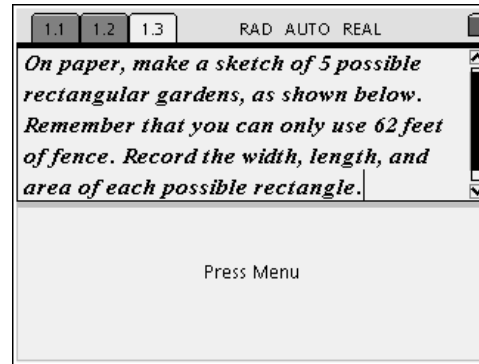


Figure 4

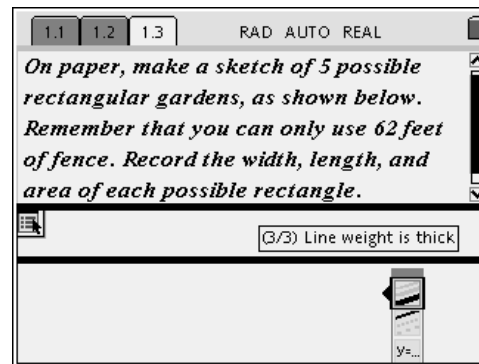


Figure 5

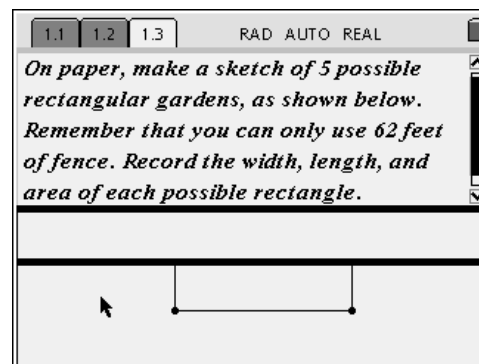


Figure 6

13. Use Text tool to add the labels *width*, *length*, *area*, and *Barn* (Figure 7).

14. Press to add a new Notes page, and enter the message shown in Figure 8.

15. Press for Menu 6:Tools, 2:Page Layout, 2:Select Layout, 2:Layout 2 (Figure 9).

- This will create a vertical split on the current page.

16. Press to switch to the right work area.

Lists & Spreadsheet: Data Collection

1. Add a Lists & Spreadsheet page.
2. Name the columns 'wid' and 'area'.
3. In each column, press for Menu 1:Actions, 2:Resize.
4. Press on the NavPad cursor control to widen the column, and press when you have the column width that you want (Figure 10).

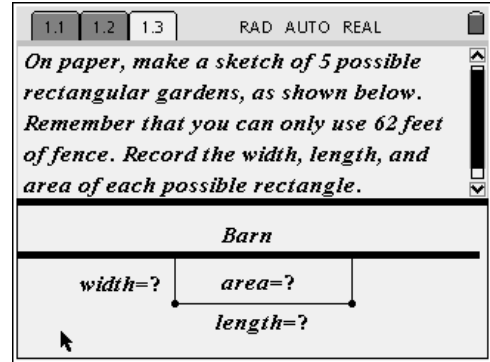


Figure 7

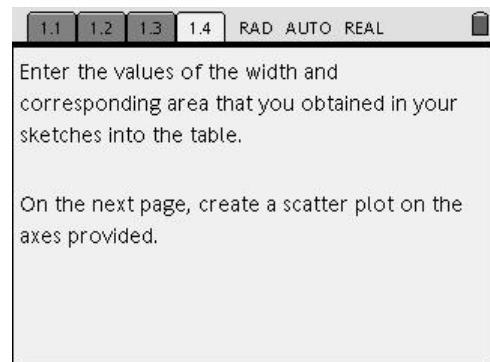


Figure 8

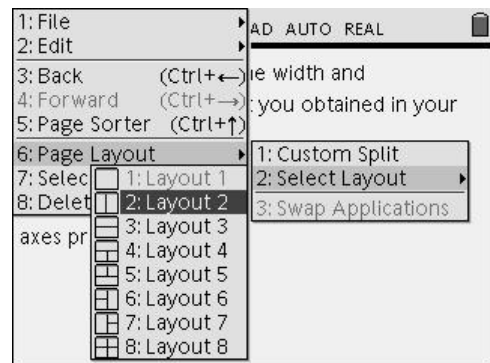


Figure 9

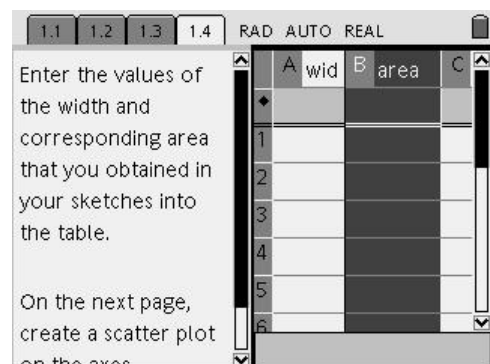


Figure 10

5. Add the data from your exploration (Figure 11).

Graphs & Geometry: Scatter plot

1. Press to open a new Graphs & Geometry page.
2. Press for Menu 4:Window, 1:Window Settings.
3. Change the values as shown in Figure 12.

4. Press for Menu 3:Graph Type, 3:Scatter Plot to change the type of graph (Figure 13).

5. Choose variables for the scatter plot (Figure 14).

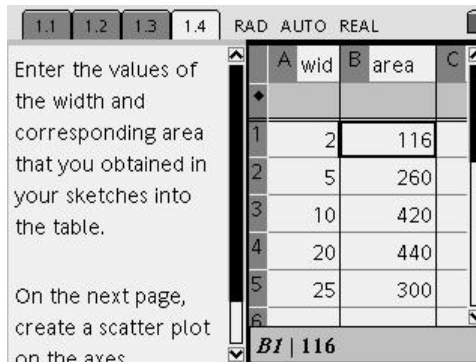


Figure 11

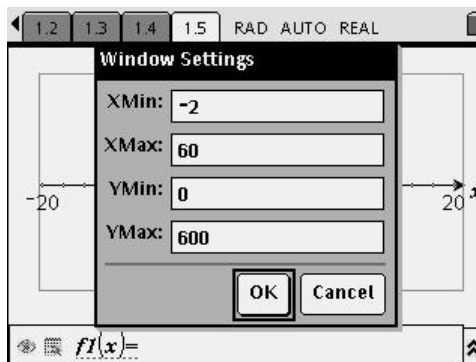


Figure 12

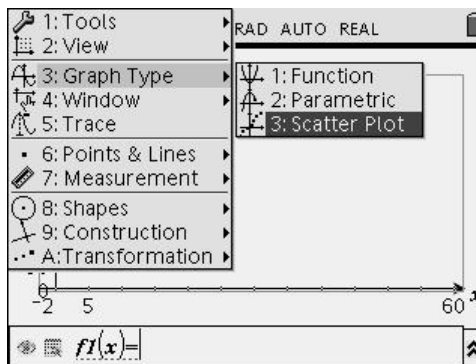


Figure 13

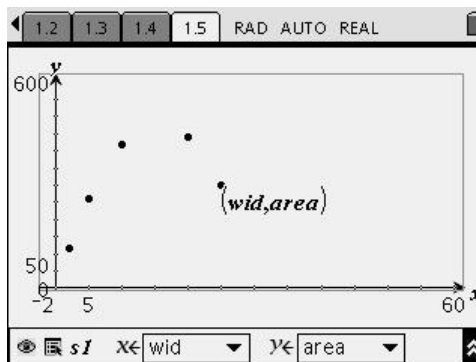


Figure 14

Lists and Spreadsheet: Quadratic Regression

- Press **ctrl** **←** to move back to Lists & Spreadsheet on page 1.4.
 - This is where we will create the regression equation.
 - Press **menu** **4** **1** **6** for Menu 4:Statistics, 1:Stat Calculations, 6:Quadratic Regression (Figure 15).
 - Complete the dialog box for quadratic regression by choosing column C for the field “1st result column” (Figure 16).
 - Tab to highlight OK and press Enter.
- The results of the quadratic regression will appear in columns C and D (Figure 17).
- You may choose to resize these columns.

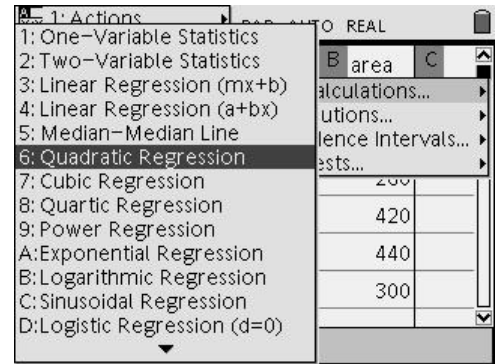


Figure 15

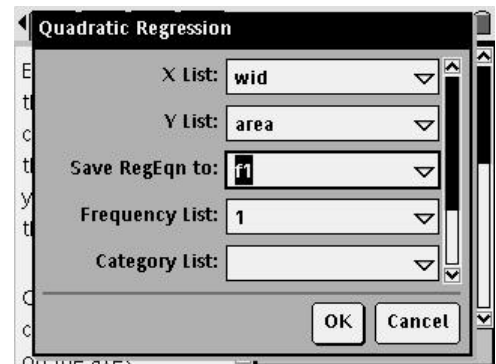


Figure 16

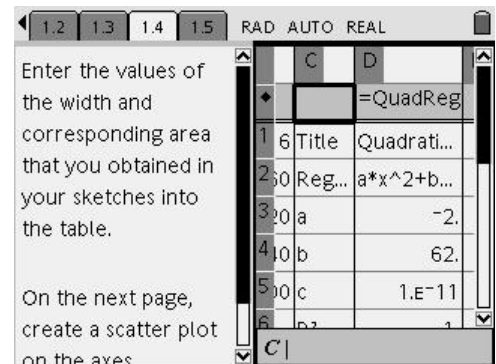


Figure 17

4. Press ctrl ▶ to return to Graphs & Geometry page (page 1.5).
5. Press menu $\{3\}$ $\{1\}$ for Menu 3:Graph Type, 1:Function.
6. In the entry line at the bottom of the Graphs & Geometry page, scroll up to $f1(x)$, and press enter .
 - The regression graph will appear on the screen (Figure 18).
7. Resize the window as needed.
8. As a first attempt, press menu $\{4\}$ $\{9\}$ for Menu 4:Window, 9:Zoom Stat (Figure 19).
 - If necessary, press menu $\{4\}$ $\{1\}$ for Menu 4:Window, 1:Window Settings to change the settings so that the maximum is available.
9. Press menu $\{5\}$ for Menu 5:Trace (Figure 20).
 - By default, the trace will begin on the scatter plot.
10. Press \blacktriangle on the NavPad to tab to the function plot.
11. Move the trace point by pressing \blacktriangleleft and \blacktriangleright on the NavPad.
12. Press enter to set the point.
13. Grab the Trace Point, and move it left and right.
 - When the Trace Point is on the local maximum, a caption with the letter “M” will appear next to the hand (Figure 21).

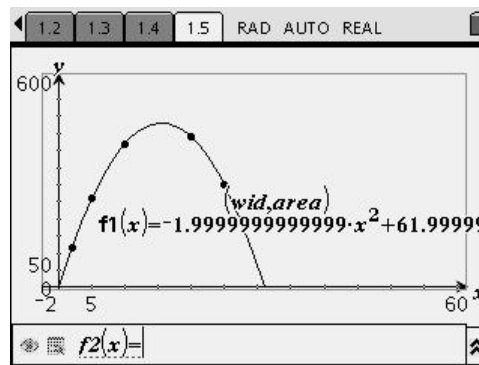


Figure 18

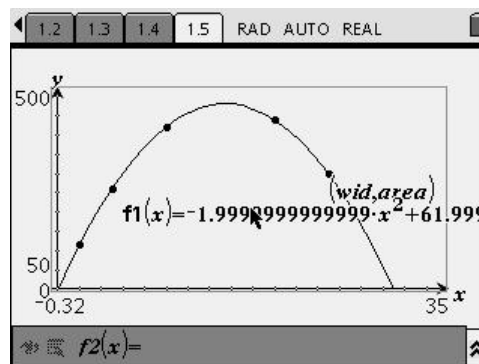


Figure 19

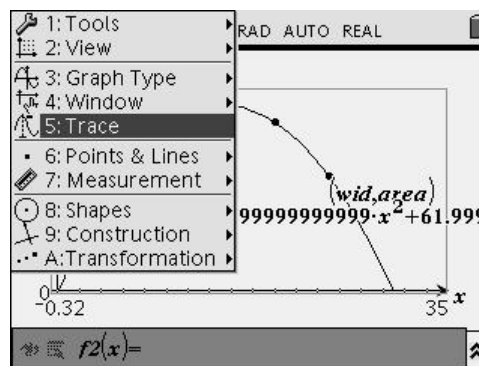


Figure 20

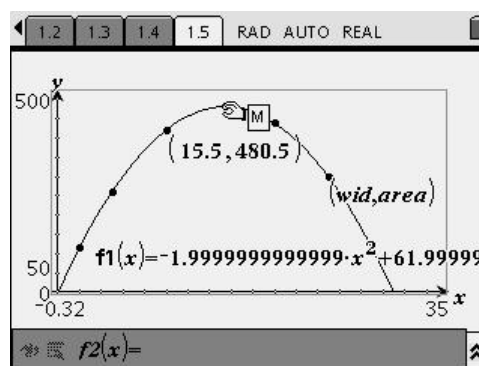


Figure 21

Optional: Using CAS

Following up on the geometric solution above, we can build an algebraic solution on a Calculator page.

1. Press ctrl home 2 8 to insert a new problem (2.1) in the same document, if desired (Figure 22).
2. When the choice of tools appears, choose 1 for 1:Add Calculator.
 - To guide students through the calculus solution, you can insert comments on the calculator page.
3. Press menu 1 6 for Menu 1:Tools, 6:Insert Comment (Figure 23).
4. Enter the comments as shown in Figure 24.
5. Press home 1 to add a new Calculator page.
6. Press menu 1 1 for Menu 1:Tools, 1:Define (Figure 25).
7. Enter the name of the function on the left side of the equal sign and the algebraic definition of the function on the right side of the equal sign.

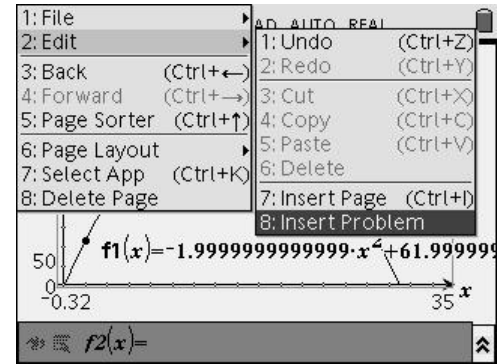


Figure 22

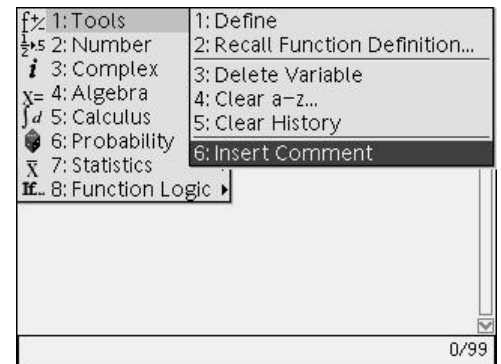


Figure 23

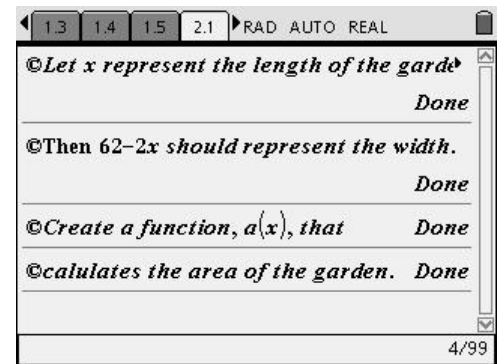


Figure 24

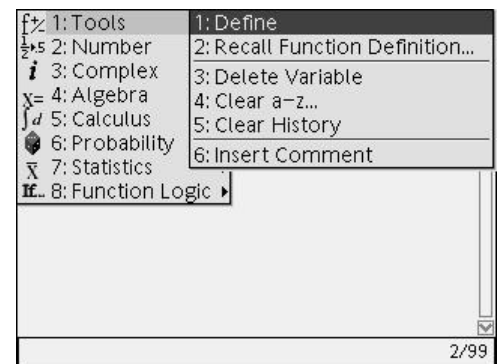


Figure 25

8. Add the next comment shown in Figure 26 to guide users.

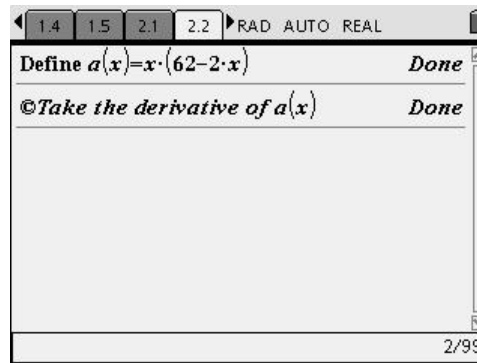


Figure 26

9. Press menu $\{5\}$ $\{1\}$ for Menu 5:Calculus, 1:Derivative (Figure 27).
- A derivative template appears.
10. Enter the variable x in the denominator of the derivative template and the function $a(x)$ in the argument field.

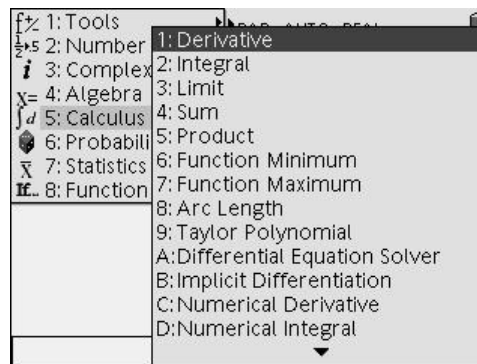


Figure 27

11. Add the next comment shown in Figure 28 to guide users.

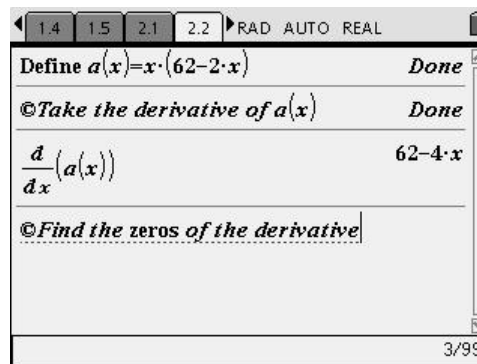


Figure 28

12. Press menu $\{4\}$ $\{1\}$ for Menu 4:Algebra, 1:Solve (Figure 29).
- Use the Solve command to find the zeros of the equation.
13. In order to insert the expression for the first derivative, either re-type it here, or scroll up to the result, and press enter .

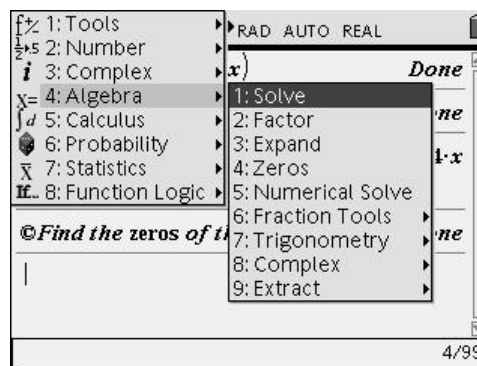








Figure 29

14. Complete the solve command (Figure 30).
15. Press   to add a new Calculator application page.

16. Add the comment as shown in Figure 31.

We are going to use the second derivative test so we will need to find another derivative.

17. Add the comments as shown in Figure 32.
18. Press   to add a new Calculator application page.

19. Add the comments as shown in Figure 33, and calculate the length and area.
20. Press   to add a new Calculator application page.

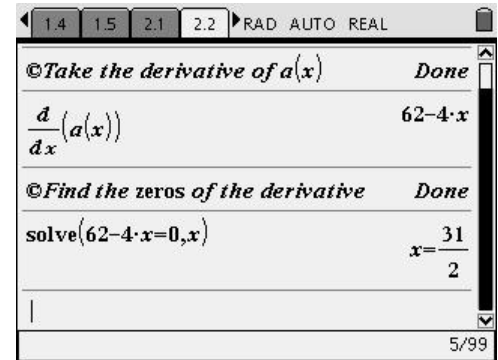


Figure 30

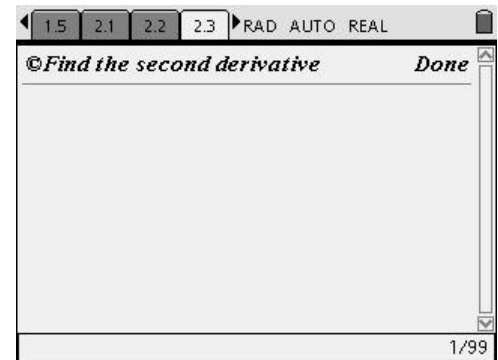


Figure 31

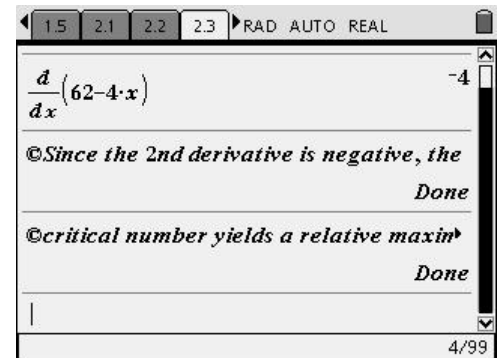


Figure 32

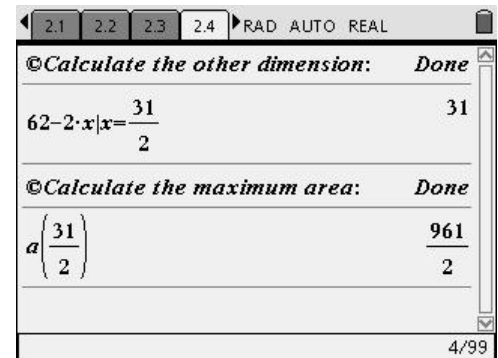


Figure 33

21. Add the comments shown in Figure 34 to conclude this part of the solution.

Note: This could also have been done using a Notes page.

Alternate Solution

There is an alternate solution using more geometry tools.

1. Press **ctrl** **⌘** **2** **8** to create a new problem if desired.
2. Add a Graphs & Geometry page.
3. Press **menu** **4** **1** for Menu 4:Window, 1:Window Settings.
4. Change the settings as shown in Figure 35.
5. Press **menu** **2** **1** for Menu 2:View, 1:Hide Axes (Figure 36).
 - Notice that you can also use this same menu to hide the entry line.
6. Hide the entry line as well.

This solution will use the scale feature.

7. Press **menu** **2** **4** for Menu 2:View, 4:Show/Hide Scale (Figure 37).
 - A line segment should appear in the upper right corner of the screen with the label “1 cm” beneath it.

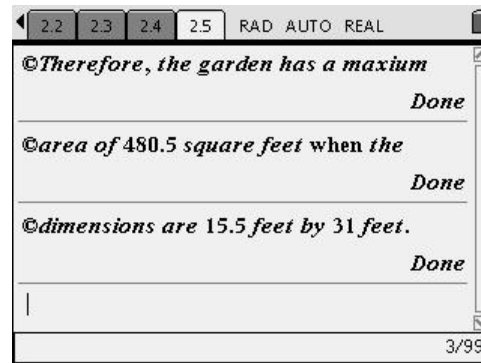


Figure 34

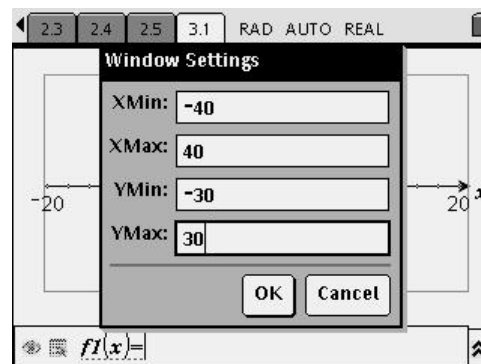


Figure 35

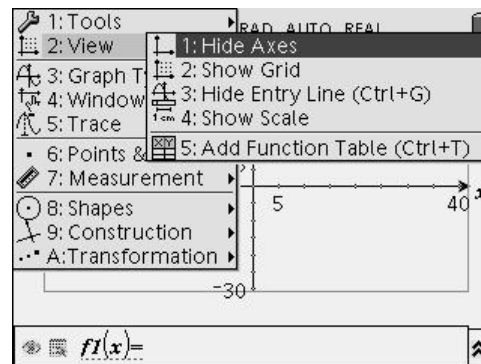


Figure 36

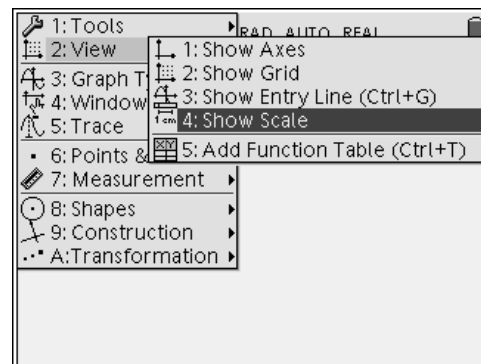


















Figure 37

8. Move the cursor to the label, and press .
 - This will allow you to edit the unit of measurement.
9. Change the “1 cm” to “5 ft” (Figure 38).
10. Press    for Menu 6:Points & Lines, 5:Segment.
11. Construct two segments – one at the top of the screen and a second one in the lower part of the screen.
12. Label point A as soon as you create it.
13. Press    for Menu 6:Points & Lines, 2:Point On.
14. Construct points B and C as shown in Figure 39, and label them immediately.
15. Press    for Menu 1:Tools, 5:Text.
16. Add the label BARN above the upper line segment.
17. Press    for Menu 7:Measurement, 1:Distance.
18. Click on point A and then point B.
19. Move the measurement off to the side (Figure 40).
 - This measurement will be used as the width of the garden.
20. Press    for Menu 9:Constructions, 1:Perpendicular.
21. Click on Point C and the line segment through point C (Figure 41).
 - A perpendicular line will appear through point C.

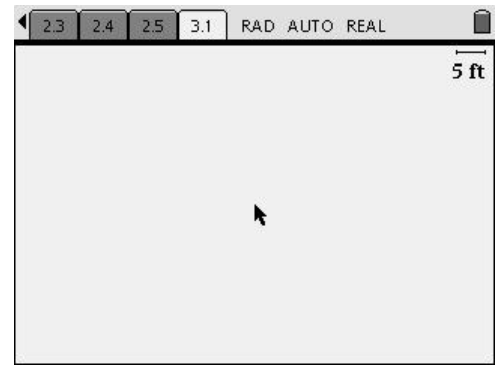


Figure 38

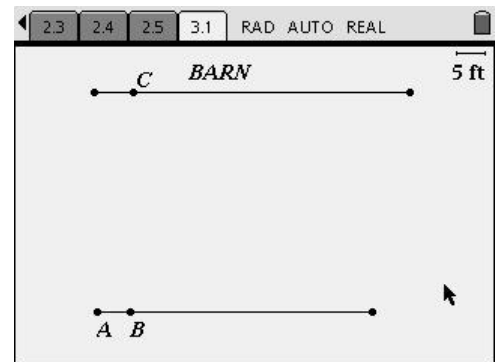


Figure 39

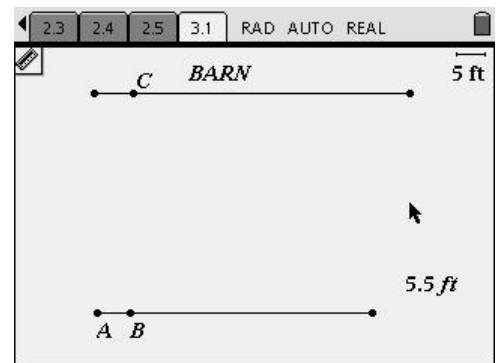


Figure 40

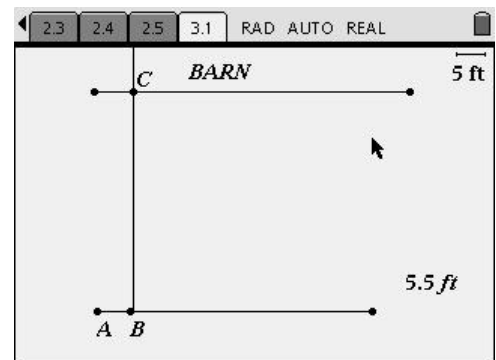






Figure 41

22. Press **menu** **6** **8** for Menu 6:Points & Lines, 8:Vector (Figure 42).
23. Construct two vectors, one south of point C and one due east of point C.
24. Once you chosen the vector tool, press  while on point C and then click  again on the line segments passing vertically through point C.
25. Before you press anything else, press  on point C and then press  again on the line passing horizontally through point C (Figure 43).

26. Use the text tool to create an expression for the length of the garden (Figure 44).

27. Press **menu** **1** **7** for Menu 1:Tools, 7:Calculate (Figure 45).
 - We will use this tool to calculate the value for the length of the garden.
28. Click on the expression “ $62-2\cdot w$ ”.
 - As you move away from this expression, you will see a “w” in the caption.

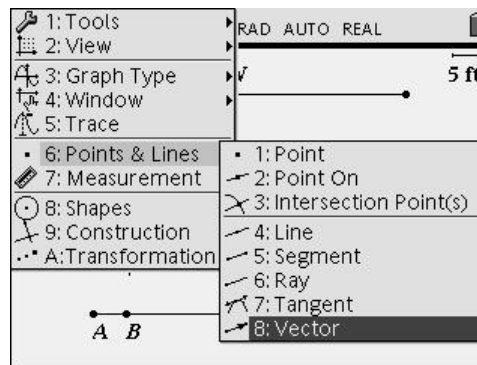


Figure 42

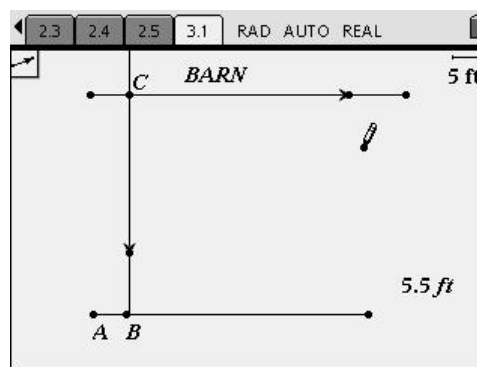


Figure 43

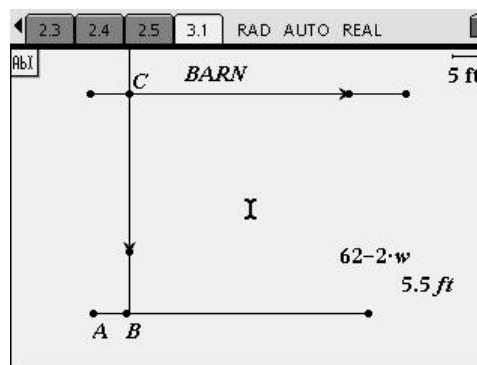


Figure 44

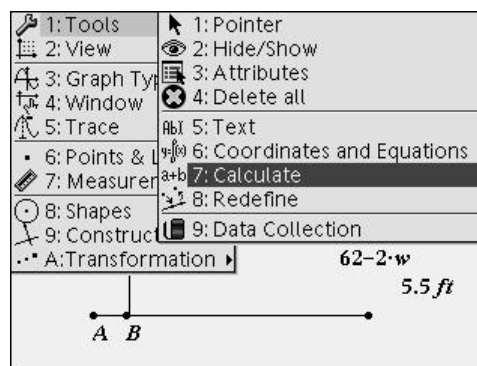





Figure 45

29. Click on the measurement, and move the result to a spot beside the formula (Figure 46).

- The measurements for width and length can be transferred to the two vectors in the diagram.

30. Press    for Menu 9:Constructions, 8:Measurement transfer (Figure 47).

31. Click on the measurement for width and then the vector pointing south.

32. Immediately label this point E (Figure 48).

33. Click on the measurement for length and then the vector pointing east.

34. Immediately label this point D (Figure 48).

35. Drag point B to see the location of these points change.

36. Use the Hide/Show tool to hide the vectors, the perpendicular line, and the points (Figure 49).

- It might be necessary to extend the horizontal line segment through C.

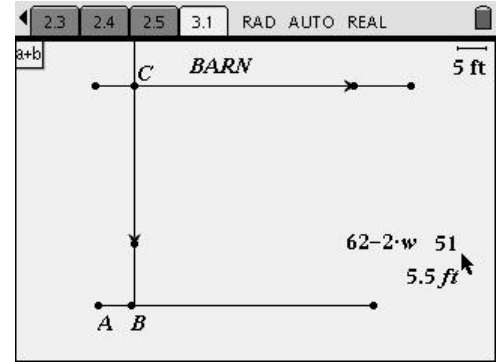


Figure 46

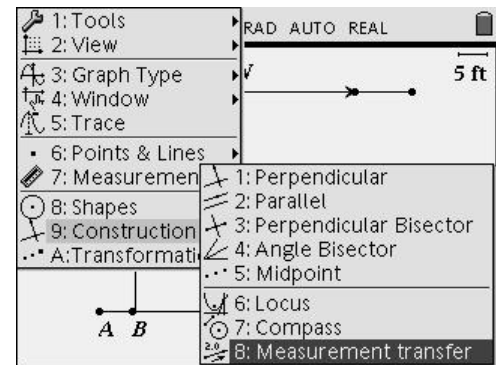


Figure 47

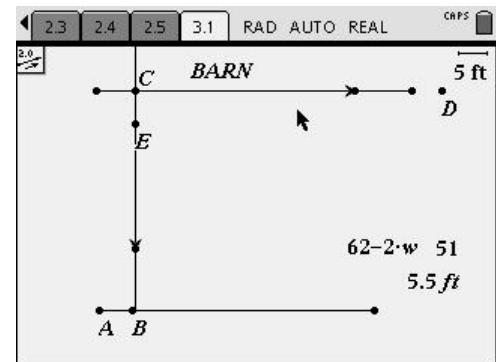


Figure 48

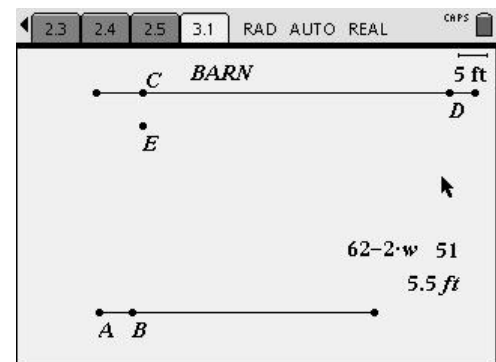


Figure 49

37. To complete the rectangle representing the garden, construct a line segment connecting C to E (Figure 50).

38. Construct a perpendicular to CE at E and a perpendicular to CD at D.

- It might be necessary to extend the line through E so that it intersects the line through D (Figure 51).

39. Press menu $\{6\}$ $\{3\}$ for Menu
6:Points & Lines, 3:Intersection Point(s).

40. Construct the intersection of the two perpendicular lines, and immediately label it as F (Figure 52).

41. Hide the perpendicular lines, and construct line segments EF and DF.

- We now have the widths CE and DF and length EF (Figure 53).
- The measurements for these are also on the screen.

42. Assign the variables “W” and “L” to the measurements for width and length on the screen.

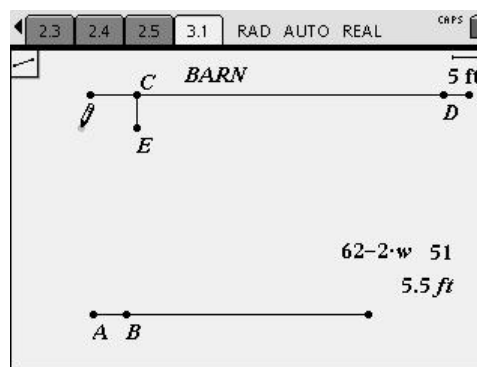


Figure 50

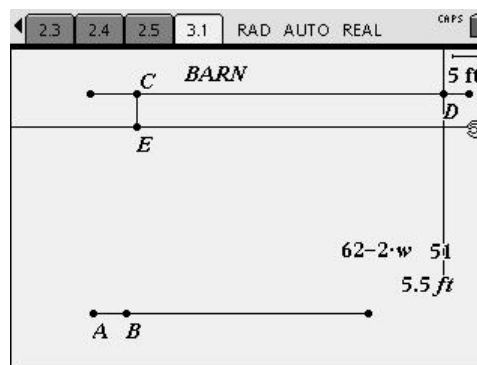


Figure 51

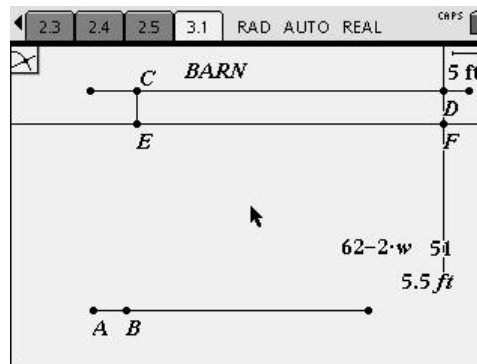


Figure 52

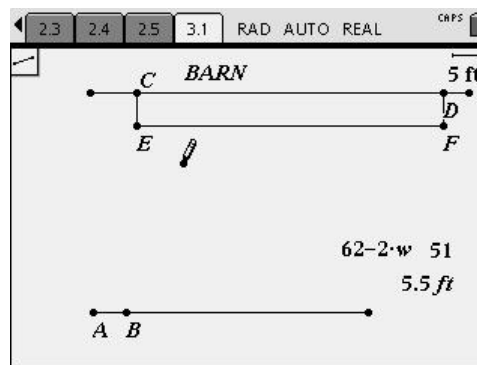


Figure 53

43. Using the Text tool, create the expression “CE*EF” (Figure 54).
- This will represent the area of the garden.
44. Use the Calculate tool to evaluate the expression “CE*EF”.

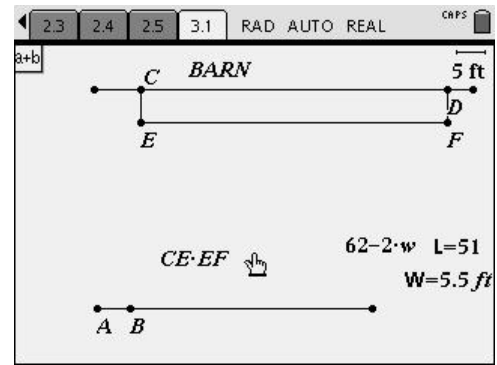


Figure 54

45. Click on the values for “W” and “L” to complete the calculation (Figure 55).

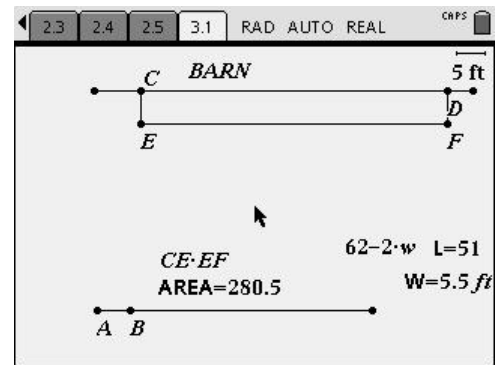


Figure 55

46. Add a new Lists & Spreadsheet page.
47. Label the first two columns “awidth” and “aarea” (Figure 56).

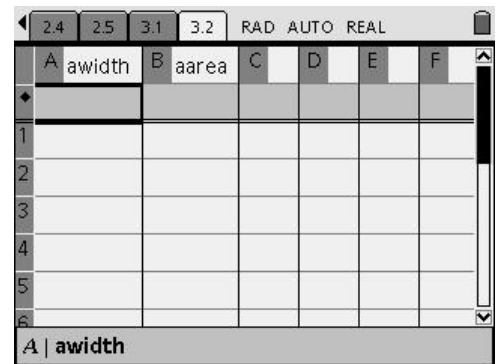


Figure 56

48. With the cursor on the formula line for column A, press MENU 3 2 1 for Menu 3:Data, 2:Data Capture, 1:Automated Data Capture (Figure 57).
- A formula will appear in column A.

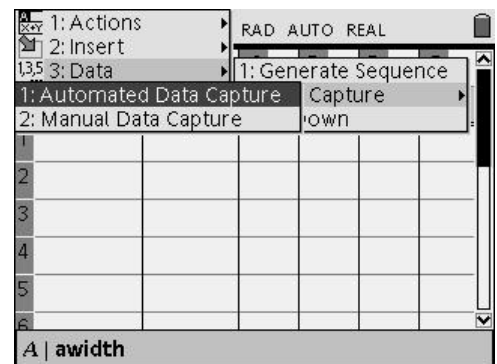






Figure 57

49. Replace the text “var” with variable W and press  (Figure 58).



- A dialog box will appear asking you to indicate if “W” is a cell or variable reference.




50. Open the dialog box, and select Variable Reference (Figure 59).

51. Press  or  to move to the OK button, and press .

52. For column B, choose the variable “area” (Figure 60).

- The numbers that appear in the first row are the current values for W and area from the construction on page 3.1.

53. Press   to add a Graphs & Geometry page.

54. Press    for Menu 3:Graph Type, 3:Scatter Plot (Figure 61).

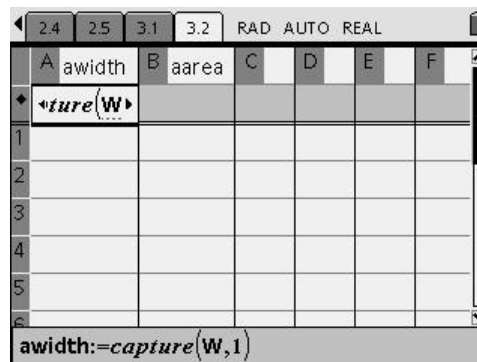


Figure 58

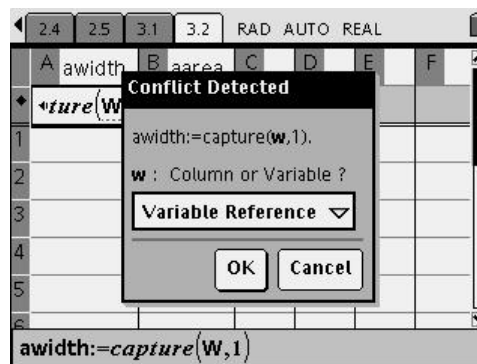


Figure 59

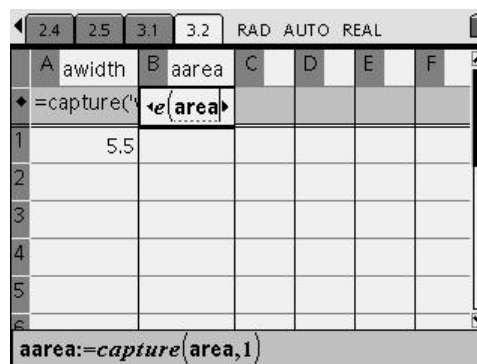


Figure 60

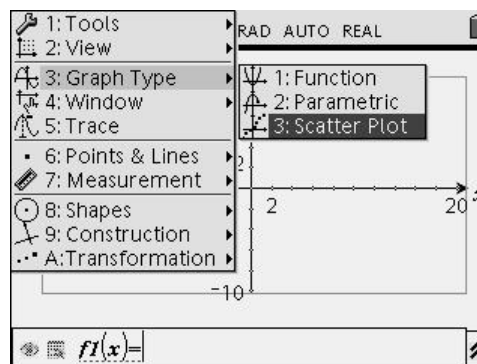


Figure 61

55. Press $\text{\textcircled{enter}}$ to open the drop down box, and select “awidth” for the x -variable (Figure 62).
56. Press $\text{\textcircled{tab}}$ to move to the y -variable drop down box and select “aarea” (Figure 62).
 - The window will need to be changed to accommodate the data.
57. Press $\text{\textcircled{menu}}$ $\text{\textcircled{4}}$ $\text{\textcircled{1}}$ for Menu 4:Window, 1:Window Settings.
58. Change the values as shown in Figure 63.

- At the moment, there is only one data point. You will see it on the screen with the label (awidth, aarea) (Figure 64).

59. Press $\text{\textcircled{ctrl}}$ $\text{\textcircled{left}}$ twice to move back two pages.

60. Grab point B at the bottom of the screen (Figure 65).

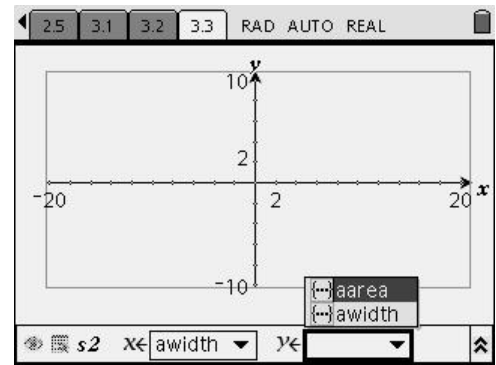


Figure 62

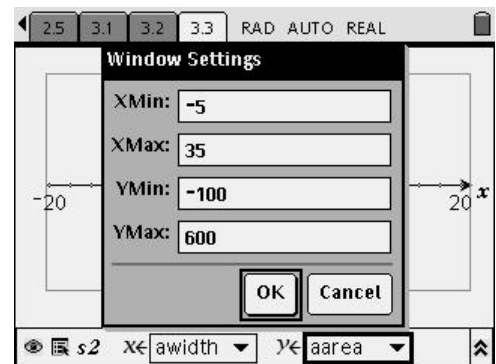


Figure 63

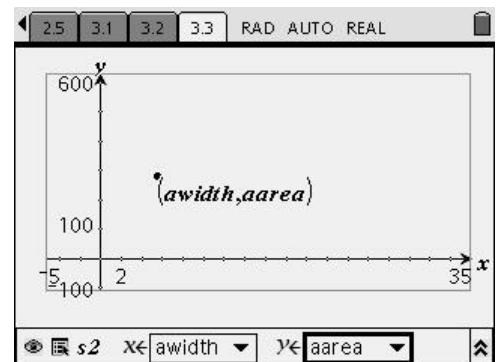


Figure 64

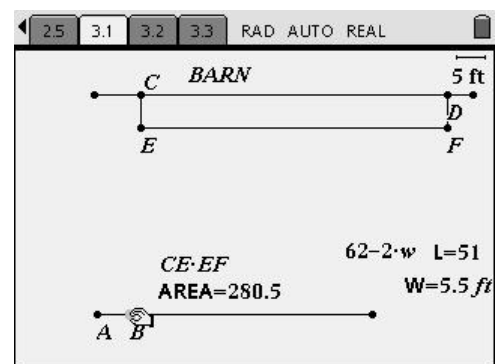


Figure 65

- As you drag point B, you will be changing the width, length, and area (Figure 66).
- Also, as point B is dragged on page 3.1, new rows of data were captured in the spreadsheet on page 3.2 and plotted on the graph on page 3.3 (Figure 67).
- The graph appears quadratic.

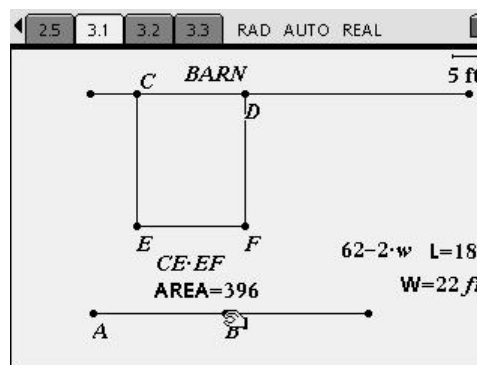


Figure 66

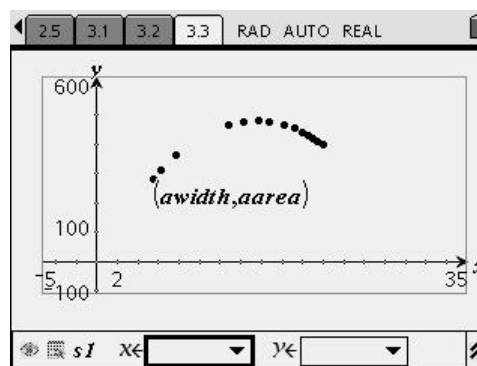


Figure 67

- To test the relationship, press **menu** **3** **1** for Menu 3;Graph Type, 1:Function (Figure 68).
- Type the equation $f1(x) = x \cdot (62 - 2x)$ and press **enter**.
- To test the relationship, press **ctrl** **←** to move back to page 3.2.

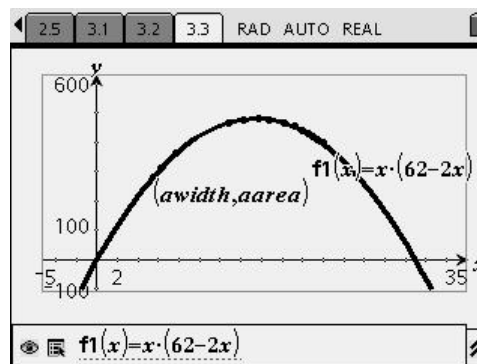


Figure 68

- In column C, move to the formula line, press **=**, and enter the formula $f1(\text{awidth})$ (Figure 69).
- Press **enter**.
 - The fact that the function values in column C match up perfectly with the captured data values in column B indicate that the function matches the data.

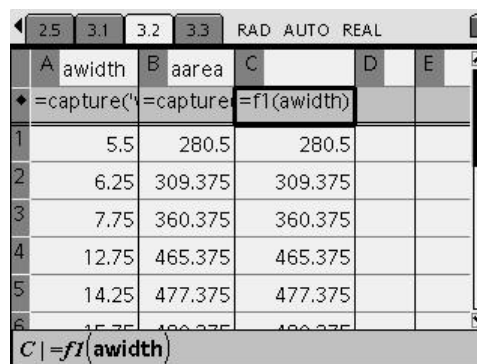


Figure 69

66. Press ctrl \rightarrow to move back to page 3.3.
67. Add a trace point, and drag it horizontally using the \blacktriangleleft and \blacktriangleright NavPad controls (Figure 70).
- When the caption with a capital “M” appears, you have found the maximum value of the area.

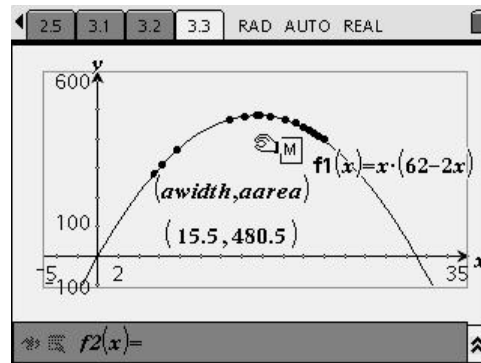


Figure 70

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Lesson Plan Notes for Maximize the Area of a Garden

Course(s): Calculus, Pre-Calculus (without the CAS part)

Topic(s): Area of a rectangle, Perimeter of a rectangle,
Representing widths and lengths of rectangles
in terms of a variable

NCTM Standards:

All students should...

Grades 9-12 Algebra Standards:

- generalize patterns using explicitly defined and recursively defined functions
- understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on more-complicated symbolic expressions
- understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions
- interpret representations of functions of two variables
- use symbolic algebra to represent and explain mathematical relationships
- judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology
- draw reasonable conclusions about a situation being modeled

Grades 9-12 Geometry Standards:

- analyze properties and determine attributes of two- and three-dimensional objects
- use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations

- investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates
- draw and construct representations of two- and three-dimensional geometric objects using a variety of tools
- use geometric models to gain insights into, and answer questions in, other areas of mathematics

Suggested Total Time: 50 minutes

Specific Pre-requisite knowledge:

Students need to be able to:

- represent the dimensions of a rectangle in terms of a single variable with the constraint of how much fence is available.
- graph equations, create regression equations, and plot data using TI-Nspire™.

Students using CAS should be familiar with these features: derivatives; solving equations; defining functions; evaluating functions.

Materials Required/Classroom Set-up/ Preparation:

There are two versions of this activity:

1. The file “D3_7MaximizeArea.tns” should be downloaded to each student’s TI-Nspire™ handheld.
2. The teacher version “D3_7MaximizeAreaTch.tns” has the same information as the student version, but it also has the answers and solutions to all questions.

Besides their TI-Nspire™ learning handheld, each student will need a piece of paper. A straightedge is optional.

Lesson Notes:

The activity is self-directed. The students need to have the activity on each of their calculators and pieces of paper to make some sketches and to explore some algebraic representations and manipulations.

Live Data Sharing

Materials

- TI-Nspire™ Computer Software for Math and Science
- D3_8LiveDataSharing.tns file

Overview

Linking of data between a scatter plot and spreadsheet is explored. With that linking the impact of moving data points on the regression line is explored.

NCTM Standards

All students should...

Grades 6-8 Algebra Standards:

1. represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules
2. relate and compare different forms of representation for a relationship
3. identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations
4. explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope
5. model and solve contextualized problems using various representations, such as graphs, tables, and equations
6. use graphs to analyze the nature of changes in quantities in linear relationships

Grades 9-12 Algebra Standards:




1. interpret representations of functions of two variables
2. use symbolic algebra to represent and explain mathematical relationships
3. judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology
4. draw reasonable conclusions about a situation being modeled

Introduction

There are two choices with this activity:

- Open the prepared TI-Nspire™ .tns document file “D3_8LiveDataSharing.tns” and move to next section of steps called Working with Linked Data, or
- Start this activity from a blank document and create from the beginning by following the steps in Constructing the Basic Document section immediately following this introduction.

Constructing the Basic Document

1. Open a new TI-Nspire™ document.
 - This example will be completed with the TI-Nspire™ Computer Software, not the TI-Nspire™ math and science learning handheld.
2. Click on the blank page, and add a Graphs & Geometry application (Figure 1).
 - This activity will be completed in split screen mode.
3. Using the Page Layout menu, choose a vertical page split  to create two work areas on one page.
4. Click on the new, blank work area, and add a Lists & Spreadsheet application (Figure 2).
5. Click in the Graphs & Geometry work area, click the Window menu , and select the Zoom - Quadrant 1  tool to reset the axes to the first quadrant (Figure 3).

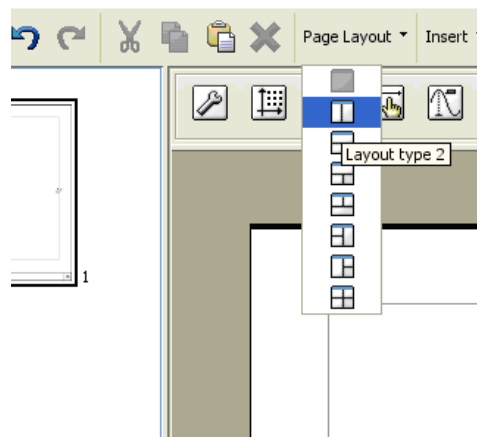


Figure 1

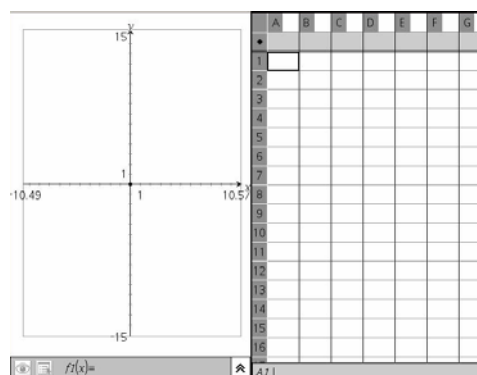


Figure 2

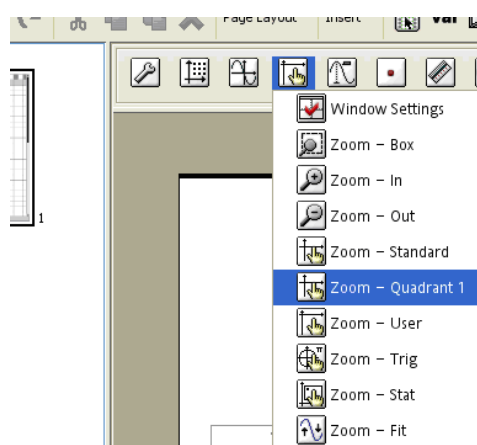






Figure 3

6. Next, click the Points and Lines menu ,
- and select the Point tool .
7. Place and label nine (9) points, A through I, randomly distributed in the first quadrant of the graph (Figure 4).
- The best way to label these points is to “label on the fly” by typing the label immediately after each point is created.
 - If you do not label points as they are created, use the Text tool in the Tools menu to create these labels later.
- Note: Labels are automatically attached to points and vertices so that the label moves when the point is dragged; you cannot drag a label off a point, only reposition it.
8. Pres Esc to exit the Points tool.
9. From the Tools menu , select the Coordinates & Equations tool  to display the coordinates of points A through I (Figure 4).
10. Mouse click on the point to display the coordinates.
11. Then mouse click again to place the coordinates in convenient places so that they are clearly associated with the correct points on the screen.

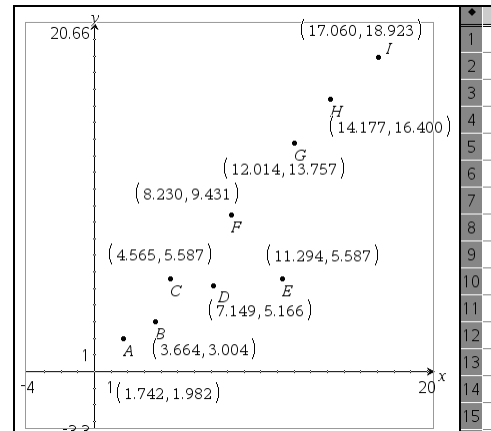


Figure 4

- You may want to readjust the position of these coordinates, the labels, the points themselves, or the two work areas.
- Press Esc to exit the Coordinates tool.
 - Select the x -coordinate of point A by clicking on the number once with the mouse.
 - Then click the Variable button **var** on the main tool bar to open the variable storage menu on the Graphs & Geometry work area (Figure 5).
 - Store this value as the variable **ax** (for the x -coordinate of point A).
 - The x -coordinate for A will become bold to indicate it is a variable.
 - Repeat this same procedure for the y -coordinate of point A storing it as the variable **ay**.
 - Repeat this process for all the other coordinate pairs on the screen, naming them appropriately for each of the individual points B through I.
 - The end result is that there are 18 identifiable stored variables that can be linked to other applications within this problem.

We are going to create two lists in the Lists & Spreadsheet application, one with all the x -coordinates of the points (column A) and one with all the y -coordinates (column B).

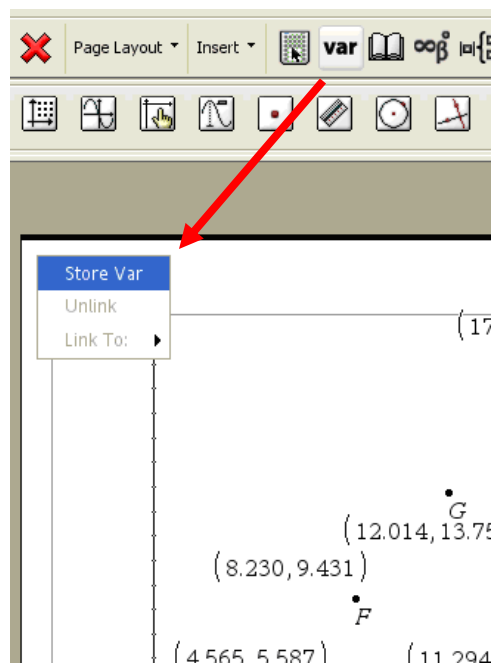


Figure 5

18. Click on the Lists & Spreadsheet work area, and select cell A1 by clicking in the cell.

- You are now ready to Link this cell to a stored variable.

Note: Do not name either of the two list columns when doing this type of data linking.

19. After the A1 cell is selected, click the Variables Menu button **var** on the main tool menu again.

20. From the Link To: list, choose **ax** (Figure 6).

- This places the x -coordinate of point A into cell A1.

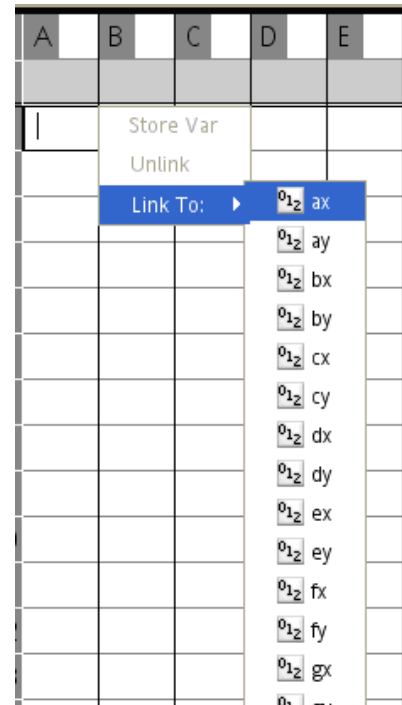


Figure 6

21. Cell B1 should contain the y -coordinate of point A, the variable named **ay** in this example (Figure 7).

22. As you move down to Row 2, the cells A2 and B2 will hold the x - and y -coordinates of point B respectively.

- Each time another coordinate is entered into a cell, select the **cell first** with the mouse and then Link to: the variable so the value will be placed into the correct cell.

23. Repeat this linking of variables into the cells of the spreadsheet until all 9 points with their 18 coordinates are linked (Figure 8).

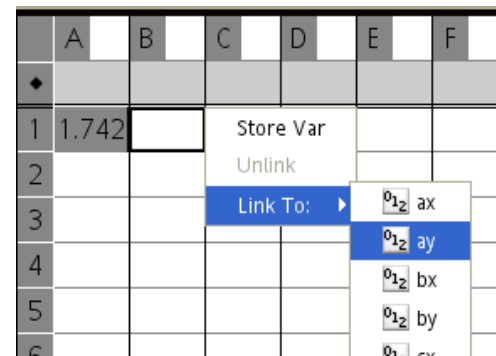


Figure 7

	A	B	C	D
1	1.742	1.982		
2	3.664	3.004		
3	4.025	5.647		
4	7.149	5.166		
5	11.294	5.587		
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				



Figure 8

- The final product of this process will be two lists of coordinates (Figure 9).

24. Click in the Graphs & Geometry work area to activate the mouse cursor.

25. Now drag different points around the graph, one at a time (Figure 10).

- Notice that the coordinates of the points change in both the graph and the spreadsheet.

26. On the Tools menu , use the Hide/Show tool  to hide all of the coordinate pairs showing on the graph (Figure 11).

- This *does not* break the connection to the values showing in the spreadsheet.
- Drag the points again to check this link.

Note: The width of a column in the spreadsheet can be changed by grabbing and moving the column divider bar between the top-most cells that might contain column names.

	A	B
1	1.742	1.982
2	3.664	3.004
3	4.025	5.647
4	7.149	5.166
5	11.294	5.587
6	8.230	9.431
7	12.014	13.757
8	14.177	16.400
9	17.060	18.923
10		

Figure 9

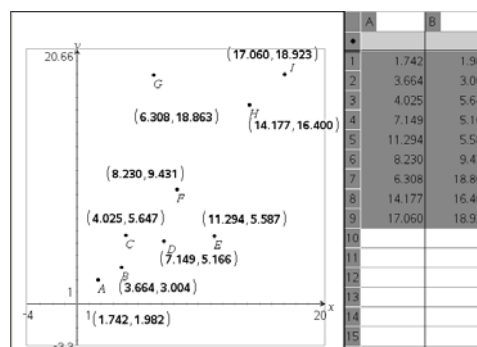


Figure 10

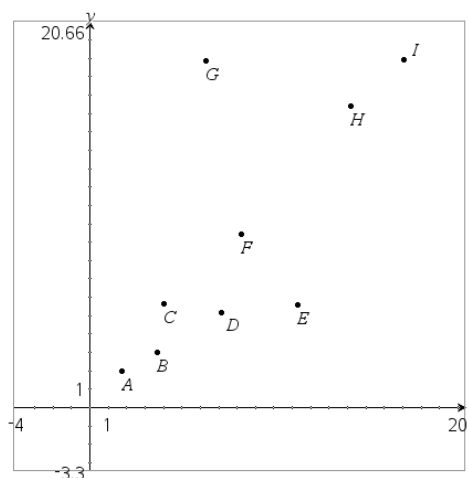


Figure 11

27. Click on the Lists & Spreadsheet work area.
28. Double-click in a cell, such as B3 representing the y -coordinate of point C, and type a new value for this coordinate (Figure 12).
29. Press Enter on your keyboard, and the point C on the graph “jumps” to the new location defined by the entered coordinate (Figure 13 as compared to Figure 11).
- The result of this process is a “live” two-way linking of the coordinates of the points on the graph and in the spreadsheet.

Working with Linked Data

If you have not constructed the file using the previous steps, you can open the TI-Nspire™ .tns document file **D3_8LiveDataSharing.tns** and start here.

Now that the live link is created between the points on the graph screen and their numerical equivalents (coordinates) in the spreadsheet, let’s treat these values as “data” by performing a regression analysis on the two “lists.”

- With your mouse, select both columns of data by clicking on the ‘A’ and dragging across the top until both columns A and B are highlighted (Figure 14).

	A	B	C
1	1.742	1.982	
2	2.343	5.527	
3	5.106	4.3251	
4	5.166	9.251	
5	8.410	7.028	
6	15.439	7.449	
7	5.226	15.198	
8	14.177	16.400	
9	16.219	18.803	
10			

Figure 12

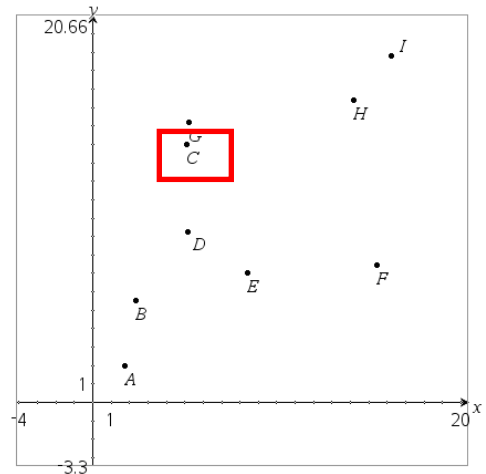



Figure 13

	A	B	C
1	1.742	1.982	
2	2.343	5.527	
3	6.187	6.007	
4	5.166	9.251	
5	8.410	7.028	
6	15.439	7.449	
7	5.226	15.198	
8	14.177	16.400	
9	16.219	18.803	
10			

Figure 14

2. From the Statistics menu , select the Stat Calculations submenu, and choose either 'Linear Regression (mx + b)' or 'Linear Regression (a + bx)' for the first analysis (Figure 15).

- Once the regression model is chosen, a dialogue box appears with the selected lists entered in the order they were selected (Figure 16).
- The results of the regression analysis will be started in the 1st Result Column for data, or "c[]" in this case.

Notes: If you do not select the two columns of values for the regression analysis by dragging before selecting a Linear Regression command, or if the columns are not adjacent to each other in the correct order on the spreadsheet, you have two choices while in the Linear Regression dialog box: 1) enter any column name you want by typing the list or column name manually into the appropriate fields; or 2) select a list name from the drop-down menus to the right of each field (Figure 16).

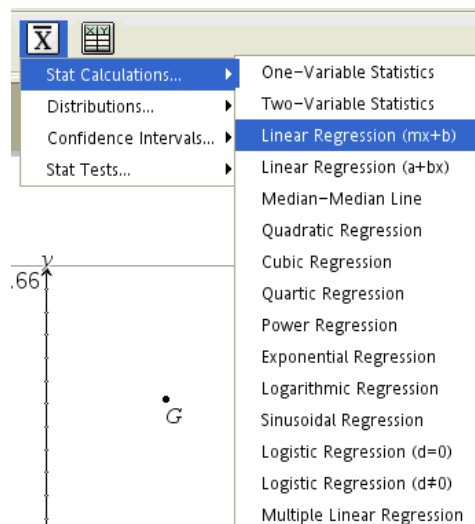


Figure 15

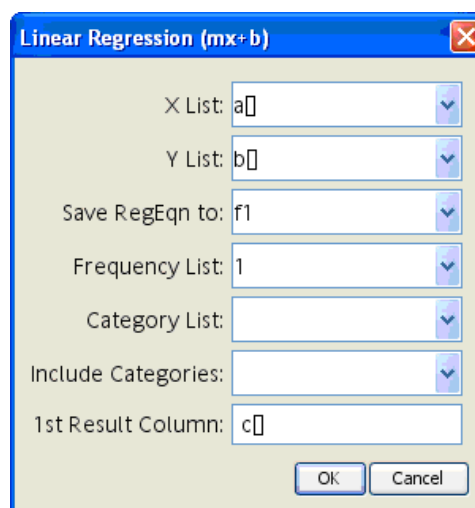


Figure 16

Note: The column names entered into the field for “x-list” and “y-list” are really characters like “a[]” and “b[]” making use of the square bracket symbols with no space between ([]). They appear so close together that you might mistake them for some alternate symbol.

You can also change the function definition where the regression equation will be automatically saved and the column name where you wish to have the results of the analysis begin—or use the defaults (See Figure 16 on the previous page).

3. Click the OK button to perform the selected regression, report the results in the spreadsheet, and store the resulting regression equation in the chosen function location in the Function Menu (Figure 17).
4. Drag any one of the points A through I around in the graph, and observe what happens to the regression model.
 - Figure 18 shows the “data” points dragged into a more linear pattern. The values for r and r^2 both respond accordingly.
 - What happens to the model when one point is dragged to a position very different than the trend of the others? Explore.
5. Click in the Function Entry Line at the very bottom of the Graphs & Geometry work area.

	A	B	C	D
•				=LinRegMx(a[
1	1.742	1.982	Title	Linear Regr...
2	2.343	5.527	RegEqn	m*x+b
3	6.187	6.007	m	.650
4	5.166	9.251	b	4.326
5	8.410	7.028	r ²	.406
6	15.439	7.449	r	.637
7	5.226	15.198	Resid	{-3.4760876...
8	14.177	16.400		
9	16.219	18.803		

Figure 17

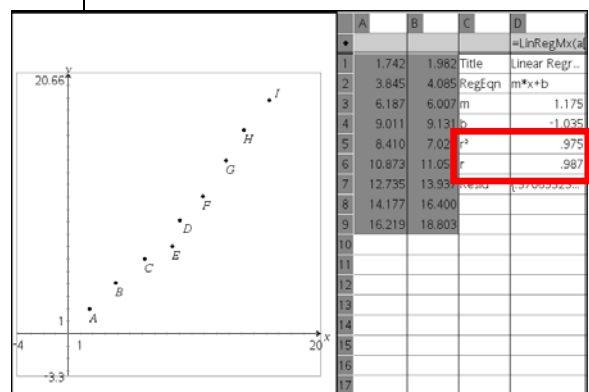


Figure 18

- The equation for the currently active regression analysis will appear in the chosen function definition location ($f1(x)$ in this case) but will not be graphed.
6. Scroll to the stored function definition in the Function Entry Line (up or down), and press Enter on your keyboard to graph this equation (Figure 19).
 7. Drag any of the “data” points A through I, and observe what happens to the graph of the regression model.
 - Explain how the position of the “data” points affects the position of the regression line and the numerical values in the regression model.

The residuals of a regression model represent the “error” terms. In other words, the residuals give insight into how closely the regression model represents the actual data set.

Also, patterns within the residuals give an indication about the appropriateness of the chosen model to adequately represent the data set.

During the regression analysis, the residual values are stored in a list called **stat.resid**. These residuals can be represented graphically as a scatter plot on the same axes with the regression line and the “data.”

8. In the Lists & Spreadsheet work area, scroll to the right to display a blank column.

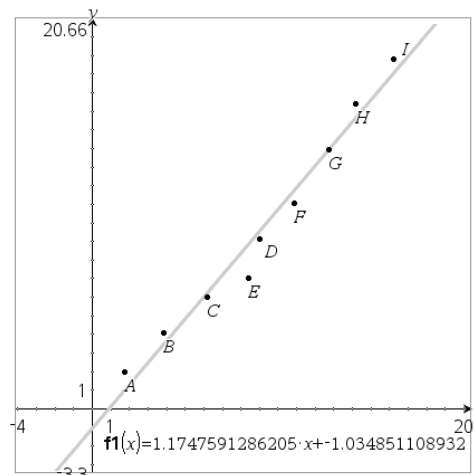



Figure 19



9. Name the empty columns **xcoords** in order to create a copy of the x -coordinates of the “data” points in a named list (Figure 20).



10. In the **xcoords** list rule box (the gray cell below the name of the column), enter the commands $=a[]$ to create the named list with a copy of the live-linked x -coordinates (Figure 21).

- Dragging any of the “data” points will confirm that this new named list is linked to the x -coordinates of the moving points.

11. Use the Page Layout tool on the main tool bar to split the screen into 4 views or work areas  (Figure 22).

12. In the lower left view, add another Graphs & Geometry application.

13. Click the Window menu , and select the Zoom - Quadrant 1  tool to show the first quadrant of the new graph.

14. In the new Graphs & Geometry application, click on Graph Type  and select Scatter Plot .

C	D	xcoords
	=LinRegMx(a	
Title	Linear Regr...	
RegEqn	$m*x+b$	
m	1.175	
b	-1.035	
r^2	.975	
r	.987	
Resid	{.97069323...	

Figure 20

C	D	E xcoords
	=LinRegMx(a	xords:=a[]
Title	Linear Regr...	
RegEqn	$m*x+b$	
m	1.175	
b	-1.035	
r^2	.975	
r	.987	
Resid	{.97069323...	

Figure 21

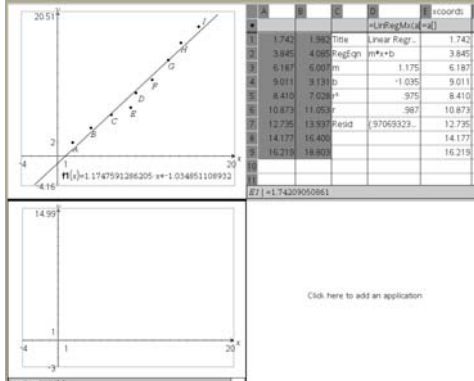


Figure 22

15. In the new work area, choose the two named lists `xcoords` and `stat.resid` for the x and y lists, respectively (Figure 23).

16. Once the scatterplot of the residuals appears, adjust the window to view all data points (Figure 24).

17. Click to the first Graphs & Geometry work area, and drag any of the “data” points A through I around the graph screen to observe the reaction of the residual plot (Figure 24).

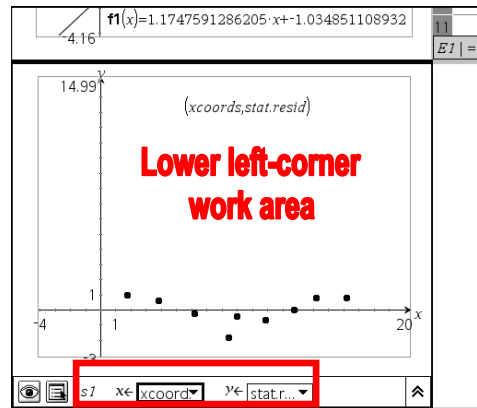


Figure 23

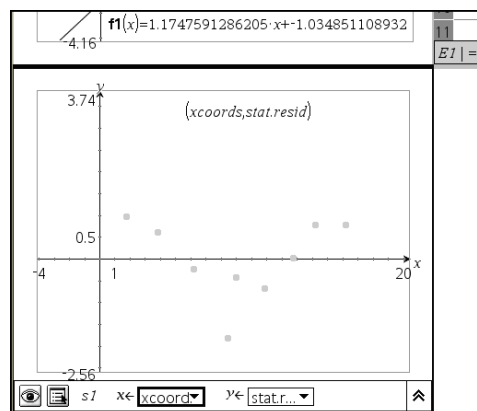


Figure 24

- Figure 25 shows the “data” points dragged so that there appears to be a pattern in the residual plot.
- As one “data” point is dragged, explain the exact connection of this point to the point representing its residual.
- Also, why do all the residual points move when only one “data” point is dragged? Explain.

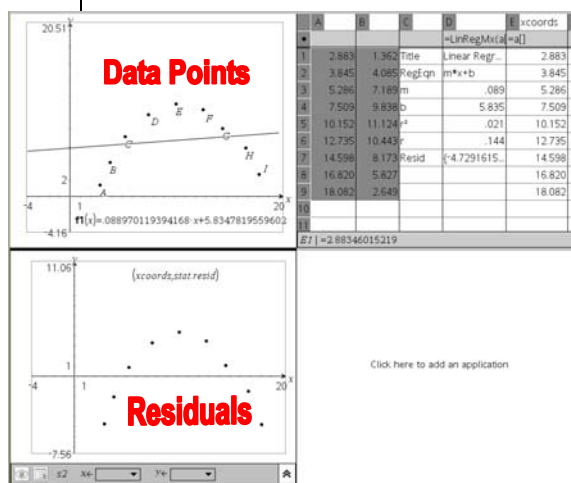


Figure 25

Extension Activity

Additional Regression and Residual Models on the Same Graph

Sometimes it is advantageous to compare two or more regression models of the same data set on the same set of coordinate axes. For example, using a linear, quadratic, and exponential model might give valuable predictive information about the future trends based on conservative, moderate, and liberal interpretations of the data.

While additional regression models are easy to add to the analysis, the automatically saved residual set stored in the generic list **stat.resid** is replaced with new values from the latest regression model executed, every time a new regression is performed.

In order to store or save the current residual list as a dynamic list under a different name, we will compute the residuals mathematically based on the dynamic variables in columns $a[]$ and $b[]$ in the spreadsheet.

The residuals of a data set are sometimes called the “error terms.” This name suggests one way to think about the residuals, as the amount the regression line “missed” the actual data points.

Each datum point is described by a coordinate pair, (x_i, y_i) . For each datum point’s x -coordinate, or each x_i , the computed regression model predicts a corresponding y -coordinate, or what is known as the \hat{y}_i value.

So for each datum point (x_i, y_i) , there is a predicted point (x_i, \hat{y}_i) . Notice that these two points have the same x -coordinate. The difference between the two y -values for each datum point constitutes the set of residuals, or the set of values $y_i - \hat{y}_i$.

Since the regression model is stored in a function definition $f(x)$, the differences we want can be computed using the regression equation as $y_i - f(x_i)$.

1. Create a list by naming a column of the spreadsheet `linresids`, and compute the set of residuals as

$$\text{linresids} := \text{b}[] - \text{f1}(\text{a}[])$$

(Figure 26).

2. Select the lower left Graphs & Geometry work area.
3. Set up Scatter Plot S2 with lists `xcoords` and `linresids` as the x - and y -coordinates, respectively (Figure 27).
 - This scatter plot will exactly match the previous plot of `stat.resid` versus `xcoords` for now.
 - When we add a new regression model, the generic scatter plot of `stat.resid` versus `xcoords` will automatically be a plot of the new model's residuals.
4. To add another regression model, repeat the same steps as before, except the new model will be stored in a different function definition and the results will be placed in different columns in the spreadsheet.

D	E xcoords	F linresids
=LinRegMx(a	=a[]	=b[]-f1(a[])
Linear Regr...	2.523	-4.357
m*x+b	3.845	-2.115
.077	5.286	.878
5.904	7.509	3.356
.018	9.852	3.781
.134	12.735	3.332
{-4.3574971...	14.598	1.145
	16.820	-1.372
	18.082	-4.648

Figure 26

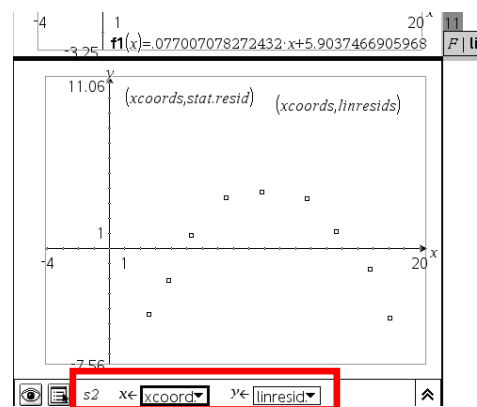



Figure 27

5. Select the data columns, a[] and b[], and choose the model from the Statistics menu , Stat Calculations submenu, Quadratic Regression in this example (Figure 28).

6. Save the new regression equation in function f2 (if appropriate for your situation), and start the report of the regression values in a blank column of the spreadsheet (column g, in this case) (Figure 29).

7. Go to the original Graphs & Geometry work area with the data set, and select function f2(x) to graph the new regression equation over the data and other models (Figure 30).
 - Notice that the new residual plot is already showing on the lower graph.

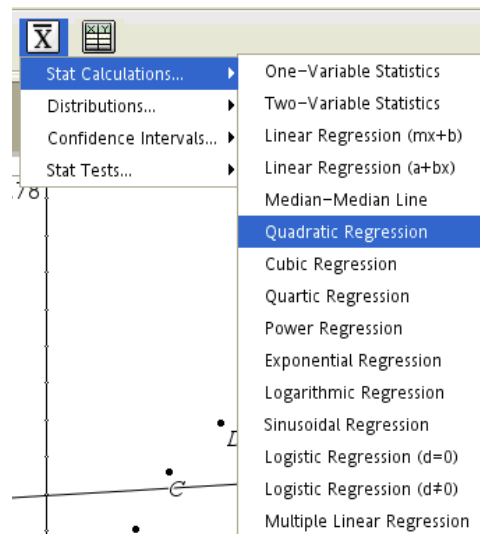


Figure 28

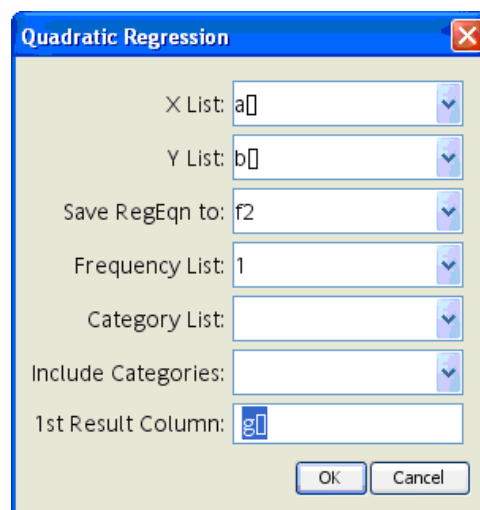


Figure 29

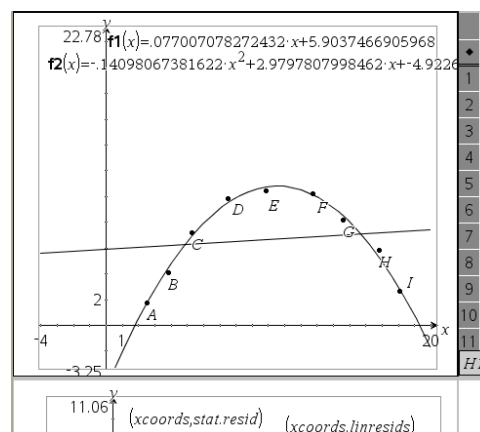


Figure 30



8. Add another Graphs & Geometry application in the lower right work area, and change the window to the first quadrant using the Window menu , and Zoom - Quadrant 1  tool (Figure 31).
9. Select the S1 scatter plot definition, and plot `stat.resid` versus `xcoords` to show only the most recent set of residuals by themselves.
10. Adjust the window as needed to show the residual plot you chose.
11. Drag the original data points around the screen, and watch the various representations change.
12. Compare the linear and quadratic models by dragging data points to new locations.
 - What do the residual plots tell you about the models? Explain.
13. Add more regression models to the data set to compare their reaction as you drag the data points around the screen.

Figure 32 shows quadratic, cubic, and quartic models overlaid on the same data set. Notice how the three models are nearly identical in the inner data range.

- Could these models be used effectively for predictions outside the data range? Explain.

Figure 33 shows how the three polynomial models can all appear nearly linear for a very “linear” data set.

- Explain the value of these models for predicting within or outside the data range.

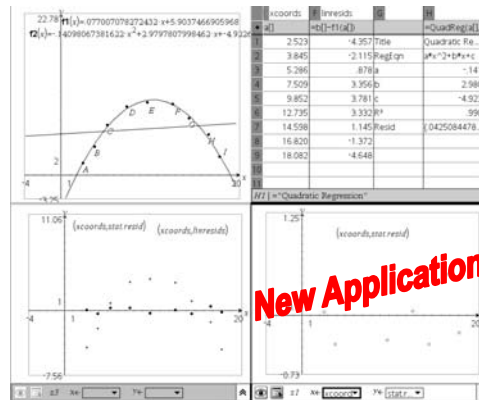


Figure 31

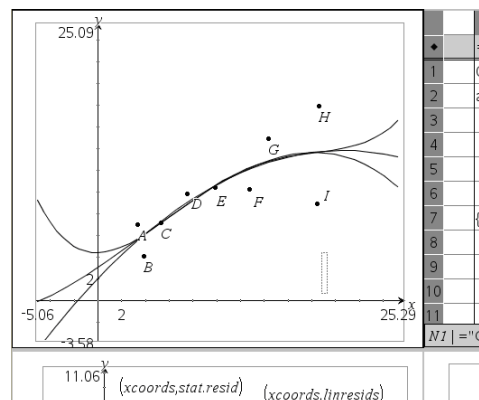


Figure 32

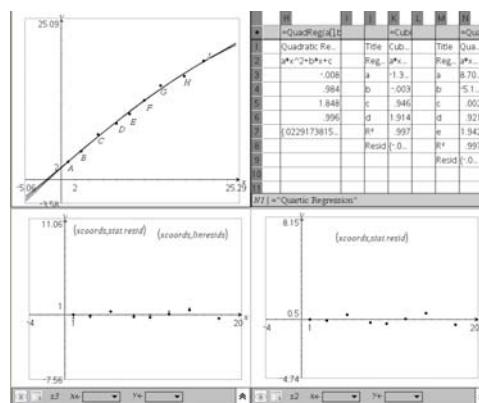


Figure 33